AP Calculus

5.5 Net or Total Change as the Integral of a Rate

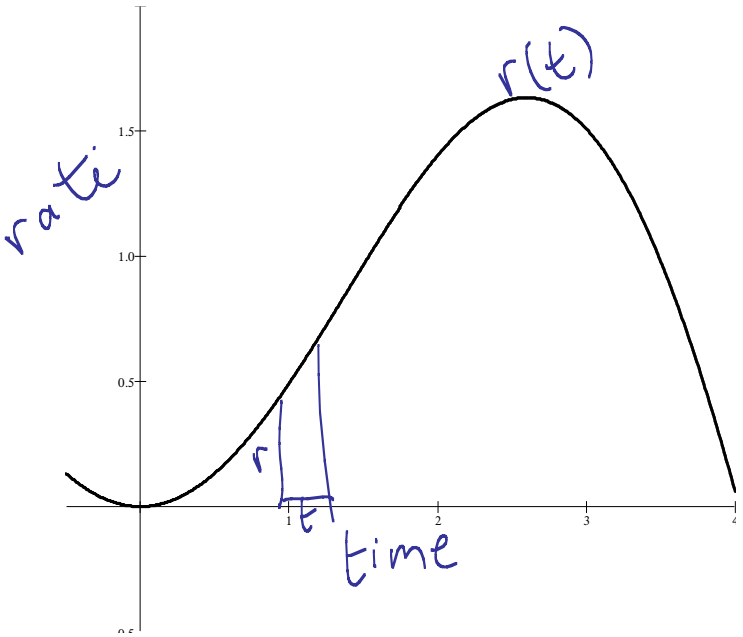
Water flows into an empty bucket at a rate of *r(t)* liters per second. How much water is in the bucket after 4 seconds?

If *r(t)=1.5 liters/second* then

(rate)(time) = Quantity

Quantity = (1.5)(4)
= 6 liters.

However the rate may not be constant. The graph represents *r(t)*.



$r(t) = s'(t)$

$$\int_0^4 s'(t)\,dt = s(4) - s(0)$$

<u>Net Change as the Integral of a Rate</u>

The net change in *s(t)* over an interval $[t_1, t_2]$ is given by

$$\int_{t_1}^{t_2} s'(t)dt = s(t_2) - s(t_1)$$

1. Water leaks out of a tank at a rate of $2 + 3t$ liters/hour, where t is the number of hours after 2pm. How much water has been lost between 4pm and 6pm.

$r(t) = -(2+3t)$

$s'(t) = -2-3t$

$[2,4]$

$\text{Quantity} = \int_2^4 (-2-3t)\,dt$

$= -2t - \frac{3t^2}{2}\Big|_2^4$

$= -2(4) - \frac{3(4)^2}{2} - \left(-2(2) - \frac{3(2)^2}{2}\right)$

$= -8 - 24 + 4 + 6 = -22$ liters

The Integral of Velocity

For an object in linear motion with velocity *v(t)*

$$Displacement\ during\ [t_1, t_2] = \int_{t_1}^{t_2} v(t)dt$$

$$Distance\ travelled\ during\ [t_1, t_2] = \int_{t_1}^{t_2} |v(t)|dt$$

2. Find the displacement over the time interval [1,6] of a helicopter whose vertical velocity at time t is $v(t) = .02t^2 + t\ \ ft/s$

$\int_1^6 (.02t^2 + t)\,dt$

$.02\frac{t^3}{3} + \frac{t^2}{2}\Big|_1^6$

$= \frac{.02}{3}(6)^3 + \frac{36}{2} - \left(\frac{.02}{3} + \frac{1}{2}\right)$

$= 1.44 + 18 - \frac{.02}{3} - \frac{1}{2} = 18.93$

3. A particle is moving along a straight line with velocity $v(t) = \cos t \ \ m/s$.

a) Find the total displacement over the time interval $[0,2\pi]$

$$= \int_0^{2\pi} \cos t \, dt$$

$$= \sin t \Big|_0^{2\pi}$$

$$\sin(2\pi) - \sin(0)$$

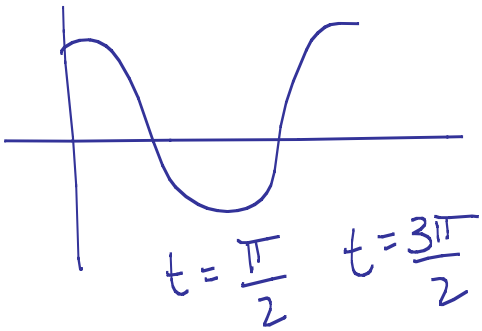
$$0 - 0$$

$$0$$

b) Find the total distance traveled over the time interval $[0,2\pi]$

$v(t) = 0$

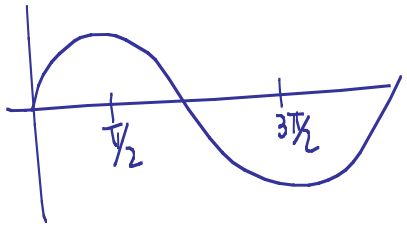
$\cos t = 0$



$$\text{distance} = \int_0^{\pi/2} \cos t\,dt + \left| \int_{\pi/2}^{3\pi/2} \cos t\,dt \right| + \int_{3\pi/2}^{2\pi} \cos t\,dt$$

$$= \sin t \Big|_0^{\pi/2} + \left| \sin t \Big|_{\pi/2}^{3\pi/2} \right| + \sin t \Big|_{3\pi/2}^{2\pi}$$

$$= \sin\frac{\pi}{2} - \sin 0 + \left| \sin\frac{3\pi}{2} - \sin\frac{\pi}{2} \right| + \sin 2\pi - \sin\frac{3\pi}{2}$$



$$= 1 - 0 + |-1 - 1| + 0 - (-1)$$

$$= 1 + |-2| + 1$$

$$= 4$$