5.6 Substitution Method

Note Title 1/19/2015

Chain Rule

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

eg $\frac{d}{dx}(x^3+5)^4 = 4(x^3+5)^3 \cdot 3x^2$

$= 12x^2(x^3+5)^3$

Substitution Method is the reverse of the chain rule

If $u = g(x)$ (inside function)

$\frac{du}{dx} = g'(x)$

$du = g'(x)\,dx$

$$\int f(g(x))\,g'(x)\,dx$$

$$= \int f(u)\,du$$

$$= F(u) + C$$

#1 $\int \underbrace{(x^2-5)}_{\text{inside}}{}^8 \cdot \underbrace{2x\,dx}$ $= \int (u)^8\,du$

$u = x^2 - 5$ Take the derivative

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$= \frac{u^9}{9} + C$

$= \frac{(x^2-5)^9}{9} + C$

#2 $\int 5\cos(5x)\,dx$

$u = 5x$

$\frac{du}{dx} = 5$

$du = 5\,dx$

$= \int \cos(u)\,du$

$= \sin u + C$

$= \sin 5x + C$

#3 $\int \sqrt{\underbrace{2x-1}_{u}}\,dx$

$u = 2x - 1$

$\frac{du}{dx} = 2$

$du = 2dx$

$\frac{du}{2} = dx$

$= \int u^{\frac{1}{2}} \cdot \frac{du}{2}$

$= \frac{1}{2}\int u^{\frac{1}{2}}\,du$

$= \frac{1}{2} \cdot \frac{u^{\frac{3}{2}}}{3/2} + C$

$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3}(2x-1)^{3/2} + C$

$= \frac{(2x-1)^{3/2}}{3} + C$

$\frac{1}{2} + 1$

$\frac{1}{2} + \frac{2}{2}$

$\frac{3}{2}$

#4 $\int \frac{x^2}{\sqrt{\underbrace{1-x^3}}}\,dx$

$u = 1 - x^3$

$\frac{du}{dx} = -3x^2$

$du = -3x^2\,dx$

$\frac{du}{-3} = x^2\,dx$

$\int \frac{1}{\sqrt{u}} \cdot \frac{du}{-3}$

$= -\frac{1}{3}\int u^{-\frac{1}{2}}\,du$

$= -\frac{1}{3} \cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= -\frac{1}{3} \cdot \frac{2}{1}(1-x^3)^{1/2} + C$

$= \frac{-2(1-x^3)^{1/2}}{3} + C$

$-\frac{1}{2} + 1$

$-\frac{1}{2} + \frac{2}{2}$

$\frac{1}{2}$

$$\#5 \quad \int x\sqrt{2x-1}\,dx \qquad = \int \frac{u+1}{2}(u)^{1/2}\cdot\frac{du}{2}$$

$u = 2x-1 \implies u = 2x-1$

$u+1 = 2x$

$\frac{du}{dx} = 2$

$\frac{u+1}{2} = x$

$\frac{du}{2} = dx$

$\frac{du}{2} = dx$

$$= \frac{1}{4}\int (u+1)\boxed{(u)^{1/2}}\,du$$

$$= \frac{1}{4}\int \left(u^{\frac{3}{2}} + u^{\frac{1}{2}}\right) du$$

$$= \frac{1}{4}\left[\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right] + C$$

$$= \frac{3\cdot 1}{3(10)}(2x-1)^{5/2} + \frac{5\cdot 1}{5(6)}(2x-1)^{3/2} + C$$

$$= \frac{3(2x-1)^{5/2} + 5(2x-1)^{3/2}}{30} + C$$

$$= \frac{(2x-1)^{3/2}}{30}\left[3(2x-1)^{1} + 5\right] + C$$

$$= \frac{(2x-1)^{3/2}(6x-3+5)}{30} + C$$

$$= \frac{(2x-1)^{3/2}(6x+2)}{30} + C$$

$$= \frac{(2x-1)^{3/2}(3x+1)}{15} + C$$

$$\#6 \quad \int \sin^2 3x \cos 3x\,dx$$

$$= \int (\sin 3x)^2 \cos 3x\,dx \qquad = \int u^2 \cdot \frac{du}{3}$$

$u = \sin 3x$

$\frac{du}{dx} = (\cos 3x)3$

$$= \frac{1}{3}\int u^2\,du$$

$$\frac{du}{3} = \cos 3x\,dx$$

$$= \frac{1}{3}\frac{u^3}{3} + C$$

$$= \frac{u^3}{9} + C$$

$$= \frac{\sin^3 3x}{9} + C$$

Compare problems

$$\int 2x\cos(x^2)\,dx$$

$u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$ ✓

$$\int \cos x^2\,dx$$

$u = x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x\,dx$

No $x\,dx$ in problem so can't be done with this method.

$$\int \sqrt{1+2x}\,dx$$

$u = 1+2x$

$\frac{du}{dx} = 2$

$\frac{du}{2} = dx$ ✓

$$\int \sqrt{1+2x^2}\,dx$$

$u = 1+2x^2$

$\frac{du}{dx} = 4x$

$\frac{du}{4} = x\,dx$

missing in Question

✗

$$\int \frac{1}{\sqrt{x^2+9}}\,dx$$ ✗

$u = x^2+9$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x\,dx$

$$\int \frac{x}{\sqrt{x^2+9}}\,dx$$ ✓

7 Evaluate

$$\int_0^1 x\,(x^2+1)^3\,dx$$

$u = x^2 + 1$ $\quad x = 0 \qquad x = 1$

$\frac{du}{dx} = 2x \quad u = 0^2 + 1 \qquad u = 1^2 + 1$

$\qquad u = 1 \qquad u = 2$

$\frac{du}{2} = x\,dx$

$$= \int_1^2 u^3 \cdot \frac{du}{2}$$

$$= \frac{1}{2} \int_1^2 u^3\,du$$

$$= \frac{1}{2} \cdot \frac{u^4}{4} \Big|_1^2$$

$$= \frac{u^4}{8} \Big|_1^2$$

$$= \frac{2^4}{8} - \frac{1^4}{8}$$

$$= \frac{16}{8} - \frac{1}{8}$$

$$= \frac{15}{8}$$

fnInt(x(x²+1)^3, x, 0, 1) = 1.875