6.1 Reciprocal, Quotient and Pythagorean Identities

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Trig Identity: A trig equation that is true for all permissible values of the variable on both sides of the equation.

1. Verify that the trig identity is true for $\theta = \frac{\pi}{3}$

$$(\tan\theta - 1)^2 = sec^2\theta - 2\tan\theta$$

Do a check using $\frac{\pi}{3}$

① Sub in $\pi/3$ for the angle

② Evaluate using special Δ's

$$\left(\tan\frac{\pi}{3} - 1\right)^2 = \sec^2\frac{\pi}{3} - 2\tan\pi/3$$

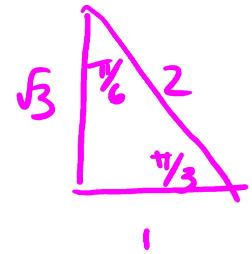
$$\left(\frac{\sqrt{3}}{1} - 1\right)^2 = (2)^2 - 2\left(\frac{\sqrt{3}}{1}\right)$$

$$(\sqrt{3}-1)(\sqrt{3}-1) = 4 - 2\sqrt{3}$$

$$3 - \sqrt{3} - \sqrt{3} + 1 = 4 - 2\sqrt{3}$$

$$4 - 2\sqrt{3} = 4 - 2\sqrt{3}$$

✓



Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

2. Simplify the identity.

$$\sin\theta\cot\theta = \cos\theta$$

Show the left and right to be the same

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\cancel{\sin\theta}\left(\frac{\cos\theta}{\cancel{\sin\theta}}\right) = \cos\theta$$

$$\cos\theta = \cos\theta$$

3. Simplify to a single trig function

$$\frac{\cot\theta}{\csc\theta\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

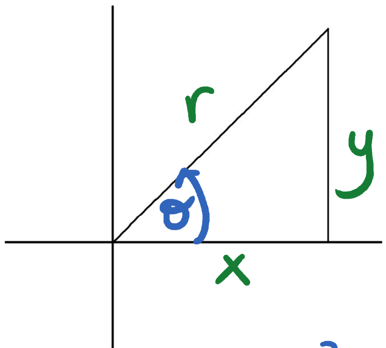
$$\csc\theta = \frac{1}{\sin\theta}$$

$$\frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}\cdot\cos\theta}$$

$$\frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}}$$

$$\frac{\cancel{\cos\theta}}{\cancel{\sin\theta}}\cdot\frac{\cancel{\sin\theta}}{\cancel{\cos\theta}} = 1$$

Pythagorean Identities



$$\sin\theta = \frac{y}{r} \qquad \tan\theta = \frac{y}{x}$$

$$\cos\theta = \frac{x}{r}$$

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2}$$

$$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$$

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$

$$\frac{x^2}{x^2} + \frac{y^2}{x^2} = \frac{r^2}{x^2}$$

$$1 + \left(\frac{y}{x}\right)^2 = \left(\frac{r}{x}\right)^2$$

$$\boxed{1 + \tan^2\theta = \sec^2\theta}$$

$$\frac{x^2}{y^2} + \frac{y^2}{y^2} = \frac{r^2}{y^2}$$

$$\left(\frac{x}{y}\right)^2 + 1 = \left(\frac{r}{y}\right)^2$$

$$\boxed{\cot^2\theta + 1 = \csc^2\theta}$$

4. Simplify to a single trig function

$$\sin\theta\,(\sin^2\theta + \cos^2\theta)\sec\theta$$

$$\sin\theta\,(1)\sec\theta$$

$$\sin\theta\sec\theta$$

$$\sin\theta\left(\frac{1}{\cos\theta}\right)$$

$$\frac{\sin\theta}{\cos\theta}$$

$$\tan\theta$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

5. Simplify to a single trig function

$$\frac{\tan\theta(sin^2\theta + cos^2\theta)}{\sec\theta}$$

$\sin^2\theta + \cos^2\theta = 1$

$\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$$\frac{\tan\theta(1)}{\sec\theta}$$

$$\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$$

$$\frac{\sin\theta}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{1} = \sin\theta$$

6. Simplify to a single trig function

$$(\tan\theta - 1)^2 + 2\sin\theta\sec\theta$$

FOIL

$(\tan\theta - 1)(\tan\theta - 1) + 2\sin\theta\sec\theta$

$\tan^2\theta - \tan\theta - \tan\theta + 1 + 2\sin\theta\sec\theta$

$\tan^2\theta - 2\tan\theta + 1 + 2\sin\theta\sec\theta$

$\sec\theta = \frac{1}{\cos\theta}$

$\tan^2\theta - 2\tan\theta + 1 + 2\sin\theta\left(\frac{1}{\cos\theta}\right)$

$\tan^2\theta - 2\tan\theta + 1 + \frac{2\sin\theta}{\cos\theta}$

$\tan^2\theta - 2\tan\theta + 1 + 2\tan\theta$

$\tan^2\theta + 1$

$\sec^2\theta$