6.1 Area Between Two Curves

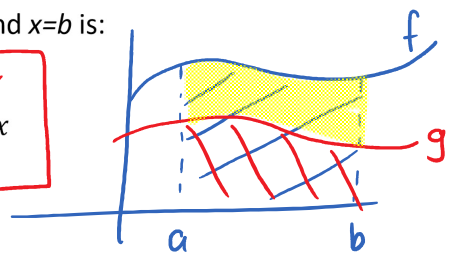
Friday, June 4, 2021 1:21 PM

AP Calculus

6.1 Area of a Region Between Two Curves

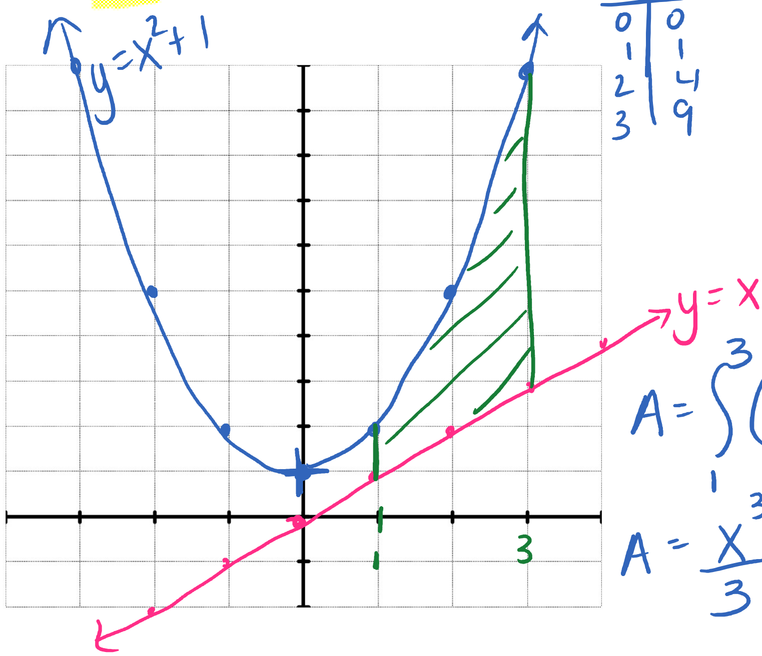
If *f* and *g* are continuous on [a,b] and $g(x) \le f(x)$ for all *x* on [a,b], then the area between the two curves bounded by the lines *x=a* and *x=b* is:

$$\boxed{A = \int_a^b [\underbrace{f(x)}_{\text{upper}} - \underbrace{g(x)}_{\text{lower}}]dx}$$



1. Find the area between the two curves $\boxed{y = x^2 + 1}$ and $\boxed{y = x}$ from $x=1$ to $x=3$.

x	y
0	0
1	1
2	4
3	9



$$A = \int_1^3 (x^2 + 1 - x)\,dx$$

$$A = \frac{x^3}{3} + x - \frac{x^2}{2} \Big|_1^3$$

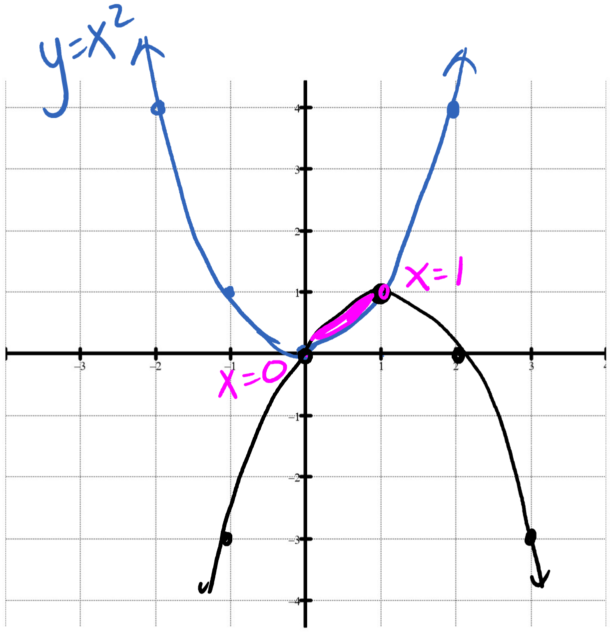
$$A = \frac{3^3}{3} + 3 - \frac{3^2}{2} - \left(\frac{1^3}{3} + 1 - \frac{1^2}{2}\right)$$

$$A = 9 + 3 - \frac{9}{2} - \frac{1}{3} - 1 + \frac{1}{2}$$

$$A = 9 + 3 - 4 - 1 - \frac{1}{3}$$

$$A = 7 - \frac{1}{3} = 20/3$$

2. Find the area of the region bounded by $y = x^2$ and $y = -x^2 + 2x$



$y = -x^2 + 2x$

$y = -(x^2 - 2x + \boxed{1}) - (-1)\boxed{1}$

$\left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$

$y = -(x-1)^2 + 1$

$\rightarrow 1 \quad \uparrow 1$

Intersection points

$x^2 = -x^2 + 2x$

$2x^2 - 2x = 0$

$2x(x-1) = 0$

$x = 0 \quad x = 1$

$$A = \int_0^1 (-x^2 + 2x - x^2)\,dx$$

$$A = \int_0^1 (-2x^2 + 2x)\,dx$$

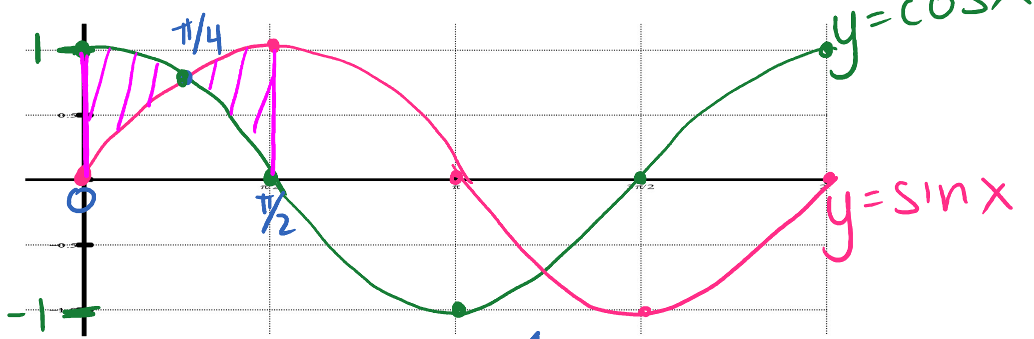
$$A = \frac{-2x^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}} \Bigg|_0^1$$

$$A = \frac{-2}{3}(1)^3 + (1)^2 - (0)$$

$$A = \frac{-2}{3} + 1$$

$$A = 1/3$$

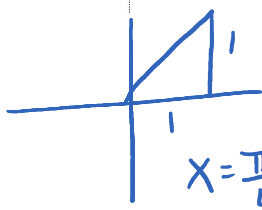
3. Find the area of the region bounded between the curves $\boxed{y = \cos x}$ and $\boxed{y = \sin x}$ from $x = 0$ to $x = \frac{\pi}{2}$



$\sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1 = \frac{1}{1}$



$x = \frac{\pi}{4}$

$A = \int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\,dx$

$A = \sin x + \cos x \Big|_0^{\pi/4} + (-\cos x) - \sin x \Big|_{\pi/4}^{\pi/2}$

$A = \sin\pi/4 + \cos\pi/4 - (\sin 0 + \cos 0) - \cos\pi/2 - \sin\pi/2 - (-\cos\frac{\pi}{4} - \sin\pi/4)$

$A = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 0 - 1 - 0 - 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$A = 4\left(\frac{1}{\sqrt{2}}\right) - 2$

$A = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} - 2$

$A = 2\int_0^{\pi/4} (\cos x - \sin x)\,dx$

$A = \frac{\cancel{4}^{2}\sqrt{2}}{\cancel{2}} - 2$

$A = 2\sqrt{2} - 2$

If your functions are such that there is not an upper and lower function

If your functions are such that there is not an upper and lower function then you may be able to integrate from right to left by rewriting the functions in terms of y

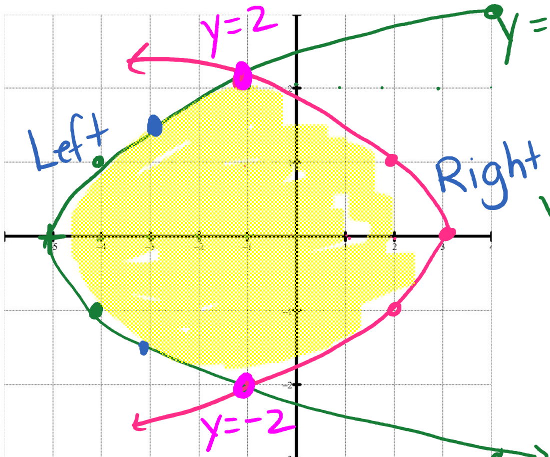
$$A = \int_{c}^{d} [f(y) - g(y)]dy$$

y=d Right left y=c

4. Find the area bounded between $y^2 = x + 5$ and $y^2 = 3 - x$

$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3



$y = \pm\sqrt{x+5}$

Right $y^2 = x+5$

$y^2 - 5 = x$

Left

$y = \pm\sqrt{-x+3}$

$y = \pm\sqrt{-(x-3)}$

$y^2 = 3 - x$

$y^2 - 3 = -x$

$-y^2 + 3 = x$

Right

$$A = \int_{-2}^{2} \left(-y^2 + 3 - (y^2 - 5)\right) dy$$

$$A = \int_{-2}^{2} (-2y^2 + 8) dy$$

$$A = \frac{-2y^3}{3} + 8y \Big|_{-2}^{2}$$

$$A = \frac{-2(2)^3}{3} + 8(2) - \left(-\frac{2}{3}(-2)^3 + 8(-2)\right)$$

$$A = \frac{-16}{3} + 16 - \frac{16}{3} + 16$$

$$A = \frac{-32}{3} + 32$$

$$A = -\frac{32}{3} + \frac{96}{3} = \frac{64}{3}$$

3 3 3