6.2 fixed

Tuesday, May 18, 2021 11:34 AM

Pre-Calculus 12

6.2 Sum Difference and Double Angle Identities

Sum Identities

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ *

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

Difference Identities

$\sin(A-B) = \sin A\cos B - \cos A\sin B$ *

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

1. Simplify to a single trig function

$$\sin\frac{\pi}{8}\cos\frac{3\pi}{5} + \cos\frac{\pi}{8}\sin\frac{3\pi}{5}$$

sin sum

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$A = \frac{\pi}{8}$ $B = \frac{3\pi}{5}$

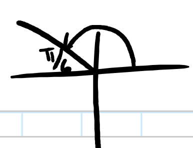
We are given the right side, ∴ create the left side of the identity

$= \sin(A+B)$

$= \sin\left(\frac{\pi}{8} + \frac{3\pi}{5}\right)$

$= \sin\left(\frac{5\pi}{40} + \frac{24\pi}{40}\right)$

$= \sin\left(\frac{29\pi}{40}\right)$



π, π, π, π, π, 5π ~~30° 60° 45° 240° 120° 150° 135°~~

$\frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6}, \frac{\pi}{2}, \pi, \frac{5\pi}{6},$ ~~30°, 60°, 45°, 240°, 120°, 150°, 135°~~

2. Find an exact value for $\cos\frac{\pi}{12}$

$\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ $\qquad \frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$

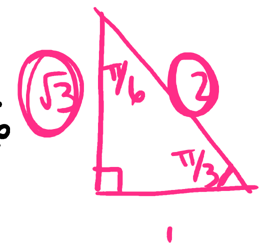
$\frac{\pi}{12} = \frac{4\pi}{12} - \frac{3\pi}{12}$ $\qquad \frac{\pi}{12} = \frac{3\pi}{12} - \frac{2\pi}{12}$

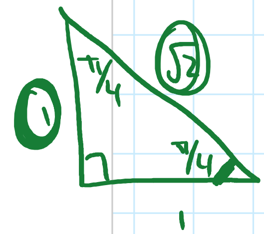
$\cos\frac{\pi}{12}$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

We have the left $\therefore$ create the right side

$$\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$$
$$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$
$$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$
$$= \frac{1+\sqrt{3}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{2(2)} = \frac{\sqrt{2}+\sqrt{6}}{4}$$





3. Simplify and create a double angle identity

$2A = A + A$

$\sin(2A)$

$= \sin(A+A)$ $\qquad 5(3)$, $3(5)$

$= \sin A\cos A + \cos A\sin A$

$= \sin A\cos A + \sin A\cos A$

$= 2\sin A\cos A$

$\sin 2A = 2\sin A\cos A$

$\cos(2A) = \cos(A+A)$

$= \cos A\cos A - \sin A\sin A$

$= \cos^2 A - \sin^2 A$ *

$= \cos^2 A - (1-\cos^2 A)$

$= 2\cos^2 A - 1$

$\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$

$\tan(2A)$

$\tan(A+A)$

$$= \frac{\tan A + \tan A}{1 - \tan A\tan A}$$
$$= \frac{2\tan A}{1-\tan^2 A}$$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = 1 - \sin^2 A - \sin^2 A$

$\cos 2A = 1 - 2\sin^2 A$

$\sin^2 A + \cos^2 A = 1$

$\cos^2 A = 1 - \sin^2 A$

Double Angle Identities

$\sin 2A = 2\sin A\cos A$

$$\cos 2A = \begin{cases} cos^2 A - sin^2 A \\ 2cos^2 A - 1 \\ 1 - 2sin^2 A \end{cases}$$

$$\tan 2A = \frac{2\tan A}{1 - tan^2 A}$$

$$\frac{3(2x+1)}{7}$$

$$3\left(\frac{2x+1}{7}\right)$$

similar to $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

$A = 20^\circ$

4. Write as a single trig function

$$\frac{6\tan 20^\circ}{1 - tan^2 20^\circ}$$

$$= \frac{3(2\tan 20^\circ)}{1-\tan^2 20^\circ}$$

$$= 3\left[\frac{2\tan 20^\circ}{1-\tan^2 20^\circ}\right] = 3\left[\tan 2(20^\circ)\right] = 3\tan 40^\circ$$

5. Evaluate without a calculator

$$10cos^2\left(\frac{\pi}{12}\right) - 5$$

$\cos 2A = 2\cos^2 A - 1$

$A = \frac{\pi}{12}$

$$5\left(2\cos^2\frac{\pi}{12} - 1\right)$$

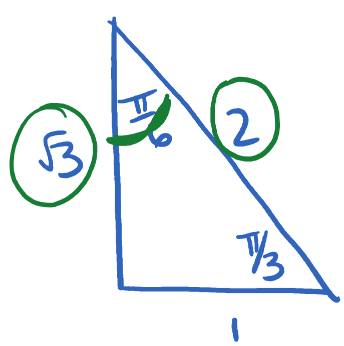
$$5\left(\cos 2\left(\frac{\pi}{12}\right)\right)$$

$$5\cos\frac{\pi}{6}$$

$$\frac{5}{1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\frac{5\sqrt{3}}{2}$$

create the left



6. Evaluate without a calculator

$\sin\frac{\pi}{8}\cos\frac{\pi}{8}$

$\sin 2A = 2\sin A\cos A$

$A = \frac{\pi}{8}$

Create the left side of the identity

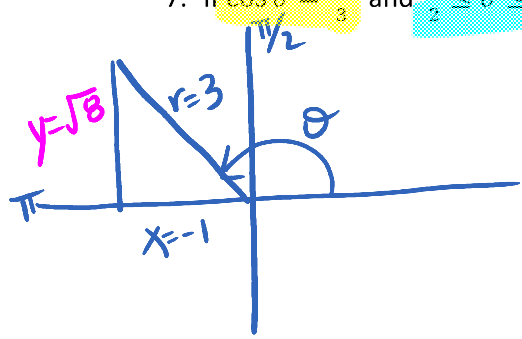
$= \frac{2}{2}\sin\frac{\pi}{8}\cos\frac{\pi}{8}$

$= \frac{1}{2}\left(2\sin\frac{\pi}{8}\cos\frac{\pi}{8}\right)$

$= \frac{1}{2}\left(\sin 2\left(\frac{\pi}{8}\right)\right)$

$= \frac{1}{2}\sin\frac{\pi}{4} = \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$

7. If $\cos\theta = \frac{-1}{3}$ and $\frac{\pi}{2} \le \theta \le \pi$, find a value for:



$\cos\theta = \frac{x}{r}$ $\quad \cos\theta = \frac{-1}{3}$

$x^2 + y^2 = r^2$

$(-1)^2 + y^2 = 3^2$

$1 + y^2 = 9$

$y^2 = 8$

$y = \pm\sqrt{8}$

a) $\sin 2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= \frac{2}{1}\left(\frac{\sqrt{8}}{3}\right)\left(\frac{-1}{3}\right)$

$= \frac{-2\sqrt{8}}{9}$

$= \frac{-2(2\sqrt{2})}{9}$

$= -\frac{4\sqrt{2}}{9}$

$\sin\theta = \frac{y}{r}$

$\sqrt{8}$

$\sqrt{4 \cdot 2}$

$\sqrt{4} \cdot \sqrt{2}$

$2\sqrt{2}$

b) $\cos(\theta + \pi)$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

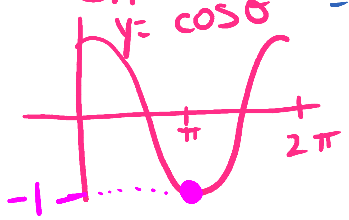
$\cos(\theta + \pi) = \cos\theta\cos\pi - \sin\theta\sin\pi$

$= \left(\frac{-1}{3}\right)(-1) - \left(\frac{\sqrt{8}}{3}\right)(0)$

$= \frac{1}{3} - 0$

$= \frac{1}{3}$

ch 5



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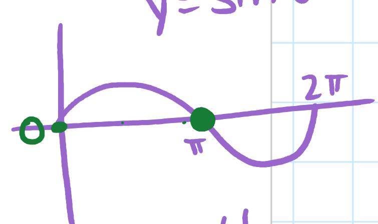

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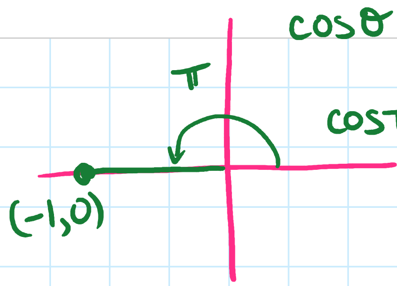
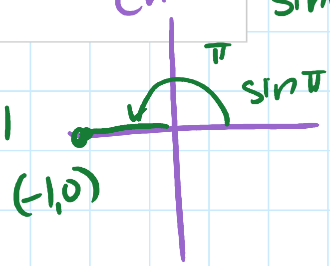


$\cos\theta = \frac{x}{r}$

$\cos\pi = \frac{-1}{1} = -1$

ch 4



$\sin\theta = \frac{y}{r}$

$\sin\pi = \frac{0}{1} = 0$