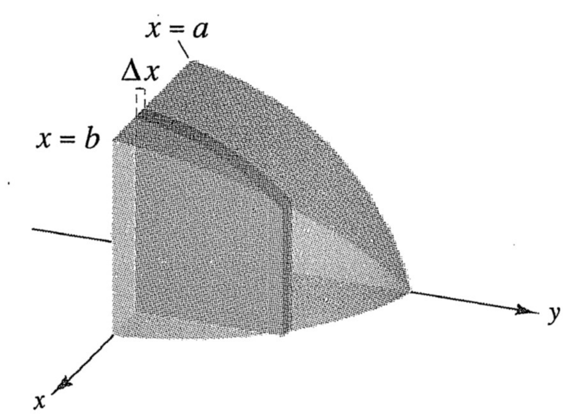
6.2 Setting Up Integrals: Volume, Density, Average Value

Volume as the integral of Cross-Sectional Area:

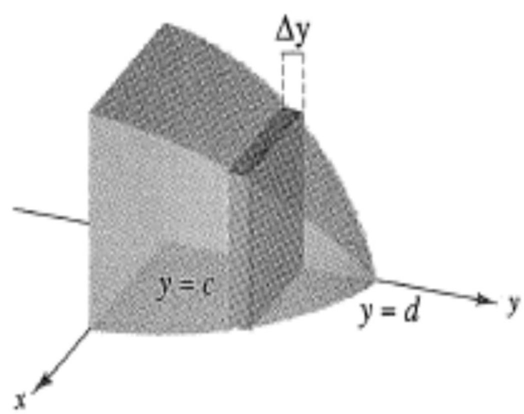
Let *A(x)* be the area of the cross-section that is perpendicular to the x-axis.

$Volume = \int_a^b A(x)dx$

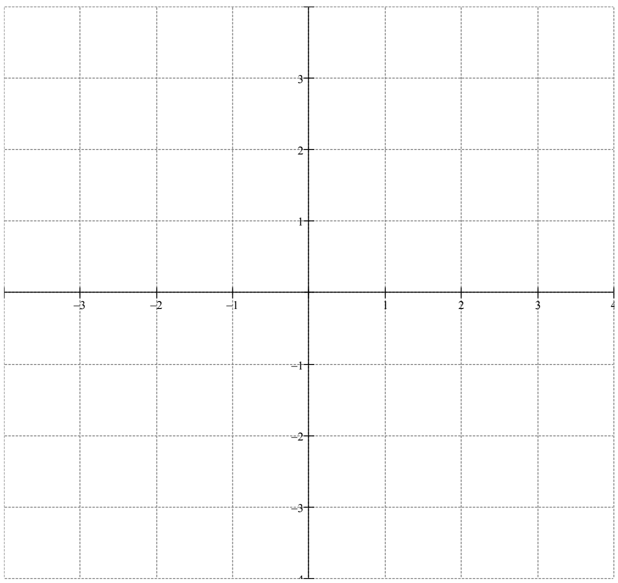


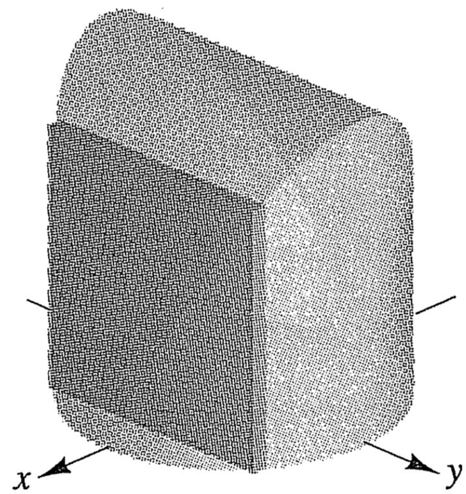
Let *A(x)* be the area of the cross-section that is perpendicular to the y-axis.

$Volume = \int_c^d A(y)dy$



1. Find the volume of the solid whose base is bounded by the circle, $x^2 + y^2 = 4$, with the cross-sections of squares perpendicular to the x-axis.





Radial Density Function: *p(r)* is a function that depends on the distance r from the center of a city. It determines the population density (people per square kilometer or mile)

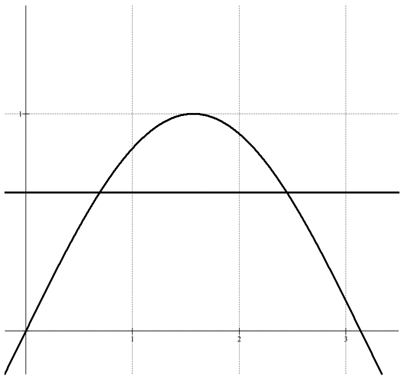
The **actual population *P*** within a radius *R* is given by

$$P = 2\pi \int_0^R rp(r)dr$$

2. A national park in the Republic of Congo has a high density of gorillas. Suppose that the radial density function is $p(r) = 52(1 + r^2)^{-2}$ gorillas per square kilometer, where *r* is the distance from a large grassy clearing with a source of food and water. Calculate the number of gorillas within a 5 km radius of the clearing.

Average Value: The average value of an integrable function f(x) on [a,b] is the quantity

$$Average\ value = \frac{1}{b-a}\int_a^b f(x)dx$$



Mean Value theorem for integrals: If f(x) is continuous on [a,b], then there exists a value c on [a,b] such that

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

3. Find the average value of $f(x) = 3x^2 - 2x$ on the interval [1,4].

4. Let *M* be the average value of $f(x) = 2x^2$ on [0,2]. Find a value *c* such that $f(c) = M$