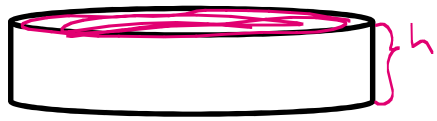
AP Calculus
6.3 Part 1

6.3 Volumes of Revolution Part 1

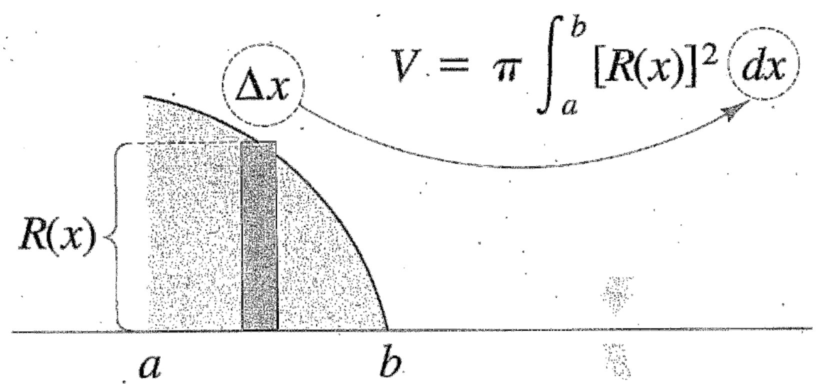
<u>Disc Method:</u>

Volume = area x thickness

$= \pi r^2 h$



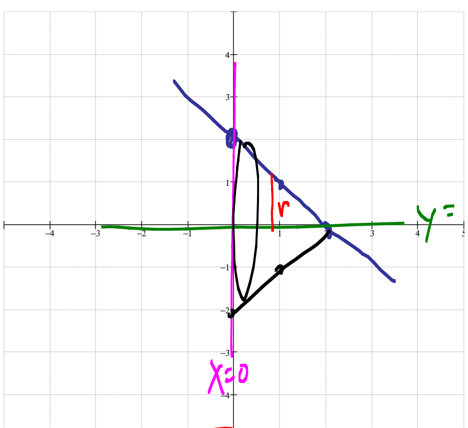
Horizontal Axis of Rotation: If $R(x)$ is continuous and $R(x) \geq 0$ on $[a, b]$ then the solid obtained by rotating the region under the graph about the x-axis has volume:



$$V = \pi \int_a^b [R(x)]^2 \, dx$$

1. Calculate the volume of the solid obtained by rotating the region under $x + y = 2$ bounded by the lines $x = 0$ and $y = 0$ about the x-axis.

$y = -x + 2$



$[0, 2]$ $\quad r = -x + 2$

$$V = \pi \int_0^2 (-x+2)^2 \, dx$$

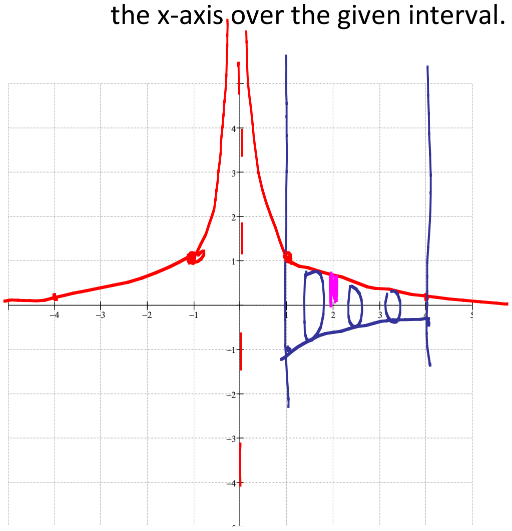
$$V = \pi \int_0^2 (x^2 - 4x + 4) \, dx$$

$$V = \pi \left[\frac{1}{3}x^3 - \frac{4x^2}{2} + 4x \right]_0^2$$

$$V = \pi \left[\frac{8}{3} - 8 + 8 - 0 \right]$$

$$V = \frac{8\pi}{3}$$

2. Find the volume of the solid obtained by rotating the region under the graph of the function about the x-axis over the given interval. $f(x) = \frac{1}{x^2}$ [1,4]



$f(1) = 1$

$f(4) = \frac{1}{16}$

$$V = \pi \int_1^4 \left(\frac{1}{x^2}\right)^2 dx$$

$$V = \pi \int_1^4 \frac{1}{x^4} dx$$

$$V = \pi \left(\frac{x^{-3}}{-3}\right)\Bigg|_1^4$$

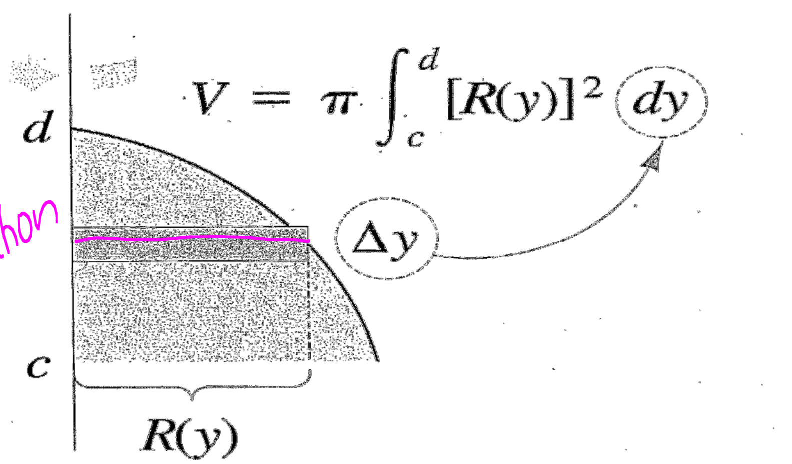
$$V = -\frac{\pi}{3}\left[\frac{1}{4^3} - \frac{1}{1^3}\right]$$

$$V = -\frac{\pi}{3}\left[\frac{1}{64} - \frac{64}{64}\right] = -\frac{\pi}{3}\left(-\frac{63}{64}\right)$$

$$= \frac{21\pi}{64}$$

Vertical Axis of Rotation: If $R(x)$ is continuous and $R(x) \geq 0$ on $[c, d]$ then the solid obtained by rotating the region under the graph about the y-axis has volume:

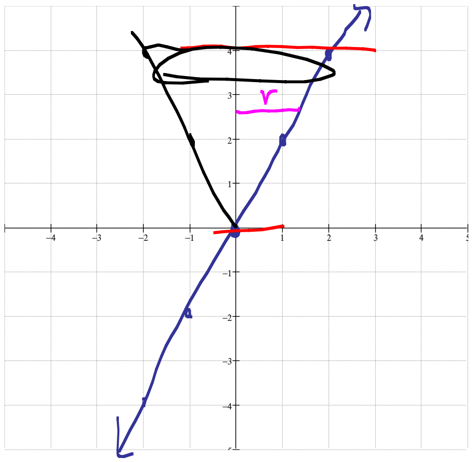
radius is $\perp$ to the axis of rotation



AP Calculus

$y = 2x$

3. Find the volume of a solid generated when the region under $f(x) = 2x$ is rotated about the y-axis from x=0 to x=2.



$x = 0$ $\quad$ $x = 2$

$y = 0$ $\quad$ $y = 4$

$y = 2x$

$\frac{y}{2} = x$

radius $= \frac{y}{2}$

$$V = \pi \int_0^4 \left(\frac{y}{2}\right)^2 dy$$

$$V = \frac{\pi}{4} \int_0^4 y^2\, dy$$

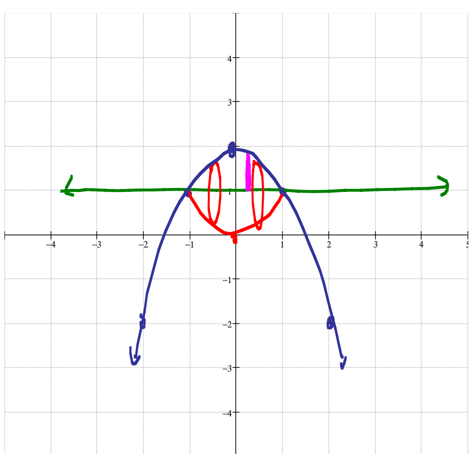
$$V = \frac{\pi}{4}\left(\frac{y^3}{3}\right)\Big|_0^4$$

$$V = \frac{\pi}{12}\left(4^3 - 0\right)$$

$$V = \frac{\pi}{12}(64)$$

$$V = \frac{16\pi}{3}$$

4. Find the volume of the solid formed by revolving the region bounded by $f(x) = 2 - x^2$ and $g(x) = 1$ about the line $y = 1$.



radius $= 2 - x^2 - 1$

$r = 1 - x^2$

$1 = 2 - x^2$

$-1 = -x^2$

$x = \pm 1$ $\quad$ $[-1, 1]$

$$V = \pi \int_{-1}^{1} (1 - x^2)^2\, dx$$

$$V = \pi \int_{-1}^{1} \left(1 - 2x^2 + x^4\right) dx$$

$$V = \pi \left[x - \frac{2x^3}{3} + \frac{1}{5}x^5\right]_{-1}^{1}$$

$$V = \pi\left[1 - \frac{2}{3} + \frac{1}{5} - \left(-1 + \frac{2}{3} - \frac{1}{5}\right)\right]$$

$$V = \pi\left[\frac{15}{15} - \frac{10}{15} + \frac{3}{15} + \frac{15}{15} - \frac{10}{15} + \frac{3}{15}\right]$$

$$V = 16\pi/15$$