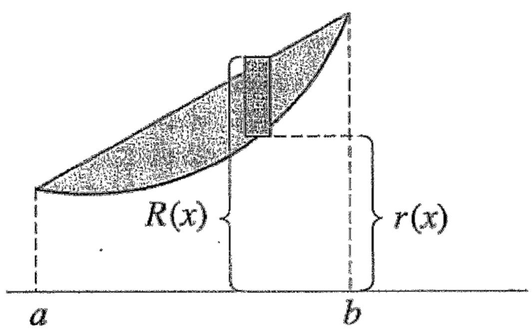
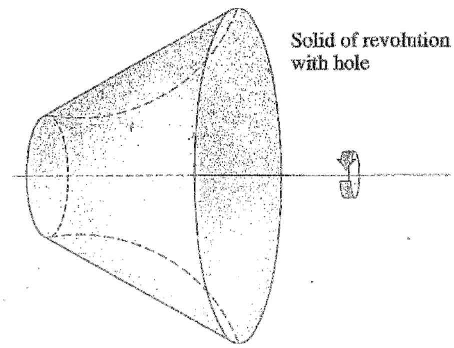
AP Calculus
6.3 Part 2

6.3 Volumes of Revolution Part 2

Washer Method:

The washer method is an extension of the disc method. Use the washer method when your 2-dimensional graph is not bounded by the axis of rotation, resulting in a 3-dimensional object with a (hole).



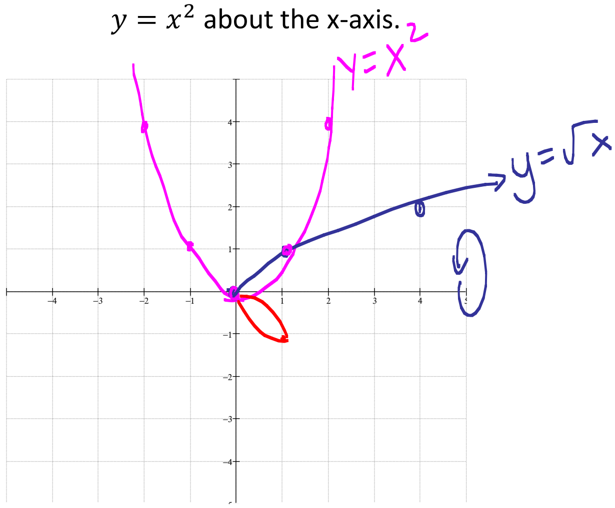


If a region is revolved about a horizontal axis where $R(x)$ is the outer radius and $r(x)$ is the inner radius then,

$$V = \pi \int_a^b [R(x)]^2 dx - \pi \int_a^b [r(x)]^2 dx$$

$$V = \pi \int_a^b \left([R(x)]^2 - [r(x)]^2 \right) dx$$

1. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x}$ and $y = x^2$ about the x-axis.



$R(x) = \sqrt{x} \quad r(x) = x^2$

$V = \pi \int_0^1 \left((\sqrt{x})^2 - (x^2)^2 \right) dx$

$V = \pi \int_0^1 (x - x^4) dx$

$V = \pi \left[\frac{1}{2}x^2 - \frac{1}{5}x^5 \right]_0^1$

$V = \pi \left[\frac{1}{2} - \frac{1}{5} - 0 \right]$

$V = \pi \left[\frac{5}{10} - \frac{2}{10} \right] = \frac{3\pi}{10}$

$\sqrt{x} = x^2$

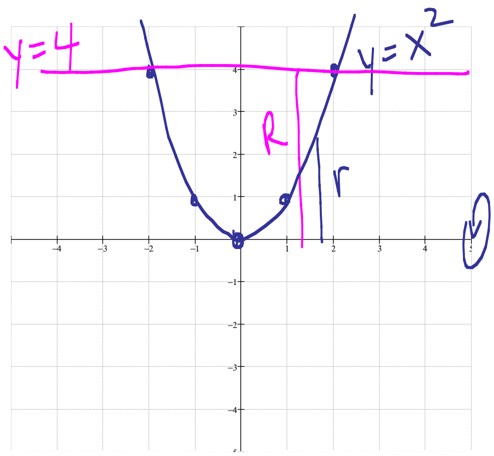
$x = x^4$

$0 = x^4 - x$

$0 = x(x^3 - 1)$

$x = 0 \quad x = 1 \qquad a = 0 \quad b = 1$

2. Find the volume of the solid formed by revolving the region bounded by the graph of $y = x^2$ and the line $y = 4$ about the x-axis.



$R(x) = 4 \qquad r(x) = x^2$

$$V = \pi \int_{-2}^{2} \left(4^2 - (x^2)^2\right) dx$$

$$V = \pi \int_{-2}^{2} (16 - x^4)\, dx$$

$$V = \pi \left[16x - \frac{1}{5}x^5\right]_{-2}^{2}$$

$x^2 = 4$

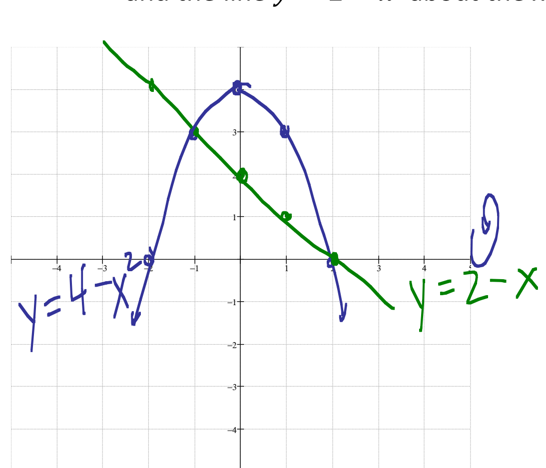
$x = \pm 2$

$a = -2 \quad b = 2$

$$V = \pi \left[32 - \frac{32}{5} - \left(-32 + \frac{32}{5}\right)\right]$$

$$V = \pi \left[32 - \frac{32}{5} + 32 - \frac{32}{5}\right] = \pi \left[\frac{320}{5} - \frac{64}{5}\right] = \frac{256\pi}{5}$$

3. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = 4 - x^2$ and the line $y = 2 - x$ about the x-axis.



$R(x) = 4 - x^2 \qquad r(x) = 2 - x$

$$V = \pi \int_{-1}^{2} \left((4-x^2)^2 - (2-x)^2\right) dx$$

$$V = \pi \int_{-1}^{2} (16 - 8x^2 + x^4 - 4 + 4x - x^2)\, dx$$

$$V = \pi \int_{-1}^{2} (x^4 - 9x^2 + 4x + 12)\, dx$$

$$V = \pi \left[\frac{1}{5}x^5 - 3x^3 + 2x^2 + 12x\right]_{-1}^{2}$$

$4 - x^2 = 2 - x$

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$x = 2 \quad x = -1$

$$V = \pi \left[\frac{32}{5} - 24 + 8 + 24 - \left(-\frac{1}{5} + 3 + 2 - 12\right)\right]$$

$$V = \pi \left[32/5 + 8 + \frac{1}{5} - 3 - 2 + 12\right] = \frac{108\pi}{5}$$

If we revolve a region about a vertical axis the volume formula becomes:

$$V = \pi \int_c^d \left([R(y)]^2 - [r(y)]^2 \right) dy$$

4. Find the volume of the solid formed by revolving the region bounded by the graph of $y = x^3$ and the lines $x = 2$ and $y = 0$ about the y-axis.

$x = 0 \quad y = 0^3 = 0$

$x = 1 \quad y = 1^3 = 1$

$x = 2 \quad y = 2^3 = 8$

$R(y) = 2$

$y = x^3$

$y^{\frac{1}{3}} = x$

$r(y) = y^{\frac{1}{3}}$

$$V = \pi \int_0^8 \left(2^2 - \left(y^{\frac{1}{3}}\right)^2 \right) dy$$

$$V = \pi \int_0^8 \left(4 - y^{\frac{2}{3}} \right) dy$$

$$V = \pi \left[4y - \frac{3y^{5/3}}{5} \right]_0^8$$

$$V = \pi \left[32 - \frac{3}{5}(32) - 0 \right]$$

$$V = \pi \left[\frac{160}{5} - \frac{96}{5} \right]$$

$$V = \frac{64\pi}{5}$$

$32 \times 5 = 160$