6.3 Part 2 New

Tuesday, April 18, 2023 2:21 PM

6.3 Part 2 Proving Identities

Recall Double angle identities

$\sin 2A = 2\sin A\cos A$

$$\cos 2A = \begin{cases} cos^2A - sin^2A \\ 2cos^2A - 1 \\ 1 - 2sin^2A \end{cases}$$

$$\tan 2A = \frac{2\tan A}{1 - tan^2A}$$

1. Expand using the double angle identities

a) $\sin 10\theta = \sin(2\cdot 5\theta)$
$= 2\sin 5\theta \cos 5\theta$

b) $\sin 6x = \sin(2\cdot 3x)$
$= 2\sin 3x \cos 3x$

c) $\cos 8\theta = \cos(2\cdot 4\theta)$
$= \cos^2 4\theta - \sin^2 4\theta$
or
$= 2\cos^2 4\theta - 1$
or
$= 1 - 2\sin^2 4\theta$

d) $\tan 4\theta = \tan(2\cdot 2\theta)$
$= \dfrac{2\tan 2\theta}{1-\tan^2 2\theta}$

2. Prove the Identity $\tan 2\theta = \frac{\sin 4\theta}{1+\cos 4\theta}$

$L.S = \dfrac{\sin(2\cdot 2\theta)}{1+\cos(2\cdot 2\theta)}$

$L.S = \dfrac{2\sin 2\theta\cos 2\theta}{1 + 2\cos^2 2\theta - 1}$

$L.S = \dfrac{\cancel{2}\sin 2\theta\cancel{\cos 2\theta}}{\cancel{2}\cos^{\cancel{2}} 2\theta}$

$LS = \dfrac{\sin 2\theta}{\cos 2\theta}$

$\tan 2\theta = \tan 2\theta$

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$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

3. Prove the Identity $\cos 2\theta = \frac{csc^2\theta - 2}{csc^2\theta}$

$$LS = \frac{\csc^2\theta}{\csc^2\theta} - \frac{2}{\csc^2\theta}$$

$\csc\theta = \frac{1}{\sin\theta}$

$\sin\theta = \frac{1}{\csc\theta}$

$\cos 2\theta = 1 - 2\sin^2\theta$

$$LS = 1 - 2 \cdot \frac{1}{\csc^2\theta}$$

$$LS = 1 - 2 \cdot \sin^2\theta$$

$$\cos 2\theta = \cos 2\theta$$

conjugate

4 Simplify

a) $\frac{18}{7} \div \frac{4}{21}$

$$\frac{\overset{9}{\cancel{18}}}{\cancel{7}} \cdot \frac{\overset{3}{\cancel{21}}}{\underset{2}{\cancel{4}}} = \frac{27}{2}$$

b) $\frac{\sqrt{3}}{5-\sqrt{2}} \cdot \frac{5+\sqrt{2}}{5+\sqrt{2}}$

$$\frac{5\sqrt{3} + \sqrt{6}}{25 + 5\sqrt{2} - 5\sqrt{2} - \sqrt{4}}$$

$$\frac{5\sqrt{3} + \sqrt{6}}{25 - 2}$$

$$\frac{5\sqrt{3} + \sqrt{6}}{23}$$

conjugate
$1 + \sin\theta$

5. Prove the identity $\dfrac{\cos\theta}{1-\sin\theta} = \dfrac{1+\sin\theta}{\cos\theta}$

$$\frac{(1+\sin\theta)\cos\theta}{(1+\sin\theta)(1-\sin\theta)} = R.S.$$

$$\frac{(1+\sin\theta)\cos\theta}{1-\sin\theta+\sin\theta-\sin^2\theta} = R.S.$$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$$\frac{(1+\sin\theta)\cos\theta}{1-\sin^2\theta} = R.S$$

$$\frac{(1+\sin\theta)\cancel{\cos\theta}}{\cos^{\cancel{2}}\theta} = R.S$$

$$\frac{1+\sin\theta}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta}$$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

6. Prove the identity $\frac{\sin\theta+\tan\theta}{1+\cos\theta} = \tan\theta$

$$\frac{\frac{\sin\theta}{1} + \frac{\sin\theta}{\cos\theta}}{1+\cos\theta} = R.S.$$

$$\frac{\frac{\cos\theta}{\cos\theta}\cdot\frac{\sin\theta}{1} + \frac{\sin\theta}{\cos\theta}}{1+\cos\theta} = RS$$

$$\frac{\frac{\cos\theta\sin\theta + \sin\theta}{\cos\theta}}{1+\cos\theta} = RS$$

$$\frac{\frac{\sin\theta(\cos\theta+1)}{\cos\theta}}{\frac{1+\cos\theta}{1}} = RS$$

$$\frac{\sin\theta(\cancel{\cos\theta+1})}{\cos\theta}\cdot\frac{1}{\cancel{1+\cos\theta}} = R.S$$

$$\frac{\sin\theta}{\cos\theta} = R.S$$

$$\tan\theta = \tan\theta$$