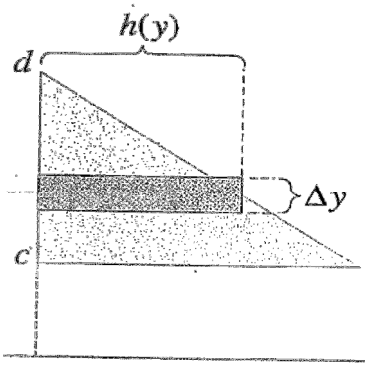
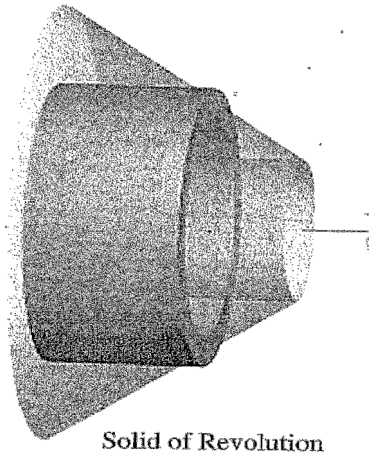
AP Calculus
6.4

6.4 Volumes of Revolution

Shell Method:

The shell method is an alternative method for finding the volume of a solid of revolution. It uses cylindrical shells instead of washers or discs.




Solid of Revolution

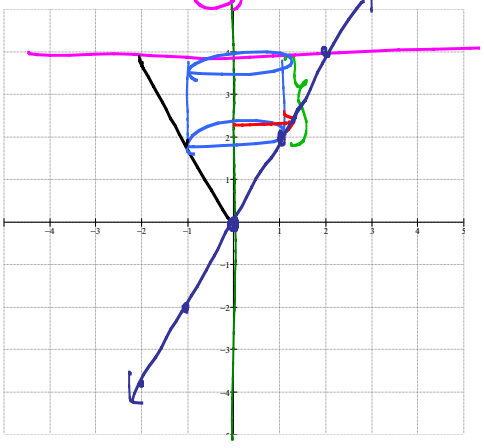
Volume of shell = (Area of rectangle)(Thickness of shell)

$= (2\pi r \cdot h)\Delta r$

Volume of revolution using the shell method: Axis of rotation (y-axis)

$$V = 2\pi \int_a^b (radius)(height\ of\ shell)dx = 2\pi \int_a^b x \cdot f(x)\,dx$$

1. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = 2x$ and the lines $x = 0$ *and* $y = 4$ about the y-axis.



$y = 2x$

$r = x$

$h = 4 - 2x$

$$V = 2\pi \int_0^2 x(4-2x)\,dx$$

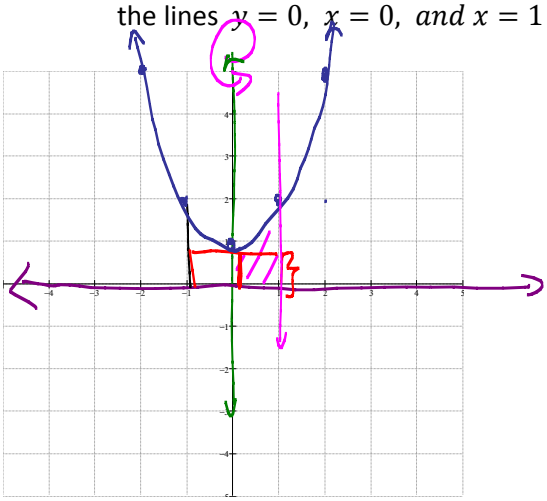
$$V = 2\pi \int_0^2 (4x - 2x^2)\,dx$$

$$V = 2\pi \left[2x^2 - \frac{2}{3}x^3\right]_0^2$$

$$V = 2\pi \left[8 - \frac{16}{3} - 0\right]$$

$$V = 2\pi \left[\frac{24}{3} - \frac{16}{3}\right] = \frac{16\pi}{3}$$

2. Find the volume of the solid formed by revolving the region bounded by the graph of $y = x^2 + 1$ and the lines $y = 0,\ x = 0,\ and\ x = 1$ about the y-axis.



Disc for $0 \le y < 1$ then washer for $1 \le y \le 2$

Use Shells instead.

$r = x$

$h = x^2 + 1$

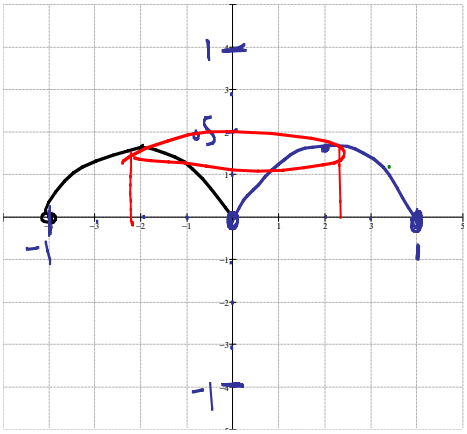
$$V = 2\pi \int_0^1 x(x^2+1)\,dx$$

$$V = 2\pi \int_0^1 (x^3 + x)\,dx$$

$$V = 2\pi \left[\frac{x^4}{4} + \frac{x^2}{2}\right]_0^1$$

$$V = 2\pi \left[\frac{1}{4} + \frac{1}{2} - 0\right] = 2\pi\left[\frac{3}{4}\right] = \frac{3\pi}{2}$$

3. Find the volume of the solid formed by revolving the region bounded by the graph of $y = x - x^3$ bounded by the x-axis $0 \le x \le 1$ about the y-axis.



$y = x(1 - x^2)$

$0 = x(1 - x^2)$

$x = 0 \quad x = 1 \quad x = -1$

$x = .5$

$y = .5 - (.5)^3$

$y = \frac{1}{2} - \frac{1}{8}$

$y = .375$

$r = x$

$h = x - x^3$

$$V = 2\pi \int_0^1 x(x - x^3)\,dx$$

$$V = 2\pi \int_0^1 (x^2 - x^4)\,dx$$

$$V = 2\pi \left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1$$

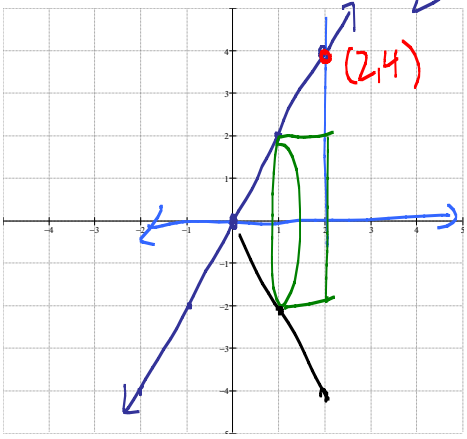
$$V = 2\pi \left[\frac{1}{3} - \frac{1}{5} - 0\right]$$

$$V = 2\pi \left[\frac{5}{15} - \frac{3}{15}\right] = \frac{4\pi}{15}$$

Volume of revolution using the shell method: Axis of rotation (x-axis)

$V = 2\pi \int_c^d (radius)(height\ of\ shell)dy =$ $2\pi \int_c^d (y \cdot f(y))dy$

4. Find the volume of the solid formed by revolving the region bounded by the graph of $y = 2x$ and the lines $x = 2$ and $y = 0$ about the x-axis.



$y = 2x$

$\frac{y}{2} = x$

$r = y$

$h = 2 - \frac{y}{2}$

$V = 2\pi \int_0^4 y\left(2 - \frac{y}{2}\right)dy$

$V = 2\pi \int_0^4 \left(2y - \frac{y^2}{2}\right)dy$

$V = 2\pi \left[y^2 - \frac{y^3}{6}\right]_0^4$

$V = 2\pi \left[16 - \frac{64}{6} - 0\right]$

$V = 2\pi \left[\frac{48}{3} - \frac{32}{3}\right]$

$V = 2\pi \left[\frac{16}{3}\right] = \frac{32\pi}{3}$