6.1 Reciprocal, Quotient and Pythagorean Identities

Wednesday, May 12, 2021 8:51 AM

Pre-Calculus 12

6.1 Reciprocal, Quotient and Pythagorean Identities

Trig Identity: A trig equation that is true for all permissible values of the variable on both sides of the equation.

1. Verify that the trig identity is true for $\theta = \frac{\pi}{3}$

$$(\tan\theta - 1)^2 = sec^2\theta - 2\tan\theta$$

Reciprocal Identities

Quotient Identities

2. Simplify the identity.

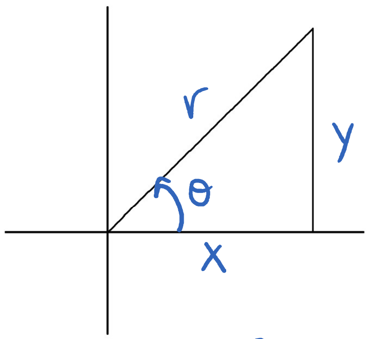
$$\sin\theta\cot\theta = \cos\theta$$

3. Simplify to a single trig function

$$\frac{\cot\theta}{\csc\theta\cos\theta}$$

Pythagorean Identities

$\sin\theta = \frac{y}{r}$ $\quad \cos\theta = \frac{x}{r}$ $\quad \tan\theta = \frac{y}{x}$



$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2}$$
$$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$$
$$\cos^2\theta + \sin^2\theta = 1$$

$$\frac{x^2}{x^2} + \frac{y^2}{x^2} = \frac{r^2}{x^2}$$
$$1 + \left(\frac{y}{x}\right)^2 = \left(\frac{r}{x}\right)^2$$
$$1 + \tan^2\theta = \sec^2\theta$$

$$\frac{x^2}{y^2} + \frac{y^2}{y^2} = \frac{r^2}{y^2}$$
$$\left(\frac{x}{y}\right)^2 + 1 = \left(\frac{r}{y}\right)^2$$
$$\cot^2\theta + 1 = \csc^2\theta$$

4. Simplify to a single trig function

$\sin\theta\,(sin^2\theta + cos^2\theta)\sec\theta$

$\sin\theta(1)\sec\theta$

$\sin\theta\sec\theta$

$\sin\theta\left(\frac{1}{\cos\theta}\right)$

$\frac{\sin\theta}{\cos\theta}$

$\tan\theta$

think

$\sin^2\theta + \cos^2\theta = 1$

$\sec\theta = \frac{1}{\cos\theta}$

5. Simplify to a single trig function

5. Simplify to a single trig function

$$\frac{\tan\theta(sin^2\theta + cos^2\theta)}{\sec\theta}$$

$\sin^2\theta + \cos^2\theta = 1$

$\frac{\tan\theta(1)}{\sec\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\frac{\tan\theta}{\sec\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$

$\frac{\sin\theta}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{1} = \sin\theta$

6. Simplify to a single trig function

$$(\tan\theta - 1)^2 + 2\sin\theta\sec\theta$$

FOIL

$(\tan\theta - 1)(\tan\theta - 1) + 2\sin\theta\sec\theta$

$\tan^2\theta - \tan\theta - \tan\theta + 1 + 2\sin\theta\sec\theta$

$\tan^2\theta - 2\tan\theta + 1 + 2\sin\theta\sec\theta$

$\tan^2\theta - 2\tan\theta + 1 + 2\sin\theta \cdot \frac{1}{\cos\theta}$

$\tan^2\theta - 2\tan\theta + 1 + 2\tan\theta$

$\tan^2\theta + 1$

$\sec^2\theta$

Pre-Calculus 12

6.2 Sum Difference and Double Angle Identities

Sum Identities

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

Difference Identities

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

1. Simplify to a single trig function

$$\sin\frac{\pi}{8}\cos\frac{3\pi}{5} + \cos\frac{\pi}{8}\sin\frac{3\pi}{5}$$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

Make the left side using our angles

$A = \frac{\pi}{8}$ $B = \frac{3\pi}{5}$

$= \sin\left(\frac{\pi}{8} + \frac{3\pi}{5}\right)$

$= \sin\left(\frac{5\pi}{40} + \frac{24\pi}{40}\right)$

$= \sin\frac{29\pi}{40}$

$$\frac{\pi}{12} = \frac{4\pi}{12} - \frac{3\pi}{12}$$

$$\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$$

2. Find an exact value for $\cos\frac{\pi}{12}$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

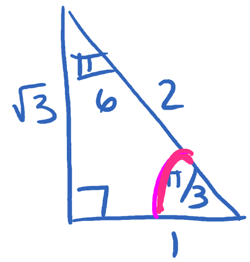
$$\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$$

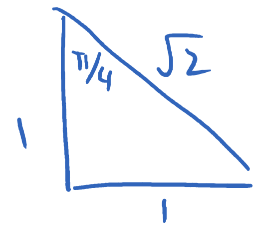
$$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{2\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$





3. Simplify and create a double angle identity

$\sin(2A) = \sin(A+A)$

$= \sin A\cos A + \cos A\sin A$

$\sin 2A = 2\sin A\cos A$

$\cos(2A) = \cos(A+A)$

$= \cos A\cos A - \sin A\sin A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan(2A) = \tan(A+A)$

$$= \frac{\tan A + \tan A}{1-(\tan A)(\tan A)}$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

Double Angle Identities

$\sin 2A = 2 \sin A \cos A$

$$\cos 2A = \begin{cases} cos^2 A - sin^2 A \\ 2cos^2 A - 1 \\ 1 - 2sin^2 A \end{cases}$$

$$\tan 2A = \frac{2 \tan A}{1 - tan^2 A}$$

4. Write as a single trig function
$$\frac{6 \tan 20°}{1 - tan^2 20°}$$

$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

$A = 20°$

$3\left(\frac{2\tan 20°}{1-\tan^2 20°}\right)$

$3(\tan 2(20°))$

$3 \tan 40°$

5. Evaluate without a calculator
$$10cos^2\left(\frac{\pi}{12}\right) - 5$$

$\cos 2A = 2\cos^2 A - 1$

$A = \frac{\pi}{12}$

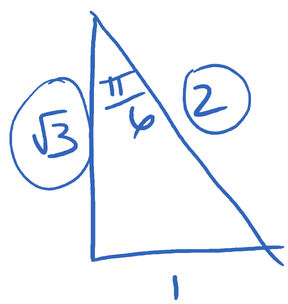
$5\left(2\cos^2 \frac{\pi}{12} - 1\right)$

$5\left(\cos 2\left(\frac{\pi}{12}\right)\right)$

$5 \cos \frac{\pi}{6}$

$\frac{5}{1}\left(\frac{\sqrt{3}}{2}\right)$

$\frac{5\sqrt{3}}{2}$



6. Evaluate without a calculator

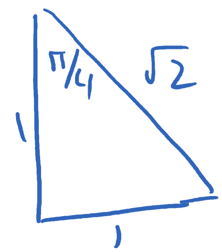
$$\sin\frac{\pi}{8}\cos\frac{\pi}{8}$$

$\sin 2A = 2\sin A\cos A$

$\frac{1}{2}$(Right) $\frac{1}{2}\left(2\sin\frac{\pi}{8}\cos\frac{\pi}{8}\right)$

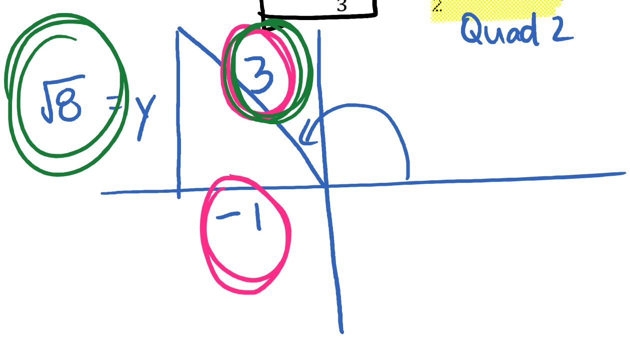
$\frac{1}{2}$(left) $\frac{1}{2}\left(\sin 2\left(\frac{\pi}{8}\right)\right)$

$\frac{1}{2}\sin\frac{\pi}{4} = \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$



7. If $\cos\theta = \boxed{\frac{-1}{3}}$ and $\frac{\pi}{2} \le \theta \le \pi$, find a value for:

Quad 2



$\cos\theta = -\frac{1}{3}$

$\cos\theta = \frac{x}{r}$

$x = -1 \quad r = 3$

$x^2 + y^2 = r^2$
$(-1)^2 + y^2 = 3^2$
$1 + y^2 = 9$
$y^2 = 8$
$y = \sqrt{8}$

a) $\sin 2\theta$

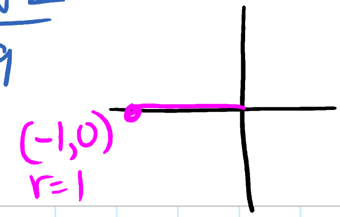
$\sin 2\theta = 2\sin\theta\cos\theta$
$= \frac{2}{1}\left(\frac{\sqrt{8}}{3}\right)\left(\frac{-1}{3}\right)$
$= -\frac{2\sqrt{8}}{9}$
$= \frac{-2(2\sqrt{2})}{9}$
$= -\frac{4\sqrt{2}}{9}$

$\sqrt{8}$
$\sqrt{4\cdot 2}$
$\sqrt{4}\cdot\sqrt{2}$
$2\sqrt{2}$

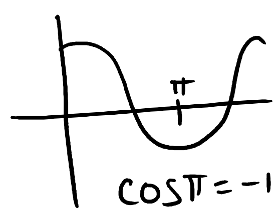
b) $\cos(\theta + \pi)$ (A B)

$\cos(A+B) = \cos A\cos B - \sin A\sin B$
$= \cos\theta\cos\pi - \sin\theta\sin\pi$
$= \left(\frac{-1}{3}\right)\left(\frac{-1}{1}\right) - \left(\frac{\sqrt{8}}{3}\right)\left(\frac{0}{1}\right)$
$= \frac{1}{3} - 0$
$= 1/3$

Ch 4



Ch 5



6.3 Part 1 New

Tuesday, April 18, 2023 12:43 PM

6.3 Part 1 Proving Identities

Review

a) $\frac{3}{7}+\frac{2}{5}$

$\frac{(5)\,3}{(5)\,7} + \frac{2\,(7)}{5\,(7)}$

$\frac{15}{35} + \frac{14}{35}$

$\frac{29}{35}$

b) expand $5(2x+3)$

$5(2x) + 5(3)$

$10x + 15$

c) factor $6x^2 - 12x$

GCF $6x$

$6x(x-2)$

$m^2 - m - 2$

$(m-2)(m+1)$

Prove the Identities

1. $\sec\theta\,(1+\cos\theta) = 1+\sec\theta$

$\sec\theta(1) + \sec\theta(\cos\theta) = R.S$

$\sec\theta + \frac{1}{\cancel{\cos\theta}}\cancel{\cos\theta} = R.S$

$\sec\theta + 1 = 1 + \sec\theta$

work on the more complicated side

Expand

$\sec\theta = \frac{1}{\cos\theta}$

2. $\tan\theta + \cot\theta = \sec\theta\csc\theta$

$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = R.S$

$\frac{\sin\theta}{\sin\theta}\cdot\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\cdot\frac{\cos\theta}{\cos\theta} = R.S$

$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = R.S$

$\frac{1}{\sin\theta\cos\theta} = R.S$

$\frac{1}{\sin\theta}\cdot\frac{1}{\cos\theta} = R.S$

$\csc\theta\sec\theta = \sec\theta\csc\theta$

Change into $\sin\theta$ and $\cos\theta$

Find a common denominator

$\sin^2\theta + \cos^2\theta = 1$ pythagorean identity

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3. $sec^4\theta - sec^2\theta = tan^4\theta + tan^2\theta$

Factor

$\sec^2\theta(\sec^2\theta - 1) = R.S$

pythagorean

$1 + \tan^2\theta = \sec^2\theta$

$(1+\tan^2\theta)(\sec^2\theta - 1) = R.S$

$(1+\tan^2\theta)(1+\tan^2\theta - 1) = R.S$

$(1+\tan^2\theta)(\tan^2\theta) = R.S$

Expand

$\tan^2\theta + \tan^4\theta = \tan^4\theta + \tan^2\theta$

4. $\frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} = 2csc^2\theta$

$\frac{(1-\cos\theta)}{(1-\cos\theta)} \cdot \frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} \cdot \frac{(1+\cos\theta)}{(1+\cos\theta)} = R.S$

$\frac{1-\cos\theta + 1 + \cos\theta}{(1-\cos\theta)(1+\cos\theta)} = R.S.$

$\frac{2}{1+\cos\theta - \cos\theta - \cos^2\theta} = R.S$

$\cos^2\theta + \sin^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\frac{2}{1-\cos^2\theta} = R.S$

$\csc\theta = \frac{1}{\sin\theta}$

$\frac{2}{\sin^2\theta} = R.S$

$2 \cdot \frac{1}{\sin^2\theta} = R.S$

$2\csc^2\theta = 2\csc^2\theta$

6.3 Part 2 New

Tuesday, April 18, 2023 2:21 PM

6.3 Part 2 Proving Identities

Recall Double angle identities

$\sin 2A = 2\sin A\cos A$

$$\cos 2A = \begin{cases} \cos^2 A - \sin^2 A \\ 2\cos^2 A - 1 \\ 1 - 2\sin^2 A \end{cases}$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

1. Expand using the double angle identities

a) $\sin 10\theta = \sin(2\cdot 5\theta)$

$= 2\sin 5\theta\cos 5\theta$

b) $\sin 6x = \sin(2\cdot 3x)$

$= 2\sin 3x\cos 3x$

c) $\cos 8\theta = \cos(2\cdot 4\theta)$

$= \cos^2 4\theta - \sin^2 4\theta$

or

$= 2\cos^2 4\theta - 1$

or $= 1 - 2\sin^2 4\theta$

d) $\tan 4\theta = \tan(2\cdot 2\theta)$

$$= \frac{2\tan 2\theta}{1 - \tan^2 2\theta}$$

2. Prove the Identity $\tan 2\theta = \frac{\sin 4\theta}{1+\cos 4\theta}$

$$L.S = \frac{\sin(2\cdot 2\theta)}{1 + \cos(2\cdot 2\theta)}$$

$$L.S = \frac{2\sin 2\theta\cos 2\theta}{\cancel{1} + 2\cos^2 2\theta \cancel{- 1}}$$

$$L.S = \frac{\cancel{2}\sin 2\theta\cancel{\cos 2\theta}}{\cancel{2}\cos^{\cancel{2}} 2\theta}$$

$$LS = \frac{\sin 2\theta}{\cos 2\theta}$$

$\tan 2\theta = \tan 2\theta$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

3. Prove the Identity $\cos 2\theta = \frac{csc^2\theta - 2}{csc^2\theta}$

$$LS = \frac{\csc^2\theta}{\csc^2\theta} - \frac{2}{\csc^2\theta}$$

$\csc\theta = \frac{1}{\sin\theta}$

$\sin\theta = \frac{1}{\csc\theta}$

$$LS = 1 - 2 \cdot \frac{1}{\csc^2\theta}$$

$$LS = 1 - 2 \cdot \sin^2\theta$$

$\cos 2\theta = 1 - 2\sin^2\theta$

$$\cos 2\theta = \cos 2\theta$$

conjugate

4 Simplify

a) $\frac{18}{7} \div \frac{4}{21}$

$$\frac{\overset{9}{\cancel{18}}}{\cancel{7}} \cdot \frac{\overset{3}{\cancel{21}}}{\underset{2}{\cancel{4}}} = \frac{27}{2}$$

b) $\frac{\sqrt{3}}{5-\sqrt{2}} \cdot \frac{5+\sqrt{2}}{5+\sqrt{2}}$

$$\frac{5\sqrt{3}+\sqrt{6}}{25+5\sqrt{2}-5\sqrt{2}-\sqrt{4}}$$

$$\frac{5\sqrt{3}+\sqrt{6}}{25-2}$$

$$\frac{5\sqrt{3}+\sqrt{6}}{23}$$

conjugate
$1+\sin\theta$

5. Prove the identity $\frac{\cos\theta}{1-\sin\theta}=\frac{1+\sin\theta}{\cos\theta}$

$$\frac{(1+\sin\theta)\cos\theta}{(1+\sin\theta)(1-\sin\theta)} = R.S.$$

$$\frac{(1+\sin\theta)\cos\theta}{1-\sin\theta+\sin\theta-\sin^2\theta} = R.S.$$

$\sin^2\theta+\cos^2\theta=1$

$\cos^2\theta=1-\sin^2\theta$

$$\frac{(1+\sin\theta)\cos\theta}{1-\sin^2\theta} = R.S$$

$$\frac{(1+\sin\theta)\cancel{\cos\theta}}{\cos^{\cancel{2}}\theta} = R.S$$

$$\frac{1+\sin\theta}{\cos\theta}=\frac{1+\sin\theta}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

6. Prove the identity $\frac{\sin\theta+\tan\theta}{1+\cos\theta} = \tan\theta$

$$\frac{\frac{\sin\theta}{1}+\frac{\sin\theta}{\cos\theta}}{1+\cos\theta} = R.S.$$

$$\frac{\frac{\cos\theta}{\cos\theta}\cdot\frac{\sin\theta}{1}+\frac{\sin\theta}{\cos\theta}}{1+\cos\theta} = RS$$

$$\frac{\frac{\cos\theta\sin\theta+\sin\theta}{\cos\theta}}{1+\cos\theta} = RS$$

$$\frac{\frac{\sin\theta(\cos\theta+1)}{\cos\theta}}{\frac{1+\cos\theta}{1}} = RS$$

$$\frac{\sin\theta(\cancel{\cos\theta+1})}{\cos\theta}\cdot\frac{1}{\cancel{1+\cos\theta}} = R.S$$

$$\frac{\sin\theta}{\cos\theta} = R.S$$

$$\tan\theta = \tan\theta$$

6.4 Part 1 New

Wednesday, April 19, 2023 9:54 AM

$\frac{3x}{x-5}$ $x \neq 5$

6.4 Solving Trig Equations Using Identities Part 1

Non-Permissible Values: Values of the variable that make the function undefined.

- the denominator is equal to zero
- the function has asymptotes ($\tan\theta$, $\csc\theta$, $\sec\theta$, $\cot\theta$) all have asymptotes

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

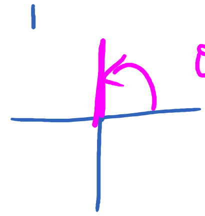
Example 1: Determine any non-permissible values for $0 \le \theta < 2\pi$

a) $\frac{\cos\theta}{1-\sin\theta}$

$1 - \sin\theta = 0$

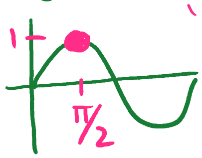
$\frac{1}{1} = \sin\theta$

Ch 4
$y = 1$
$r = 1$



$\theta \neq \pi/2$

or ch 5



1
$\pi/2$

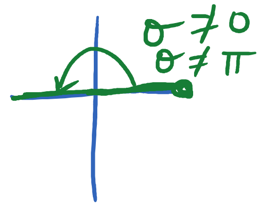
b) $3\cot\theta$

cot θ has asymptotes when $\sin\theta = \frac{0}{1}$

$3\cot\theta$

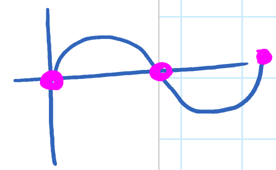
$\frac{3\cos\theta}{\sin\theta}$

ch 4
$y = 0$
$r = 1$



$\theta \neq 0$
$\theta \neq \pi$

ch 5



Example 2: Solve $\cos 2\theta + 1 - \cos\theta = 0$ over the domain $0 \le \theta < 2\pi$. Express your answer as exact values.

$\cos 2\theta + 1 - \cos\theta = 0$

$\cos 2\theta = 2\cos^2\theta - 1$
$\cos^2\theta - \sin^2\theta$
$1 - 2\sin^2\theta$

$2\cos^2\theta - 1 + 1 - \cos\theta = 0$

$2\cos^2\theta - \cos\theta = 0$

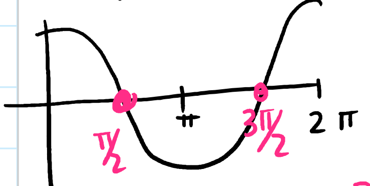
$\cos\theta(2\cos\theta - 1) = 0$

$2m^2 - m = 0$
$m(2m-1) = 0$
$m = 0$ $2m - 1 = 0$

$\cos\theta = 0$

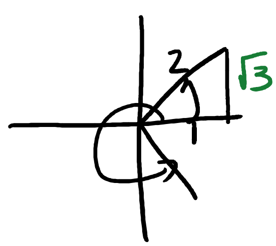
$2\cos\theta - 1 = 0$
$\cos\theta = \frac{1}{2}$ $x = 1$ $r = 2$

Graph



$\pi/2$ π $3\pi/2$ 2π

$\theta_1 = \pi/2$ $\theta_2 = 3\pi/2$



2 $\sqrt{3}$

ref $\angle = \frac{\pi}{3}$

$\theta_3 = \pi/3$

$\theta = 2\pi - \frac{\pi}{3}$

$\theta_4 = 5\pi/3$

Example 3: Solve $1 - \cos^2\theta = 3\sin\theta - 2$ over the domain $0 \le \theta < 2\pi$. Express your answer as exact values.

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\sin^2\theta = 3\sin\theta - 2$

$\sin^2\theta - 3\sin\theta + 2 = 0$

$\times = 2$
$+ = -3$
$-2, -1$

$\sin^2\theta - \sin\theta - 2\sin\theta + 2 = 0$

$\sin\theta(\sin\theta - 1) - 2(\sin\theta - 1) = 0$

$(\sin\theta - 1)(\sin\theta - 2) = 0$

$\sin\theta - 1 = 0$

$\sin\theta = 1$

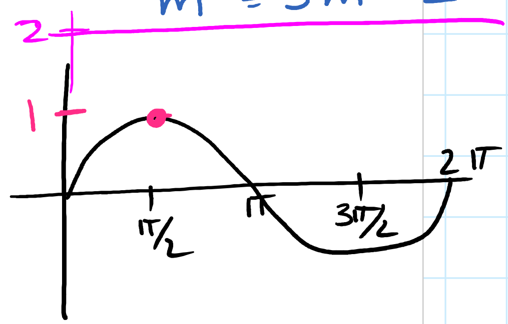
$\boxed{\theta = \pi/2}$

$\sin\theta - 2 = 0$

$\sin\theta = 2$

No Solution

$m^2 = 3m - 2$



Example 4: Solve $\cos^2\theta = \cot\theta\sin\theta$ over the domain $0° \le \theta < 360°$. Express your answer as exact values.

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\cos^2\theta = \cot\theta\sin\theta$

$\cos^2\theta = \frac{\cos\theta}{\cancel{\sin\theta}} \cdot \cancel{\sin\theta}$

$\cos^2\theta = \cos\theta$

$m^2 = m$

$m^2 - m = 0$

$\cos^2\theta - \cos\theta = 0$

$\cos\theta(\cos\theta - 1) = 0$

$\cos\theta = 0$

$\theta = 90°$

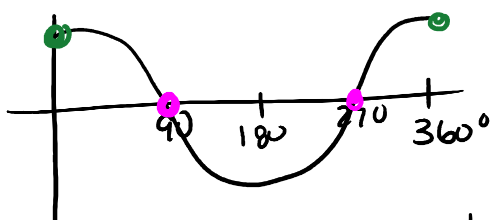
$\theta = 270°$

$\cos\theta - 1 = 0$

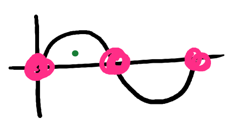
$\cos\theta = 1$

$\cancel{\theta = 0°}$

$\cancel{\theta = 360°}$



cot θ has asymptotes when sin θ = 0



$\theta \ne 0°$
$\theta \ne 180°$
$\theta \ne 360°$

Practice: p. 296 #1, p. 320 #1 – 4,

See google classroom

6.4 part 2

Friday, April 21, 2023 9:00 AM

6.4 Solving Trig Equations Using Identities – part 2

General Solutions

Example 1: Solve $\sin 2\theta = \sqrt{2}\cos\theta$. Give the general solution in radians.

Solve $0 \le \theta < 2\pi$
Use the solutions to form the general solution.

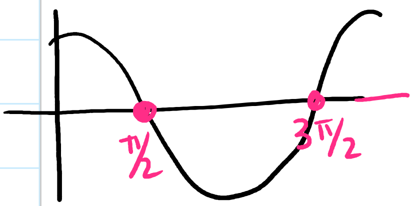
$\sin 2\theta = 2\sin\theta\cos\theta$

$2\sin\theta\cos\theta = \sqrt{2}\cos\theta$

$2\sin\theta\cos\theta - \sqrt{2}\cos\theta = 0$

$\cos\theta(2\sin\theta - \sqrt{2}) = 0$

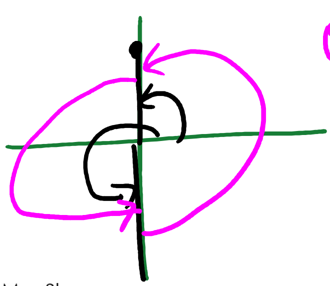
$\cos\theta = 0$



$\theta = \pi/2 \quad \theta = 3\pi/2$

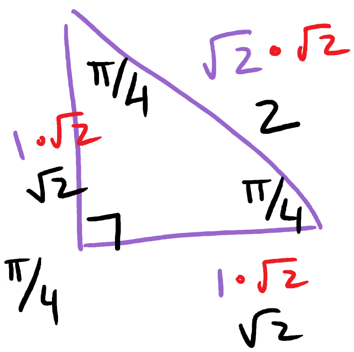
period of $\cos\theta$ is 2π

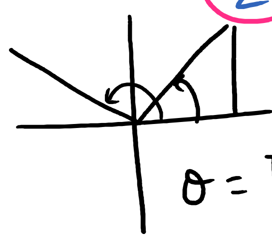
$\theta = \pi/2 + 2\pi n \quad n \in I$
$\theta = 3\pi/2 + 2\pi n$



$\theta = \frac{\pi}{2} + \pi n \quad n \in I$

$2\sin\theta - \sqrt{2} = 0$
$2\sin\theta = \sqrt{2}$
$\sin\theta = \frac{\sqrt{2}}{2}$ ← Hyp





ref $\angle = \pi/4$

$\theta = \pi/4$
$\theta = \pi - \pi/4$
$\theta = 3\pi/4$

$\sin\theta = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

period of $\sin\theta = 2\pi$

$\theta = \pi/4 + 2\pi n \quad n \in I$
$\theta = \frac{3\pi}{4} + 2\pi n$

Example 2: Solve $2\sin\theta = 7 - 3\csc\theta$. Give the general solution in radians.

$2\sin\theta = 7 - 3\cdot\frac{1}{\sin\theta}$

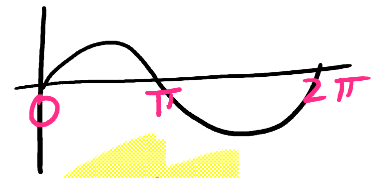
$\csc\theta$ has asymptotes

$\csc\theta = \frac{1}{\sin\theta}$

when $\sin\theta = 0$

$\sin\theta(2\sin\theta) = 7\sin\theta - \frac{3}{\cancel{\sin\theta}}\cancel{\sin\theta}$ $\quad m2m = 7m - \frac{3}{\cancel{m}}\cancel{m}$

$2\sin^2\theta = 7\sin\theta - 3$



$2m^2 = 7m - 3$

$2\sin^2\theta - 7\sin\theta + 3 = 0$

$\_\,x\,\_ = 6$

$-1 + -6 = -7$

$\theta \neq 0$

$\theta \neq \pi$

$\underbrace{2\sin^2\theta - 1\sin\theta}\ \underbrace{-6\sin\theta + 3} = 0$

$\sin\theta(2\sin\theta - 1) - 3(2\sin\theta - 1) = 0$

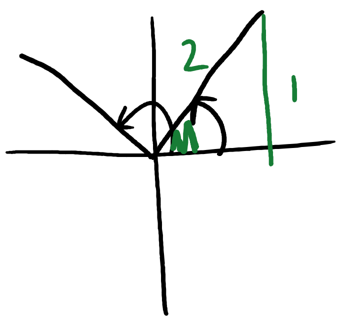
$(2\sin\theta - 1)(\sin\theta - 3) = 0$

$2\sin\theta - 1 = 0$

$\sin\theta = \frac{1}{2}$ $\quad y = 1$, $r = 2$

$\sin\theta - 3 = 0$

$\sin\theta = 3$

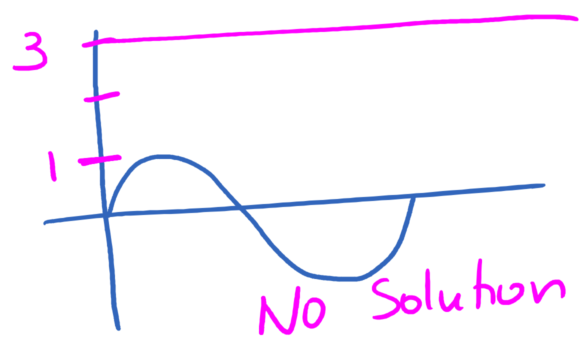


ref $= \frac{\pi}{6}$

$\theta = \frac{\pi}{6}$

$\theta = \pi - \frac{\pi}{6}$

$\theta = 5\pi/6$



No Solution

$\theta = \frac{\pi}{6} + 2\pi n \qquad n \in I$

$\theta = \frac{5\pi}{6} + 2\pi n$

Practice: p.320 #8 – 12, 16

Identity Chart

Friday, April 6, 2018 12:28 PM

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Reciprocal Identities 3	$\frac{1}{\sin\theta} = \csc\theta$ $\quad \frac{1}{\tan\theta} = \cot\theta$ $\frac{1}{\cos\theta} = \sec\theta$
Quotient Identities 2	$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$
Pythagorean Identities 3	$\sin^2\theta + \cos^2\theta = 1$ $\tan^2\theta + 1 = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$
Sum and Difference Identities 6	$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\tan(A+B) = \frac{\tan A + \tan B}{1-(\tan A)(\tan B)}$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$ $\tan(A-B) = \frac{\tan A - \tan B}{1+(\tan A)(\tan B)}$
Double Angle Identities 5	$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$ $= 2\cos^2\theta - 1$ $= 1 - 2\sin^2\theta$ $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

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