7.1 Exponential Functions

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AP Calculus 12

7.1 Calculus of Exponential Functions

Use n = 0.0001 to find estimates for the following

$\lim_{n\to 0}\frac{2^n-1}{n}$	$\lim_{n\to 0}\frac{2.7^n-1}{n}$	$\lim_{n\to 0}\frac{e^n-1}{n}$	$\lim_{n\to 0}\frac{2.8^n-1}{n}$	$\lim_{n\to 0}\frac{3^n-1}{n}$
0.6932	0.9933	1.0001	1.0297	1.0987
$\ln 2 = .6932$	$\ln 2.7 = .9933$	$\ln e = 1$	$\ln 2.8$	$\ln 3$

$$\lim_{n\to 0}\frac{b^n-1}{n} = \ln b$$

1. Use the definition of a derivative to find $f'(x)$ if $f(x) = e^x$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{e^{x+h}-e^x}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{e^x\cdot e^h - e^x}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{e^x(e^h-1)}{h}$$

$$f'(x) = e^x\lim_{h\to 0}\frac{e^h-1}{h}$$

$$f'(x) = e^x(\ln e)$$

$$f'(x) = e^x(1)$$

$$f'(x) = e^x$$

$$\frac{d}{dx}e^x = e^x \qquad \frac{d}{dx}e^u = e^u\cdot\frac{du}{dx}$$

2. Find the derivative of the following

a) $f(x) = e^{x^2}$

$f'(x) = e^{x^2} \cdot \frac{d}{dx} x^2$

$f'(x) = e^{x^2} \cdot 2x$

$f'(x) = 2xe^{x^2}$

b) $f(x) = e^{-3/x}$

$f(x) = e^{-3x^{-1}}$

$f'(x) = e^{-3x^{-1}} \cdot (-3(-1)x^{-2})$

$f'(x) = \frac{3}{x^2} e^{-3/x}$

c) $f(x) = x^2 e^x$ product Rule

$f'(x) = x^2 \cdot e^x + 2xe^x$

$f'(x) = xe^x(x+2)$

3. Find the local extrema of $f(x) = xe^x$ Find the Max/Min

$f'(x) = x \cdot e^x + 1 \cdot e^x$

$0 = xe^x + e^x$

$0 = e^x(x+1)$

$e^x = 0$ Never

$x+1 = 0$

$x = -1$

test sign $f'(x)$: $(-\infty, -1)$ $x=-2$ ⊖ ; $(-1, \infty)$ $x=0$ ⊕

Min at $x = -1$

$f(-1) = (-1)e^{-1}$

$f(-1) = -\frac{1}{e}$

Min $\left(-1, -\frac{1}{e}\right)$

Integration of e^x

$$\int e^x dx = e^x + C$$

$$\int e^u du = e^u + C$$

4. $\int e^{3x+1} dx$

$u = 3x + 1$

$\frac{du}{dx} = 3$

$\frac{du}{3} = dx$

$$\int e^u \frac{du}{3}$$

$$= \frac{1}{3}\int e^u du$$

$$= \frac{1}{3}e^u + C$$

$$= \frac{1}{3}e^{3x+1} + C$$

5. Evaluate

$$\int \frac{e^{\frac{1}{x}}}{x^2} dx$$

$u = \frac{1}{x}$

$u = x^{-1}$

$\frac{du}{dx} = -1x^{-2}$

$\frac{du}{dx} = \frac{-1}{x^2}$

$\frac{du}{-1} = \frac{1}{x^2}dx$

$$\int e^{\frac{1}{x}} \cdot \frac{dx}{x^2}$$

$$= \int e^u \cdot \frac{du}{-1}$$

$$= -\int e^u du$$

$$= -e^u + C$$

$$= -e^{\frac{1}{x}} + C$$

6. $\int 5xe^{-x^2}\,dx$

$u = -x^2$

$\frac{du}{dx} = -2x$

$\frac{du}{-2} = x\,dx$

$= 5\int e^u \cdot \frac{du}{-2}$

$= -\frac{5}{2}\int e^u\,du$

$= -\frac{5}{2}e^u + C$

$= -\frac{5}{2}e^{-x^2} + C$

7. Evaluate the definite integral

$$\int_{-1}^{0} e^x \cos(e^x)\,dx$$

$u = e^x$

$\frac{du}{dx} = e^x$

$du = e^x dx$

$x = -1$, $u = e^{-1}$, $u = 1/e$

$x = 0$, $u = e^0$, $u = 1$

$\int_{1/e}^{1} \cos u\,du$

$= \sin u \Big|_{1/e}^{1}$

$= \sin(1) - \sin(\frac{1}{e})$

$= 0.482$