7.1 Arithmetic Sequences (Part 1)

A sequence is an ordered list of objects.

It contains elements, called __**terms**__, that follow a regular pattern or rule that determines the next term

An Arithmetic Sequence is an ordered list of terms in which the difference between two consecutive terms is __**constant**__. $t_2 - t_1$ $t_5 - t_4$

The constant is called the __**common difference   d**__.

The first term of a sequence is __t_1__.

The term number of a sequence is __n__.

The general term of a sequence is __t_n__. This term depends on the value of __n__.

___The formula for the general term___ determines any term of sequence. This formula is a rule that indicates the relationship between t_n and n.

$$t_n = t_1 + (n-1)d$$

An arithmetic sequence is a <u>linear relation</u> that can be modeled by the formula of the general term with a starting term, t_1 , and an <u>common difference</u>, d .

$$y = mx + b$$

Example 1: Indicates sequences that are arithmetic. For each arithmetic sequence, determines the value of t_1, d, and the next three terms.

$t_1 \; t_2 \; t_3 \; t_4 \; t_5$

a) $3, 6, 12, 24, 48, …$

$+3 \; +6 \; +12 \; +24$

Not a constant value being added
NOT Arithmetic

b) $1, 8, 15, 22, 29, …$

$+7 \; +7 \; +7$

Arithmetic
Adding on a constant amount

$t_1 = $ first term
$t_1 = 1$

$d = 8 - 1$
$d = 7$

$t_6 = 29 + 7 = 36$
$t_7 = 36 + 7 = 43$
$t_8 = 43 + 7 = 50$

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When represented graphically, an arithmetic sequence is a discrete **linear relation** with a constant rate of change equal to the common difference.

Slope = m = common difference

Example 2: For the given sequence $t_1 = 8$ and $d = -3$:

a) Determine the general term, t_n

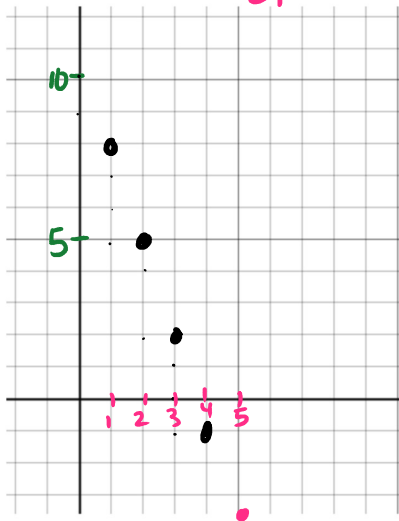$$t_n = t_1 + (n-1)d$$
$$t_n = 8 + (n-1)(-3)$$
$$t_n = 8 - 3n + 3$$

$$t_n = 11 - 3n$$
or
$$t_n = -3n + 11$$

b) Represent the sequence graphically

$t_1 = 8$ when $n = 1$

$$y = mx + b$$
$$t_n = -3n + 11$$

$$m = slope = -3 = \frac{-3}{1}$$

$$3 \rightarrow 1$$

$$y\text{-int} = 11$$

Example 3: Determine the value of the 17th term of the arithmetic sequence: 4, 7, 10, 13, 16, ...

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ 52 $\quad$ 17th

$$t_{17} = ?$$
$$t_1 = 4$$
$$d = t_2 - t_1$$
$$d = 7 - 4$$
$$d = 3$$
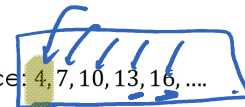$$n = 17$$

$$t_n = t_1 + (n-1)d$$
$$t_{17} = 4 + (17-1)(4)$$
$$t_{17} = 4 + (16)(4)$$
$$t_{17} = 4 + 48$$
$$t_{17} = 52$$

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Example 4: Given the arithmetic sequence: $-3, 2, 7, 12, ...$

$\frac{212}{44th}$

a) Determine t_{20}

$t_1 = -3$

$d = t_3 - t_2$

$d = 7 - 2$

$d = 5$

$n = 20$

$t_n = t_1 + (n-1)d$

$t_{20} = -3 + (20-1)(5)$

$t_{20} = -3 + 19(5)$

$t_{20} = -3 + 95$

$t_{20} = 92$

b) Determines which term has a value of 212

$t_1 = -3$

$d = 5$

$t_n = 212$

$n = ?$

$t_n = t_1 + (n-1)d$

$212 = -3 + (n-1)(5)$

$212 = -3 + 5n - 5$

$212 = 5n - 8$

$\quad +8 \qquad\qquad +8$

$\dfrac{220}{5} = \dfrac{5n}{5}$

$44 = n$

$t_{44} = 212$

44th term is 212

Example 5: Determine the first term of the arithmetic sequence when the 20th term is 84 and the common difference is 4.

$t_1 = ?$

$t_{20} = 84$

$n = 20$

$d = 4$

$t_n = t_1 + (n-1)d$

$t_{20} = t_1 + (n-1)d$

$84 = t_1 + (20-1)4$

$84 = t_1 + 19(4)$

$84 = t_1 + 76$

$76 \qquad\qquad -76$

$8 = t_1$

1st term = 8

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