7.2 Pg 354

1. $f(x) = 7x - 4$

inverse

$x = 7y - 4$

$x + 4 = 7y$

$\frac{x+4}{7} = y$

$f^{-1}(x) = \frac{x}{7} + \frac{4}{7}$

let $f^{-1}(x) = g(x) = \frac{x}{7} + \frac{4}{7}$

Find $f(g(x))$

$f\left(\frac{x}{7} + \frac{4}{7}\right)$

$7\left(\frac{x}{7} + \frac{4}{7}\right) - 4$

$x + 4 - 4$

x

$f(g(x)) = x$

Find $g(f(x))$

$g(7x - 4)$

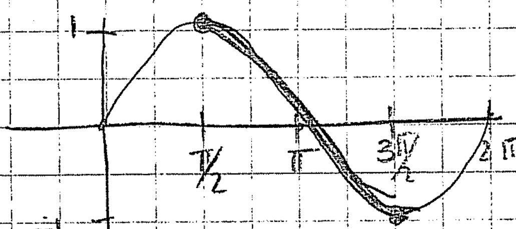
$\frac{7x - 4}{7} + \frac{4}{7}$

$x - \frac{4}{7} + \frac{4}{7}$

x

$g(f(x)) = x$

3. $f(x) = \sin x$ one to one



$\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

or

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ or $\left[\frac{3\pi}{2}, \frac{5\pi}{2}\right]$ ...

4. $f(x) = \frac{x-2}{x+3}$

$x = \frac{y-2}{y+3}$

$x(y+3) = y - 2$

$xy + 3x = y - 2$

$xy - y = -3x - 2$

$y(x-1) = -3x - 2$

$g(x) = y = \frac{-3x-2}{x-1}$

Inverse

$f(g(x))$

$$\frac{\frac{-3x-2}{x-1} - 2}{\frac{-3x-2}{x-1} + 3}$$

$$\frac{\frac{-3x-2-2(x-1)}{x-1}}{\frac{-3x-2+3(x-1)}{x-1}}$$

$$\frac{-3x-2-2x+2}{-3x-2+3x-3}$$

$$\frac{-5x}{-5}$$

$= x$

$g(f(x))$

$$\frac{-3\left(\frac{x-2}{x+3}\right) - 2}{\frac{x-2}{x+3} - 1}$$

$$\frac{\frac{-3x+6-2(x+3)}{x+3}}{\frac{x-2-(x+3)}{x+3}}$$

$$\frac{-3x+6-2x-6}{x-2-x-3}$$

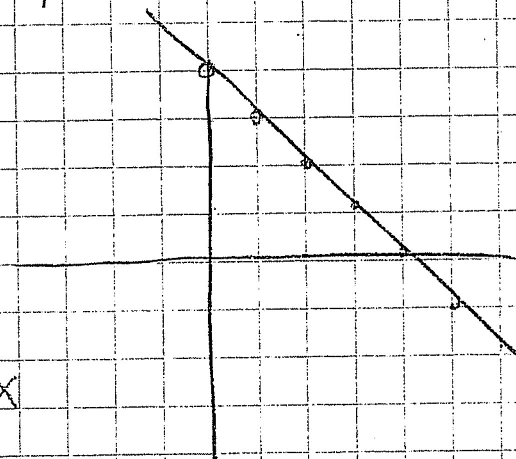
$$\frac{-5x}{-5}$$

x

#4 continued

$f(x)$	$g(x)$
$\{x : x \neq -3 \ x \in \mathbb{R}\}$	$\{x / x \neq 1 \ x \in \mathbb{R}\}$
$\{y : y \neq 1 \ y \in \mathbb{R}\}$	$\{y / y \neq -3 \ y \in \mathbb{R}\}$

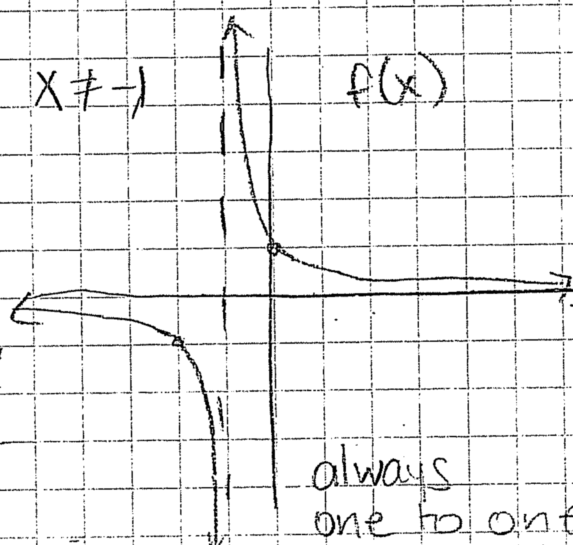
9. $f(x) = 4 - x$

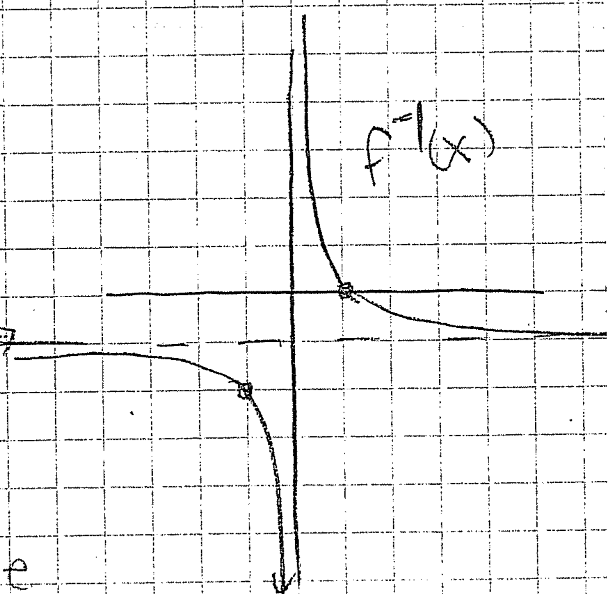


$x = 4 - y$
$x - 4 = -y$
$y = 4 - x$
$f^{-1}(x) = 4 - x$

No restrictions
one to one
always

10. $f(x) = \dfrac{1}{x+1}$ $\quad x \neq -1$





$x = \dfrac{1}{y+1}$

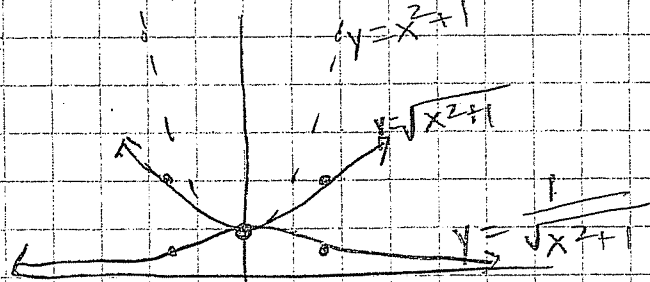
$x(y+1) = 1$

$y+1 = \dfrac{1}{x}$

$y = \dfrac{1}{x} - 1$

always
one to one
No restrictions

13. $f(x) = \frac{1}{\sqrt{x^2+1}}$



- draw parabola first
- then take square root of the y-values
- 3rd step reciprocal

Not one to one
restrict domain
$x \geq 0$

$f^{-1}(x)$

$x = \frac{1}{\sqrt{y^2+1}}$

$x^2 = \frac{1}{y^2+1}$

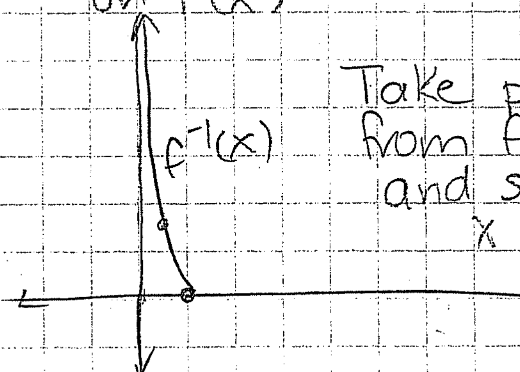
$(y^2+1)x^2 = 1$

$y^2+1 = \frac{1}{x^2}$

$y^2 = \frac{1}{x^2} - 1$

$y = \sqrt{\frac{1}{x^2} - 1}$

No $\pm$ due to restriction on $f(x)$



Take points from f(X) and switch x and y

17. $f^{-1} = f$ if the graph is reflected over $y=x$ and there is no change.

A No
B Yes
C Yes
D No

21. $f(x) = x^2 - 2x$

$f(x) = x^2 - 2x + \boxed{1} - \boxed{1}$
$= (x-1)^2 - 1$

f(x) will pass the horizontal line test if we restrict the domain to $x \geq 1$

$f^{-1}(x)$
$x = (y-1)^2 - 1$
$x+1 = (y-1)^2$
$\sqrt{x+1} = y-1$
$\sqrt{x+1} + 1 = y$
$f^{-1}(x) = \sqrt{x+1} + 1$

or if restriction on f(x) is $x \leq 1$
$f^{-1}(x) = -\sqrt{x+1} + 1$

25. $f(x) = 7x + 6$
$x = 7y + 6$
$x - 6 = 7y$
$g(x) = \frac{x-6}{7}$
$f'(x) = 7$

$g'(x) = \frac{1}{f'(g(x))}$

$g'(x) = \frac{1}{7}$

27. $f(x) = x^{-5}$
$x = y^{-5}$
$\frac{1}{x^{1/5}} = g(x)$
$f'(x) = -5x^{-6}$

$g'(x) = \frac{1}{-5\left(\frac{1}{x^{1/5}}\right)^{-6}}$
$= \frac{1}{-5\left(\frac{1}{x^{-6/5}}\right)}$
$= \frac{1}{-5x^{6/5}}$

29. $f(x) = \frac{x}{x+1}$

$$f'(x) = \frac{(x+1)(1) - x(1)}{(x+1)^2}$$

$$= \frac{x+1-x}{(x+1)^2}$$

$$= \frac{1}{(x+1)^2}$$

$$x = \frac{y}{y+1}$$

$$x(y+1) = y$$
$$xy + x = y$$
$$x = y - xy$$
$$x = y(1-x)$$
$$\frac{x}{1-x} = y$$
$$g(x) = \frac{x}{1-x}$$

$$g'(x) = \frac{1}{\frac{1}{\left(\frac{x}{1-x}+1\right)^2}}$$

$$= 1 \cdot \frac{\left(\frac{x}{1-x}+1\right)^2}{1}$$

$$= \left(\frac{x}{1-x} + \frac{1-x}{1-x}\right)^2$$

$$= \left(\frac{1}{1-x}\right)^2$$

$$= \frac{1}{(1-x)^2}$$

31. $f(x) = x^3 + 2x + 4$

$g(x) = f^{-1}(x)$

(7, 1) is on g(x)
(1, 7) is on f(x)

$g(7)$
$7 = x^3 + 2x + 4$
$0 = x^3 + 2x - 3$

$$g'(7) = \frac{1}{f'(g(7))}$$

$$= \frac{1}{f'(1)} = \frac{1}{5}$$

-1	1	0	2	-3
		-1	-1	-3
	1	1	3	0

$f'(x) = 3x^2 + 2$
$f'(1) = 3 + 2$
$= 5$

(x − 1) is a factor
x = 1 is a root
g(7) = 1

33. $f(x) = x + \cos x \quad b = 1$

$1 = x + \cos x$

$x = 0$

$(0,1)$ is on $f(x)$

$(1,0)$ is on $g(x)$

$f'(x) = 1 - \sin x$

$f'(0) = 1 - \sin(0)$

$= 1$

$$g'(1) = \frac{1}{f'(g(1))}$$

$$= \frac{1}{f'(0)}$$

$$= 1$$

35. $f(x) = \sqrt{x^2+6x} \quad x \geq 0 \quad b = 4$

$4 = \sqrt{x^2+6x}$

$(2,4)$ is on $f(x)$

$(4,2)$ is on $g(x)$

$16 = x^2 + 6x$

$0 = x^2 + 6x - 16$

$0 = (x+8)(x-2)$

$\cancel{x = -8} \quad x = 2$

$f'(x) = \frac{1}{2}(x^2+6x)^{-\frac{1}{2}}(2x+6)$

$f'(2) = \frac{1}{2}\left(\frac{1}{\sqrt{2^2+6(2)}}\right)(2(2)+6)$

$= \frac{10}{2\sqrt{16}}$

$= \frac{10}{8}$

$= \frac{5}{4}$

$$g'(4) = \frac{1}{f'(g(4))}$$

$$= \frac{1}{f'(2)}$$

$$= \frac{1}{\frac{5}{4}}$$

$$= 4/5$$

37. $f(x) = \frac{1}{x+1}$ $b = \frac{1}{4}$

$\frac{1}{4} = \frac{1}{x+1}$

$(3, \frac{1}{4})$ is on $f(x)$
$(\frac{1}{4}, 3)$ is on $g(x)$

$\cancel{4}(x+1)\left(\frac{1}{\cancel{4}}\right) = \left(\frac{1}{\cancel{x+1}}\right)(4)\cancel{(x+1)}$

$x + 1 = 4$
$x = 3$

$g'(\frac{1}{4}) = \frac{1}{f'(g(\frac{1}{4}))}$

$f'(x) = \frac{(x+1)(0) - (1)(1)}{(x+1)^2}$

$= \frac{1}{-\frac{1}{16}}$

$= \frac{-1}{(x+1)^2}$

$= -16$

$f'(3) = \frac{-1}{(3+1)^2} = -\frac{1}{16}$

38. $f(x) = e^x$ $b = e$

$(1, e)$ is on $f(x)$
$(e, 1)$ is on $g(x)$

$e = e^x$
$x = 1$

$f'(x) = e^x$

$g'(e) = \frac{1}{f'(g(e))}$

$f'(1) = e^1$
$= e$

$= \frac{1}{f'(1)}$

$= \frac{1}{e}$