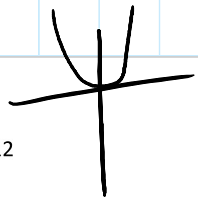
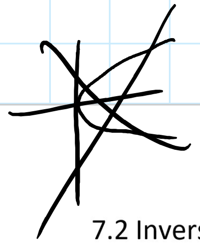
AP Calculus 12

7.2 Inverse Functions

Review: If $f(x)$ and $g(x)$ are functions and $f(g(x)) = x$ and $g(f(x)) = x$ then $f(x)$ and $g(x)$ are inverses and are said to be invertible.

1. Show that $f(x) = 2x^3 - 1$ and $g(x) = \sqrt[3]{\frac{x+1}{2}}$ are inverses.

$f(g(x))$
$= f\left(\sqrt[3]{\frac{x+1}{2}}\right)$
$= 2\left(\sqrt[3]{\frac{x+1}{2}}\right)^3 - 1$
$= \cancel{2}\left(\frac{x+1}{\cancel{2}}\right) - 1$
$= x + 1 - 1$
$= x$

$g(f(x))$
$= g(2x^3 - 1)$
$= \sqrt[3]{\frac{2x^3 - 1 + 1}{2}}$
$= \sqrt[3]{\frac{\cancel{2}x^3}{\cancel{2}}}$
$= \sqrt[3]{x^3}$
$= x$

$f(g(x)) = x = g(f(x))$
so they are inverses

2. Find the inverse of $f(x) = \frac{x}{x+3}$

$y = \frac{x}{x+3}$

$x = \frac{y}{y+3}$

$(y+3)x = \left(\frac{y}{\cancel{y+3}}\right)\cancel{(y+3)}$

$(y+3)x = y$

$xy + 3x = y$

$xy - y = -3x$

$\frac{y(\cancel{x-1})}{\cancel{x-1}} = \frac{-3x}{x-1}$

$y = \frac{-3x}{x-1}$

$x = 1 \quad y = -3$
Asymptotes

$f(x) = \frac{x}{x+3}$

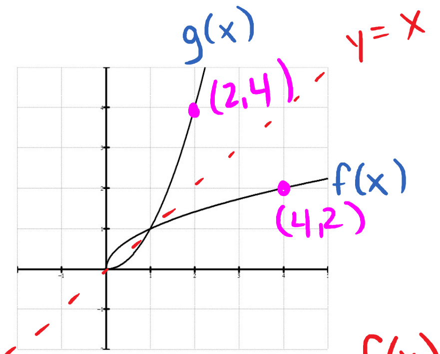
$x = -3 \quad y = 1$
Asymptotes

Reflective Property of Inverses

If $f(x)$ and $g(x)$ are inverses and $f(a) = b$ then

then $g(b) = a$

(a, b) is on $f(x)$
(b, a) is on $g(x)$



Domain of $f(x)$ = Range of $g(x)$

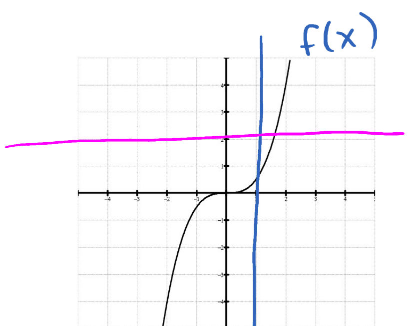
Range of $f(x)$ = Domain of $g(x)$

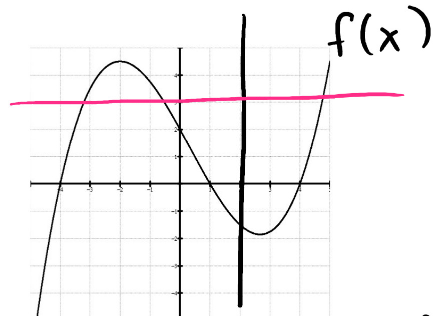
$f(x)$ and $g(x)$ are reflections of each other over the line $y = x$

Existence of an Inverse

Not every function has an inverse function.

For a relation to be a function and to have an inverse function, the relation must pass the vertical line test and the horizontal line test. Functions that have an inverse function are called *one to one.* Both are functions





$f(x)$ is a function
$f(x)$ has an inverse (function)
Passed vertical and horizontal line test

$f(x)$ is a function
does not have an inverse
Passes Vertical line test
fails Horizontal line test

Calculus and Inverses

How are the slopes of tangent lines related with inverse functions?

change of base

How are the slopes of tangent lines related with inverse functions?

If $f(x) = 2^x$ then $g(x) = inverse$. Find $g(x)$

change of base

$\log_c x = \frac{\log_{10} x}{\log_{10} c}$

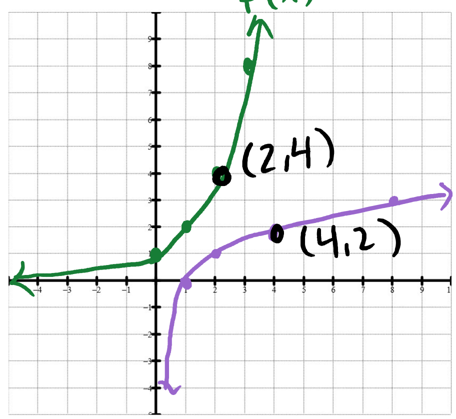
$y = 2^x$

$x = 2^y$

$y = \log_2 x$

$y = \frac{\log_{10} x}{\log_{10} 2}$

Graph $f(x) and\ g(x)$ using a graphing calculator.



on f(x) (2,4)

Slope of tangent line at (2,4)

$f'(2) = 2.773$

on g(x) (4,2)

Slope of tangent line at (4,2)

$g'(4) = .3606$

$A \cdot B$

$(2.773)(.3606) = 1$

$\frac{2}{3} \cdot \frac{3}{2} = 1$

∴ slopes are reciprocals of each other

How are derivatives of inverse functions related?

If $f(x)$ and $g(x)$ are inverse functions

$$f(g(x)) = x$$

$$\frac{d}{dx} f(g(x)) = \frac{d}{dx} x$$

$$\frac{f'(g(x)) \cdot g'(x)}{f'(g(x))} = \frac{1}{f'(g(x))}$$

$$\boxed{g'(x) = \frac{1}{f'(g(x))}}$$

If $f(a) = b$ (a, b) is on $f(x)$

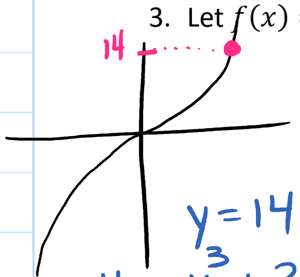
$g(b) = a$ (b, a) is on $g(x)$

$$\frac{f'(a) \cdot g'(b)}{f'(a)} = \frac{1}{f'(a)}$$

$$g'(b) = \frac{1}{f'(a)}$$

$$\boxed{g'(b) = \frac{1}{f'(g(b))}}$$

3. Let $f(x) = x^3 + 3x$ $g(x)$ is the inverse of $f(x)$. Find $g'(14)$



14 is an x-value on $g(x)$

14 is a y-value on $f(x)$

$y = 14$

$14 = x^3 + 3x$

$0 = x^3 + 3x - 14$

$x = 2$ factor $x - 2$

-2	1	0	3	-14
		-2	-4	-14
	1	2	7	0

$0 = (x-2)(x^2 + 2x + 7)$

$x = 2$ $(2, 14)$ on $f(x)$

$$g'(b) = \frac{1}{f'(g(b))}$$

$g(14) = 2$

$f'(x) = 3x^2 + 3$

$f'(2) = 3(2)^2 + 3$

$f'(2) = 15$

$$g'(14) = \frac{1}{15}$$