7.2 Quadratics Inequalities in One Variable - Solving Graphically

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7.2 Quadratic Inequalities in One Variable – Solving Graphically

A quadratic inequality with one variable may be in one of the following forms:

$$ax^2 + bx + c > 0 \qquad ax^2 + bx + c \geqslant 0$$

$$ax^2 + bx + c < 0 \qquad ax^2 + bx + c \leq 0$$

Where a, b and c are real numbers and $a \neq 0$.

Example 1: Does $x = -3$ satisfy the following inequalities?

a) $x^2 - 4x - 5 > 0$

$$(-3)^2 - 4(-3) - 5 > 0$$
$$9 + 12 - 5 > 0$$
$$16 > 0$$
$$\text{True}$$

b) $2x^2 + 6x - 1 \geq 0$

$$2(-3)^2 + 6(-3) - 1 \geqslant 0$$
$$18 - 18 - 1 \geqslant 0$$
$$-1 \geqslant 0$$
$$\text{False}$$

Example 2: Solve graphically. Express your final answer in set notation and in interval notation.
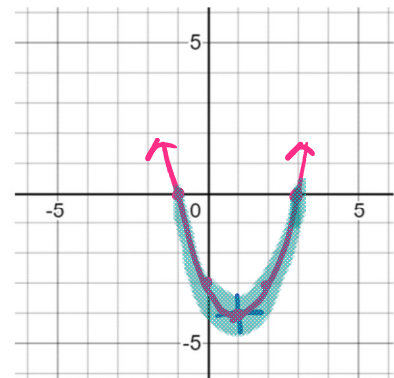
a) $x^2 - 2x - 3 \leq 0$

Graph $y = x^2 - 2x - 3$

$$y = x^2 - 2x + \boxed{1} - 3 - \boxed{1}$$
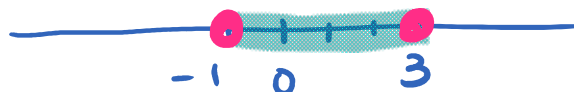
$$\left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$$

$$y = (x-1)^2 - 4$$

$$y = x^2$$

0	0
1	1
2	4

When is the parabola below or on the x-axis

$$\{x \mid -1 \leq x \leq 3 \quad x \in \mathbb{R}\} \quad \text{set notation}$$

$$[-1, 3] \quad \text{interval notation}$$
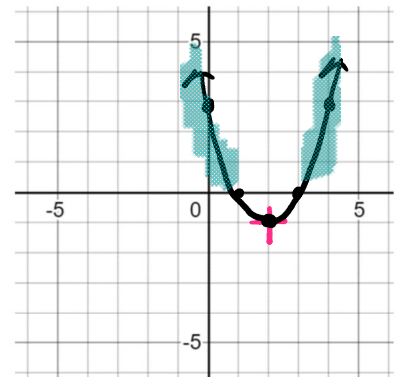
b) $x^2 - 4x > -3$

$$x^2 - 4x + 3 > 0$$

$$y = x^2 - 4x + 3$$

$$y = x^2 - 4x + \boxed{4} + 3 - \boxed{4} \qquad \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$$

$$y = (x-2)^2 - 1$$

0	0
1	1
2	4

When is the parabola above the x-axis

Set Notation $\{x \mid x < 1 \text{ or } x > 3 \; x \in \mathbb{R}\}$

Interval Notation $(-\infty, 1) \cup (3, \infty)$

← union symbol

both parts are part of the final solution

0 1 3

Example 3: Solve graphically. Express your final answer in set notation and in interval notation.

a) $2x^2 - 4x + 2 \leq 0$

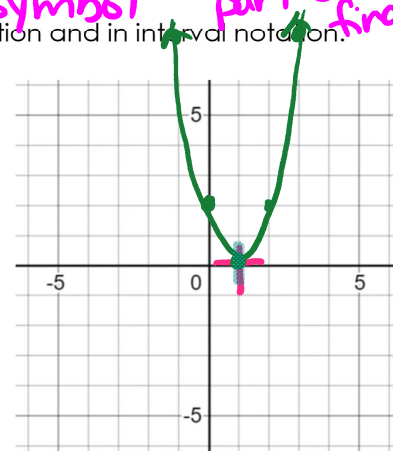$$y = 2x^2 - 4x + 2$$

$$y = 2(x^2 - 2x) + 2$$

$$y = 2(x^2 - 2x + \boxed{1}) + 2 - 2\boxed{1}$$

$$\left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$$

$$y = 2(x-1)^2 + 2 - 2$$

$$y = 2(x-1)^2 + 0$$

0	0	0
1	1	2
2	4	8

When is the parabola below or on the x-axis

At the vertex it is on the x-axis

$\{x \mid x = 1 \; x \in \mathbb{R}\}$ set notation

No interval notation since there is no interval