Pre-Calculus 12

7.2 Transformations of Exponential Functions

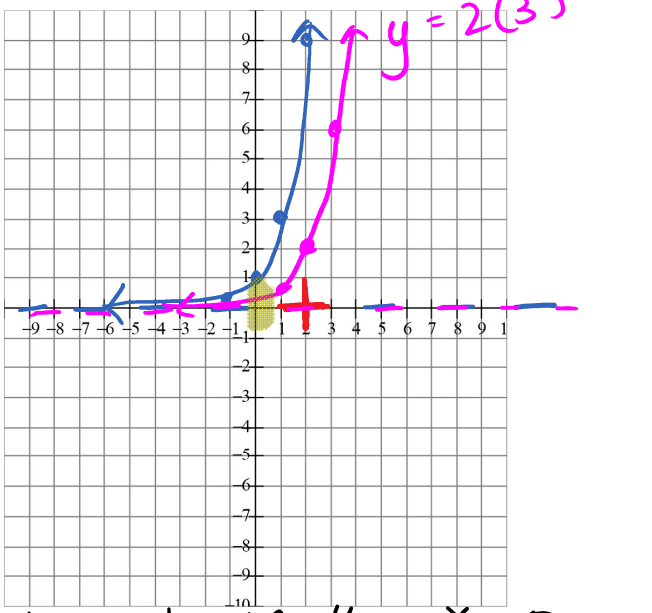
$$y = a(c)^{b(x-h)} + k$$

- a Vertical stretch factor $|a|$, $a<0$ Reflection over x-axis
- b Horizontal stretch factor $|\frac{1}{b}|$, $b<0$ Reflection y-axis
- h Horizontal translation
- k Vertical translation

old $y = 3^x$

Ex. #1: Graph the base function $y = 3^x$ and the transformed function $y = 2(3)^{x-2}$ on the same grid. Describe the transformations.

new $y = 2(3)^{x-2}$



Transformations:

Vertical stretch factor 2

Horizontal translation right 2

$y = 3^x$

-1	$3^{-1} = \frac{1}{3}$
0	$3^0 = 1$
1	$3^1 = 3$
2	$3^2 = 9$

$a = 2$ Mult y's by 2

-1	2/3
0	2
1	6
2	18

Find the intercepts of $y = 2(3)^{x-2}$

No x-intercept

y-intercept Make $x = 0$

$y = 2(3)^{0-2}$

$y = 2(3)^{-2}$

$y = 2(\frac{1}{3^2})$

$y = 2(\frac{1}{9})$

$y = \frac{2}{9}$

$(0, \frac{2}{9})$

$y = 2\left(\frac{1}{3^2}\right)$ $(0, 2/9)$

old

Ex. #2: Graph the base function $y = \left(\frac{1}{2}\right)^x$ and the transformed function $y = -3\left(\frac{1}{2}\right)^{2x} + 4$ on the same grid. State the asymptotes, domain and range of the transformed function.

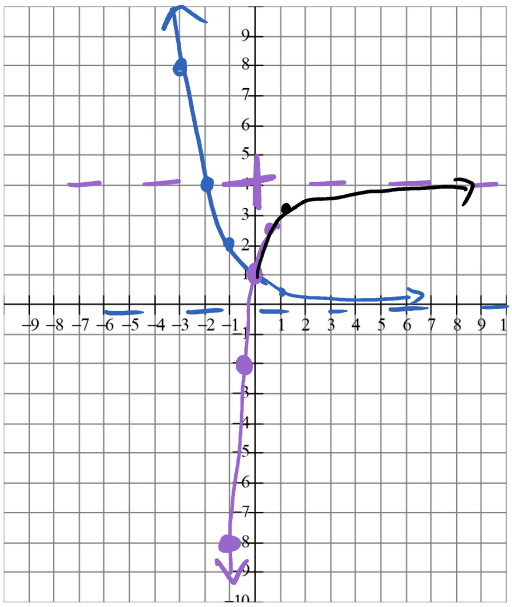
$y = \left(\frac{1}{2}\right)^x$

-3	8
-2	4
-1	$(\frac{1}{2})^{-1} = 2$
0	1
1	$\frac{1}{2}$

$a = -3$ Mult y's by (-3)

$b = 2$ Divide x's by 2

~~-3/2~~	~~-24~~
-1	-12
-1/2	-6
0	-3
1/2	-3/2



Asymptote: $y = 4$

Domain : $\{x \mid x \in \mathbb{R}\}$

Range : $\{y \mid y < 4, y \in \mathbb{R}\}$

Ex. #3: The base equation is $y = 3^x$. Write an equation for the transformed function.

$y = 3^x$

-1	1/3
0	1
1	3
2	9
3	27

Translate vertically UP by 4

Reflection over x-axis $a < 0$

Reflection over y-axis $b < 0$

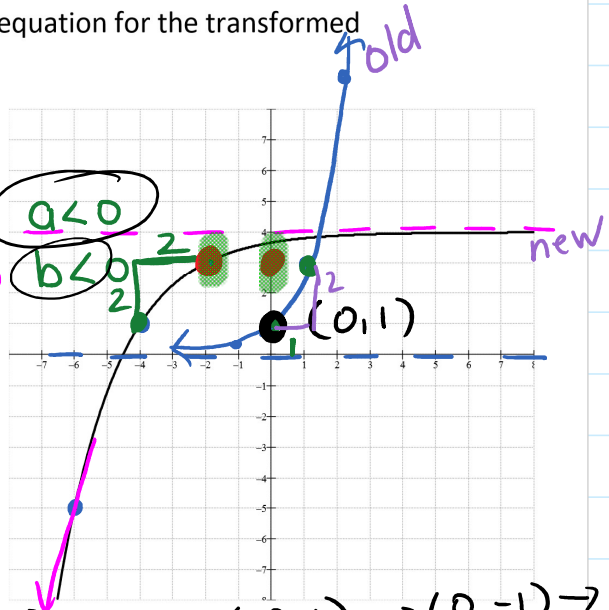
Find b-value

$\frac{\text{old x's}}{b} = \text{new x's}$

$b\left(\frac{1}{b}\right) = (2)b$

$1 = 2b$

$\frac{1}{2} = b$



$(0,1) \rightarrow (0,1) \rightarrow (0,-1) \rightarrow (0,3)$

$b = -\frac{1}{2}$ $a = -1$ $\frac{1}{4}$

$-\frac{1}{2}(x+2)$ left by 2

$\frac{1}{2} = b$

$y = -1(3)^{-\frac{1}{2}(x+2)} + 4$

left by 2

Pre-Calculus 12

Ex. #4: A scientist finds that when you consume caffeine it takes about 4 hours for the amount of caffeine to drop by 40%.

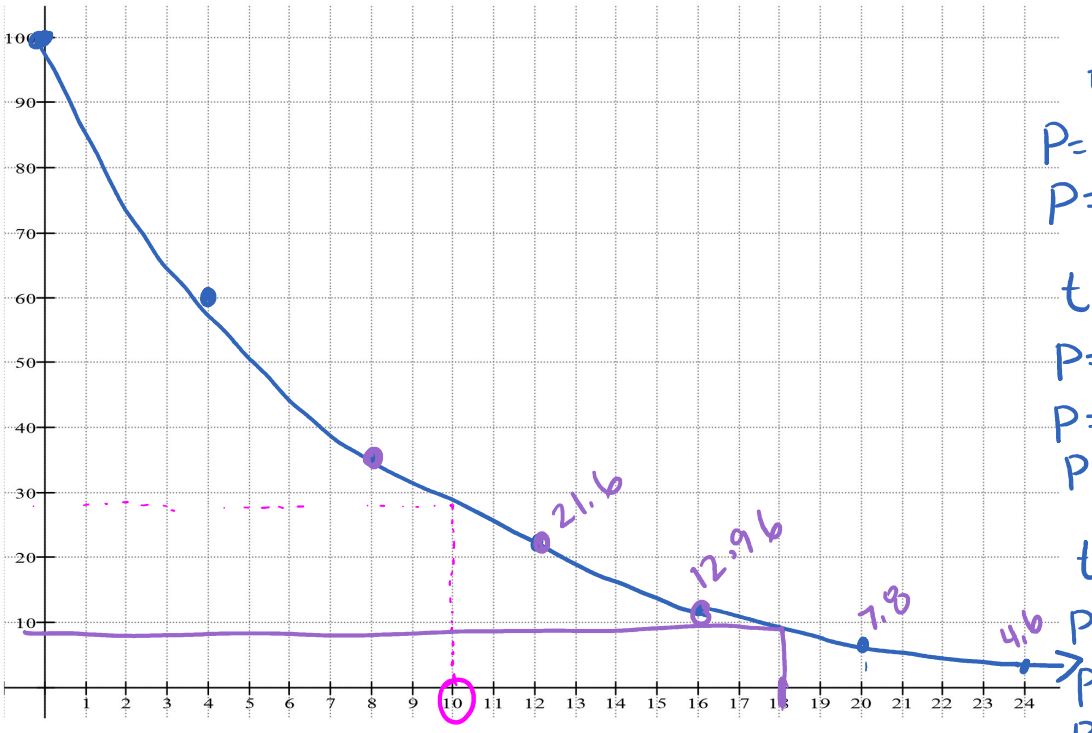
C = rate = 100% − 40% = 60% = 0.6

a) Write an equation to represent the percentage, P, of the caffeine left in the body after time t. Where $P = a(c)^{\frac{t}{T}}$

$P = 100(0.6)^{t/4}$

P = Final % of caffeine
a = Initial % of caffeine
C = rate
t = time
T = How often the rate changes

b) Graph the function.



t = 0
$P = 100(0.6)^0$
P = 100

t = 4
$P = 100(0.6)^{4/4}$
$P = 100(0.6)^1$
P = 60

t = 8
$P = 100(0.6)^{8/4}$
$P = 100(0.6)^2$
P = 100(.36)
P = 36

c) Using the graph estimate the percent of caffeine that remains after 10 hours.

28 %

d) Use your formula to calculate the percent of caffeine that remains after 18 hours.

t = 18

$P = 100(0.6)^{t/4}$

$P = 100(0.6)^{18/4}$

P = 10.04 %