7.3 Quadratic Inequalities in one Variable

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Pre-Calculus 11

7.3 Quadratic Inequalities in One Variable – Solving Algebraically

Quadratic inequalities in one variable can be solved algebraically. There are two methods.

Method 1: Roots and Test Points

Example 1: Use roots and test points to solve the following inequality: $x^2 - 2x - 3 \leq 0$. Express your answer in set notation and interval notation.

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x-3=0 \quad x+1=0$

$x=3 \quad x=-1$

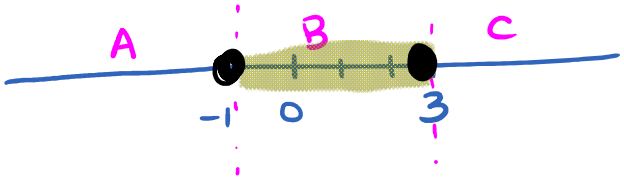
$-x- = -3$

$-+- = -2$

① Solve the roots for the corresponding equation

② Set up a number line use the roots as boundries

③ Test a point in each region to see if it satisfies the inequality



A $x=-2$

$(-2)^2 - 2(-2) - 3 \leq 0$

$4 + 4 - 3 \leq 0$

$5 \leq 0$

False

B $x=0$

$0^2 - 2(0) - 3 \leq 0$

$0 - 0 - 3 \leq 0$

$-3 \leq 0$

True

Region B is part of the solution

$\{x \mid -1 \leq x \leq 3, x \in \mathbb{R}\}$ set notation

$[-1, 3]$ interval notation

C $x=4$

$4^2 - 2(4) - 3 \leq 0$

$16 - 8 - 3 \leq 0$

$4^2 - 2(4) - 3 \leq 0$

$16 - 8 - 3 \leq 0$

$5 \leq 0$ False

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Method 2: Sign Analysis

Example 2: Use sign analysis to solve the following inequality: $-x^2 + x + 12 < 0$. Express your answer in set notation and interval notation.

$-x^2 + x + 12 < 0$

$-1(x^2 - x - 12) < 0$

$-1(x+3)(x-4) < 0$

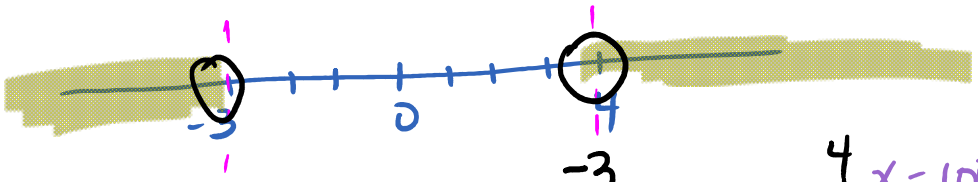
$x \cdot -4 = -12$

$3 + -4 = -1$

① Factor the inequality

② Find the roots and place them on a number line

Roots $x = -3$ $x = 4$



	$x = -100$	$x = 0$	$x = 100$
$-(x+3)$	+	−	−
$(x-4)$	−	−	+
$-(x+3)(x-4)$	−	+	−

$-x^2 + x + 12 < 0$

When is the parabola negative

$\{x \mid x < -3 \text{ or } x > 4 \quad x \in \mathbb{R}\}$

$(-\infty, -3) \cup (4, \infty)$

Example 3: Solve the following inequality algebraically. Use both methods to find a solution. Express your answer in set notation and interval notation.

$$2x^2 - 12x > -10$$

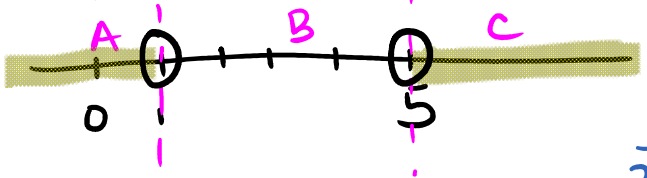
$2x^2-12x+10=0$

$2(x^2-6x+5)=0$

$2(x-1)(x-5)=0$

Root

$x=1$ $x=5$



	x=0	x=2	x=100
2(x-1)	−	+	+
(x-5)	−	−	+
2(x-1)(x-5)	+	−	+

x=1 x=5

$2x^2-12x+10>0$ positive

A $x=0$

$2(0)^2-12(0)>-10$

$0-0>-10$

$0>-10$ T

B $x=2$

$2(2)^2-12(2)>-10$

$8-24>-10$ F

$-16>-10$

C $x=6$

$2(6)^2-12(6)>-10$

$72-72>-10$ T

$0>-10$

$\{x \mid x<1 \text{ or } x>5 \quad x \in \mathbb{R}\}$

$(-\infty,1) \cup (5,\infty)$

$\{x \mid x<1 \text{ or } x>5 \quad x \in \mathbb{R}\}$

$(-\infty,1) \cup (5,\infty)$