7.3 Pg 362 Pt 1

1. $\log_3 27$
$= \log_3 3^3$
$= 3 \log_3 3$
$= 3$

3. $\ln 1$
$= \log_e 1$
$= 0$

if $m = \log_e 1$
$e^m = 1$
$m = 0$

5. $\log_2 2^{5/3}$
$= \frac{5}{3} \log_2 2$
$= \frac{5}{3}$

7. $\log_{64} 4$
$m = \log_{64} 4$
$64^m = 4$
$(4^3)^m = 4$
$4^{3m} = 4^1$
$3m = 1$
$m = \frac{1}{3}$

9. $\log_8 2 + \log_4 2$
$\frac{\log_2 2}{\log_2 8} + \frac{\log_2 2}{\log_2 4}$
$\frac{1}{3} + \frac{1}{2}$
$\frac{2}{6} + \frac{3}{6} = 5/6$

11. $\log_4 48 - \log_4 12$
$= \log_4 \frac{48}{12}$
$= \log_4 4$
$= 1$

13 $\ln(e^3) + \ln(e^4)$
$= 3 \ln e + 4 \ln e$
$= 3 + 4$
$= 7$

15. $7^{\log_7(29)}$
$= 29$

19. $7e^{5t} = 100$
$e^{5t} = \frac{100}{7}$
$\ln e^{5t} = \ln 100/7$
$5t = \ln(100/7)$
$t = \frac{\ln(100/7)}{5}$

21. $2^{x^2-2x} = 8$

$2^{x^2-2x} = 2^3$

$x^2 - 2x = 3$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3 \quad x = -1$

23. $\ln(x^4) - \ln(x^2) = 2$

$\ln\left(\frac{x^4}{x^2}\right) = 2$

$\ln x^2 = 2$

$e^2 = x^2$

$x = e$

27. $P(t) = 2.4e^{0.06t}$

$4.8 = 2.4e^{0.06t}$

$2 = e^{0.06t}$

$\ln 2 = 0.06t \ln e$

$\frac{\ln 2}{0.06} = t$

$t = 11.55$ years

29. $y = x \ln x$

$y' = 1 \cdot \ln x + x \cdot \frac{1}{x}$

$y' = \ln x + 1$

31. $y = (\ln x)^2$

$y' = 2 \ln x \left(\frac{1}{x}\right)$

$y' = \frac{2 \ln x}{x}$

33. $y = \ln(9x^2 - 8)$

$y' = \frac{1}{9x^2-8} \cdot 18x$

$y' = \frac{18x}{9x^2-8}$

35. $y = \ln(\sin t + 1)$

$y' = \frac{1}{\sin t + 1} \cdot \cos t$

$y' = \frac{\cos t}{\sin t + 1}$

37. $y = \frac{\ln x}{x}$

$y' = \frac{x\left(\frac{1}{x}\right) - \ln x(1)}{x^2}$

$= \frac{1 - \ln x}{x^2}$

39. $y = \ln(\ln x)$

$$y' = \frac{1}{\ln x} \cdot \frac{d}{dx}(\ln x)$$

$$y' = \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$y' = \frac{1}{x \ln x}$$

41. $y = (\ln(\ln x))^3$ chain Rule. (3 functions)

$$y' = 3(\ln(\ln x))^2 \cdot \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$y' = \frac{3(\ln(\ln x))^2}{x \ln x}$$

43. $y = \ln((x+1)(2x+9))$

$$y' = \frac{1}{(x+1)(2x+9)} \cdot \left[(x+1)(2) + (1)(2x+9)\right]$$

$$y' = \frac{2x+2+2x+9}{(x+1)(2x+9)}$$

$$y' = \frac{4x+11}{(x+1)(2x+9)}$$

45. $y = 11^x$

$$y' = \ln(11) \cdot 11^x$$

$$y' = 11^x \ln(11)$$

47. $y = \dfrac{2^x - 3^{-x}}{x}$

$$y' = \frac{x(2^x \ln 2 - 3^{-x}\ln(3)(-1)) - (1)(2^x - 3^{-x})}{x^2}$$

$$y' = \frac{x(2^x \ln 2 + 3^{-x}\ln 3) - 2^x + 3^{-x}}{x^2}$$

49. $f(x) = \log_2 x$

$$f'(x) = \frac{1}{\ln 2\, x}$$

51. $f(t) = \log_3(\sin t)$

$$f'(t) = \frac{1}{\ln 3\,(\sin t)} \cdot \cos t = \frac{\cot t}{\ln 3}$$

53. $f(x) = 6^x \qquad x = 2$

$f(2) = 6^2 = 36 \qquad (2, 36)$

$f'(x) = 6^x \ln(6)$

$f'(2) = 6^2 \ln(6) = 36\ln 6$

$y - 36 = 36\ln 6\,(t-2)$

$y = 36\ln 6\,(t-2) + 36$

59. $s(t) = \ln(8-4t) \qquad t = 1$

$s(1) = \ln(8-4) = \ln(4)$

$(1, \ln(4))$

$s'(t) = \dfrac{1}{8-4t} \cdot -4$

$s'(1) = \dfrac{-4}{8-4} = -\dfrac{1}{1}$

$y - \ln 4 = -1(t-1)$

$y = -1(t-1) + \ln 4$

61. $R(z) = \log_5(2z^2+7)$ $\quad z=3$

$$R'(z) = \frac{1}{\ln 5} \cdot \frac{1}{2z^2+7} \cdot 4z$$

$$R'(3) = \frac{1}{\ln 5} \cdot \frac{1}{2(3)^2+7} \cdot 4(3) = \frac{1}{\ln 5} \cdot \frac{1}{25} \cdot 12$$

$$R(3) = \log_5(2(3)^2+7) \qquad (3,2) \qquad m = \frac{12}{25\ln 5}$$

$$= \log_5(25)$$

$$= 2$$

$$y-2 = \frac{12}{25\ln 5}(x-3)$$

$$y = \frac{12}{25\ln 5}(x-3)+2$$

63. $f(w) = \log_2 w \qquad w = \frac{1}{8}$

$$f'(w) = \frac{1}{\ln 2} \cdot \frac{1}{w}$$

$$f'\left(\frac{1}{8}\right) = \frac{1}{\ln 2} \cdot \frac{1}{\frac{1}{8}}$$

$$= \frac{8}{\ln 2}$$

$$f\left(\frac{1}{8}\right) = \log_2\left(\frac{1}{8}\right)$$

$$= \log_2 8^{-1}$$

$$= -\log_2 8$$

$$= -\log_2 2^3$$

$$= -3$$

$$\left(\frac{1}{8}, -3\right)$$

$$y-(-3) = \frac{8}{\ln 2}\left(x-\frac{1}{8}\right)$$

$$y = \frac{8}{\ln 2}\left(x-\frac{1}{8}\right)-3$$

67. $y = (x-1)(x-12)(x+7)$

$\ln y = \ln[(x-1)(x-12)(x+7)]$

$\ln y = \ln(x-1) + \ln(x-12) + \ln(x+7)$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x-1} + \frac{1}{x-12} + \frac{1}{x+7}$

$\frac{dy}{dx} = y\left[\frac{(x-12)(x+7) + (x-1)(x+7) + (x-1)(x-12)}{(x-1)(x-12)(x+7)}\right]$

$\frac{dy}{dx} = y\left[\frac{x^2-5x-84 + x^2+6x-7 + x^2-13x+12}{(x-1)(x-12)(x+7)}\right]$

$\frac{dy}{dx} = (x-1)(x-12)(x+7)\left[\frac{3x^2-12x-79}{(x-1)(x-12)(x+7)}\right]$

$\frac{dy}{dx} = 3x^2 - 12x - 79$

69. $y = \frac{x(x^2+1)}{\sqrt{x+1}}$

$\ln y = \ln\left[\frac{x(x^2+1)}{\sqrt{x+1}}\right]$

$\ln y = \ln x + \ln(x^2+1) - \ln(x+1)^{1/2}$

$\ln y = \ln x + \ln(x^2+1) - \frac{1}{2}\ln(x+1)$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{1}{x^2+1}(2x) - \frac{1}{2}\cdot\frac{1}{x+1}$

$\frac{dy}{dx} = y\left[\frac{1}{x} + \frac{2x}{x^2+1} - \frac{1}{2(x+1)}\right]$

$\frac{dy}{dx} = \frac{x(x^2+1)}{\sqrt{x+1}}\left[\frac{1}{x} + \frac{2x}{x^2+1} - \frac{1}{2(x+1)}\right]$

73. $f(x) = x^{3x}$

$y = x^{3x}$

$\ln y = \ln x^{3x}$

$\ln y = 3x \ln x$

product rule

$\frac{1}{y}\frac{dy}{dx} = 3 \cdot \ln x + 3x\left(\frac{1}{x}\right)$

$\frac{dy}{dx} = y(3\ln x + 3)$

$\frac{dy}{dx} = x^{3x}(3\ln x + 3)$

75. $f(x) = x^{e^x}$

$y = x^{e^x}$

$\ln y = \ln x^{e^x}$

$\ln y = e^x \ln x$

$\frac{1}{y}\frac{dy}{dx} = e^x \cdot \ln x + e^x \cdot \frac{1}{x}$

$\frac{dy}{dx} = y\left[e^x \ln x + \frac{e^x}{x}\right]$

$\frac{dy}{dx} = x^{e^x}\left(e^x \ln x + \frac{e^x}{x}\right)$

77. $f(x) = x^{3^x}$

$y = x^{3^x}$

$\ln y = \ln x^{3^x}$

$\ln y = 3^x \ln x$

$\frac{1}{y}\frac{dy}{dx} = 3^x \ln 3 \cdot \ln x + 3^x \cdot \frac{1}{x}$

$\frac{dy}{dx} = y\left(3^x (\ln 3)(\ln x) + \frac{3^x}{x}\right)$

$\frac{dy}{dx} = x^{3^x} \cdot 3^x \left((\ln 3)(\ln x) + \frac{1}{x}\right)$