7.3 Part 2

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7.3 Part 2 Integration and Logs

1. Find the absolute maximum of $f(x) = \frac{\ln x}{x}$ $\ln x$ if $x > 0$

$f'(x) = \frac{x \cdot (\frac{1}{x}) - (1) \cdot \ln x}{x^2}$

$f'(x) = \frac{1 - \ln x}{x^2}$

$f'(x) = 0$

$1 - \ln x = 0$

$1 = \ln x$

$1 = \log_e x$

$e^1 = x$

$e = x$

$f'(x)$ undefined

$x^2 = 0$

$x = 0$

$(0, e)$ (e, ∞)

$x = 1$ $x = e^2$

$+$ $-$

$f(e) = \frac{\ln e}{e}$

$f(e) = \frac{1}{e}$

Max $(e, \frac{1}{e})$

Recall

$$\frac{d}{dx}\ln x = \frac{1}{x} \quad and \quad \frac{d}{dx}\ln u = \frac{1}{u}\frac{du}{dx}$$

$$\int \frac{1}{x}\,dx = \ln|x| + c$$

$$\int \frac{1}{u}\,du = \ln|u| + c$$

2. $\int \frac{2}{x}\,dx$

$= 2\int \frac{1}{x}\,dx$

$= 2\ln|x| + C$

$= \ln x^2 + C$

3. $\int \frac{1}{4x-1}\,dx$

$u = 4x-1$

$\frac{du}{dx} = 4$

$\frac{du}{4} = dx$

$= \int \frac{1}{u} \cdot \frac{du}{4}$

$= \frac{1}{4}\int \frac{1}{u}\,du$

$= \frac{1}{4}\ln|u| + C$

$= \frac{1}{4}\ln|4x-1| + C$

4. $\int \frac{3x^2+1}{x^3+x}\,dx$

$u = x^3+x$

$du = (3x^2+1)\,dx$

$= \int \frac{1}{u} \cdot du$

$= \ln|u| + C$

$= \ln|x^3+x| + C$

5. $\int \tan x\,dx$

$= \int \frac{\sin x\,dx}{\cos x}$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$\frac{du}{-1} = \sin x\,dx$

$= \int \frac{1}{u} \cdot \frac{du}{-1}$

$= -\int \frac{1}{u}\,du$

$= -\ln|u| + C$

$= -\ln|\cos x| + C$

6. $\int_0^3 \frac{x}{x^2+1}dx$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x\,dx$

$x = 0$
$u = 0^2+1$
$u = 1$

$x = 3$
$u = 3^2+1$
$u = 10$

$$= \int_1^{10} \frac{1}{u} \cdot \frac{du}{2}$$

$$= \frac{1}{2}\int_1^{10} \frac{1}{u}\,du$$

$$= \frac{1}{2}\ln|u|\Big|_1^{10}$$

$$= \frac{1}{2}\ln 10 - \frac{1}{2}\ln 1$$

$$= \frac{1}{2}\ln 10 - 0$$

$$= \frac{1}{2}\ln 10 = \ln 10^{\frac{1}{2}} = \ln\sqrt{10}$$

Recall

$$\frac{d}{dx}b^u = b^u \ln b \frac{du}{dx}$$

$$\int \frac{d}{dx}b^u = \int b^u \ln b \frac{du}{dx}$$

$$b^u = \ln b \int b^u \frac{du}{dx}$$

$$\frac{b^u}{\ln b} = \int b^u \frac{du}{dx}$$

$$\int b^u\,du = \frac{b^u}{\ln b} + C$$

7. $\int 5^{2x}\,dx$

$u = 2x$

$\frac{du}{dx} = 2$

$\frac{du}{2} = dx$

$= \int 5^u \cdot \frac{du}{2}$

$= \frac{1}{2}\int 5^u\,du$

$= \frac{1}{2}\cdot\frac{5^u}{\ln 5} + C$

$= \frac{5^{2x}}{2\ln 5} + C$ $\quad = \frac{5^{2x}}{\ln 25} + C$

8. $\int 6^{x^2+1}x\,dx$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x\,dx$

$= \int 6^u \cdot \frac{du}{2}$

$= \frac{1}{2}\int 6^u\,du$

$= \frac{1}{2}\cdot\frac{6^u}{\ln 6} + C$

$= \frac{6^{x^2+1}}{2\ln 6} + C$

$= \frac{6^{x^2+1}}{\ln 36} + C$