Pg 363 7.3 Part 2

79. $g(x) = \frac{\ln x}{x}$

$g'(x) = \frac{x\left(\frac{1}{x}\right) - (1)\ln x}{x^2}$

$g'(x) = \frac{1 - \ln x}{x^2}$

$g''(x) = \frac{x^2\left(-\frac{1}{x}\right) - 2x(1 - \ln x)}{(x^2)^2}$

$= \frac{-x - 2x + 2x\ln x}{x^4}$

$= \frac{-3x + 2x\ln x}{x^4}$

$g''(x) = \frac{-3 + 2\ln x}{x^3}$

$g'(x) = 0 \quad 1 - \ln x = 0$

$1 = \ln x$

$e^1 = x$

$x = e$

$g'(x)$ undefined if $x = 0$

but $g(x)$ not defined at $x = 0$

$g''(e) = \frac{-3 + 2\ln e}{e^3}$

$= \frac{-3 + 2}{e^3}$

$g''(e) < 0$ concave down

$g(e)$ Maximum

81. $g(x) = \frac{\ln x}{x^3}$

$g'(x) = \frac{x^3\left(\frac{1}{x}\right) - 3x^2(\ln x)}{(x^3)^2}$

$= \frac{x^2 - 3x^2\ln x}{x^6}$

$= \frac{x^2(1 - 3\ln x)}{x^6}$

$= \frac{(1 - 3\ln x)}{x^4}$

$g'(x) = 0 \quad 1 - 3\ln x = 0$

$1 = 3\ln x \quad x = e^{1/3}$

$g''(x) = \frac{x^4\left(-3\frac{1}{x}\right) - 4x^3(1 - 3\ln x)}{x^8}$

$= \frac{-3x^3 - 4x^3(1 - 3\ln x)}{x^8}$

$= \frac{-3 - 4 + 12\ln x}{x^5}$

$= \frac{-7 + 12\ln x}{x^5}$

$g''(e^{1/3}) = \frac{-7 + 12\ln(e^{1/3})}{(e^{1/3})^5}$

$= \frac{-7 + 4}{e^{5/3}} < 0$

Max

83. $f(x) = \frac{10\ln x}{x^2}$ $[1, 4]$

$$f'(x) = \frac{x^2(10\cdot\frac{1}{x}) - 2x(10\ln x)}{x^4}$$

$$= \frac{10x - 20x\ln x}{x^4}$$

$$= \frac{10(1-2\ln x)}{x^3}$$

$f'(x) = 0$

$0 = 1 - 2\ln x$

$2\ln x = 1$

$\ln x = \frac{1}{2}$

$e^{1/2} = x$

$$f''(x) = \frac{x^3(10)(-\frac{2}{x}) - 3x^2\cdot 10(1-2\ln x)}{x^6}$$

$$= \frac{-20x^2 - 30x^2 + 60x^2\ln x}{x^6}$$

$$= \frac{-50 + 60\ln x}{x^4}$$

$f''(x) = 0$

$-50 + 60\ln x = 0$

$60\ln x = 50$

$\ln x = \frac{5}{6}$

$e^{5/6} = x$

$$f''(e^{1/2}) = \frac{-50 + 60\ln(e^{1/2})}{(e^{1/2})^4}$$

$$= \frac{-50 + 60(\frac{1}{2})(1)}{e^2}$$

$$= \frac{-20}{e^2} < 0$$ Max at $x = e^{1/2}$

$(1, e^{5/6})$ $(e^{5/6}, 4)$

$x = e^{1/2}$ $x = e^1$

sign $f''(x)$ $-$ $+$

inflection point at $x = e^{5/6}$

83 continued

$f(1) = \frac{10\ln(1)}{1^2} = 0$

$f(4) = \frac{10\ln(4)}{4^2} \approx .866$

$f(e^{1/2}) = \frac{10\ln(e^{1/2})}{(e^{1/2})^2}$

$= \frac{10(1/2)}{e}$

$= 5/e$

≈ 1.84

$(1.65, 1.84)$

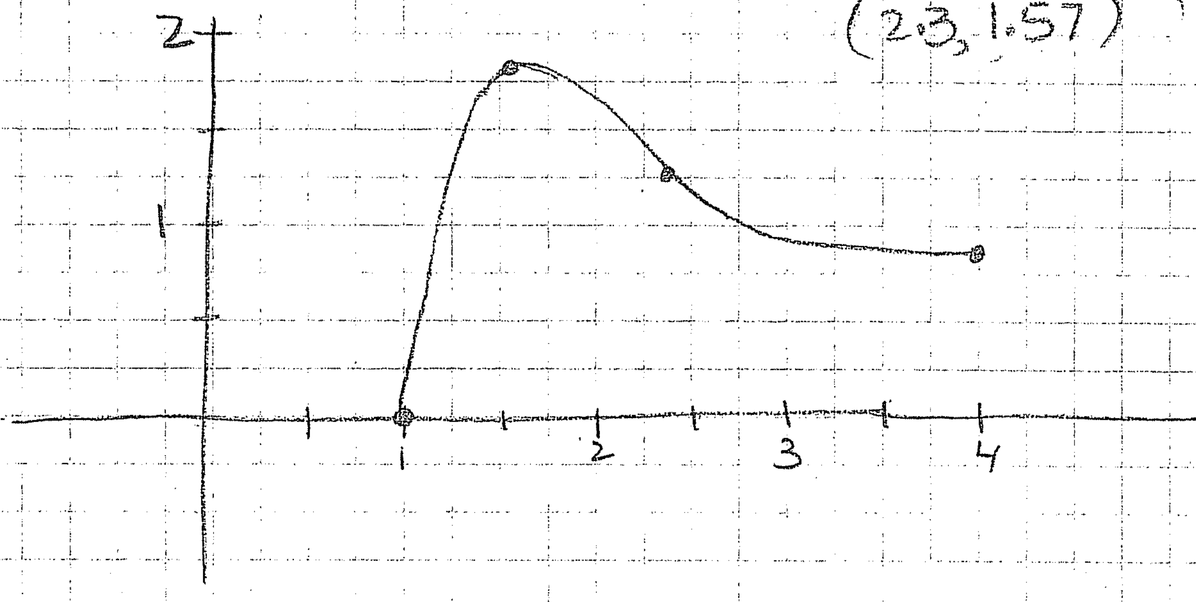
$f(e^{5/6}) = \frac{10\ln(e^{5/6})}{(e^{5/6})^2}$

$= \frac{10(5/6)}{e^{5/3}}$

$= \frac{25}{3e^{5/3}}$

≈ 1.57

$(2.3, 1.57)$



85. $\int \frac{7dx}{x}$

$= 7\int \frac{1}{x}\,dx$

$= 7\ln|x| + C$

87. $\int \frac{dx}{2x+4} = \int \frac{1}{u}\,\frac{du}{2}$

$u = 2x+4$ $\quad = \frac{1}{2}\int \frac{1}{u}\,du$

$\frac{du}{dx} = 2$ $\quad = \frac{1}{2}\ln|u| + C$

$\frac{du}{2} = dx$ $\quad = \frac{1}{2}\ln|2x+4| + C$

$= \ln\sqrt{2x+4} + C$

89. $\int \frac{t\,dt}{t^2+4} = \int \frac{1}{u}\cdot\frac{du}{2}$

$u = t^2+4$ $\quad = \frac{1}{2}\int \frac{1}{u}\,du$

$\frac{du}{dt} = 2t$ $\quad = \frac{1}{2}\ln|u| + C$ $\quad t^2+4 \geq 0$ always

$\frac{du}{2} = t\,dt$ $\quad = \frac{1}{2}\ln|t^2+4| + C$

$= \frac{1}{2}\ln(t^2+4) + C$

$= \ln\sqrt{t^2+4} + C$

91. $\int \frac{(3x-1)}{9-2x+3x^2}\,dx = \int \frac{1}{u}\,\frac{du}{2}$

$u = 9-2x+3x^2$ $\qquad = \frac{1}{2}\int \frac{1}{u}\,du$

$\frac{du}{dx} = -2+6x$ $\qquad = \frac{1}{2}\ln|u| + C$

$du = 2(-1+3x)\,dx$

$\frac{du}{2} = (3x-1)\,dx$ $\qquad = \frac{1}{2}\ln|9-2x+3x^2| + C$

$\qquad = \ln\sqrt{9-2x+3x^2} + C$

93. $\int \cot x\,dx$

$= \int \frac{\cos x}{\sin x}\,dx = \int \frac{1}{u}\,du$

$= \ln|u| + C$

$u = \sin x$

$\frac{du}{dx} = \cos x$ $\qquad = \ln|\sin x| + C$

$du = \cos x\,dx$

95. $\int \frac{\ln x}{x}\,dx = \int u\,du$

$u = \ln x$ $\qquad = \frac{1}{2}u^2 + C$

$du = \frac{1}{x}\,dx$ $\qquad = \frac{1}{2}(\ln x)^2 + C$

$\qquad = \frac{1}{2}\ln^2 x + C$

103. $\int x 3^{x^2} dx = \int 3^u \cdot \frac{du}{2}$

$u = x^2$ $\qquad = \frac{1}{2}\int 3^u \, du$

$\frac{du}{dx} = 2x$ $\qquad = \frac{1}{2} \cdot 3^u \cdot \frac{1}{\ln 3} + C$

$\frac{du}{2} = x\,dx$

$= \frac{3^{x^2}}{2\ln 3} + C$

105. $\int \left(\frac{1}{2}\right)^{3x+2} dx = \int \left(\frac{1}{2}\right)^u \cdot \frac{du}{3}$

$u = 3x + 2$ $\qquad = \frac{1}{3}\int \left(\frac{1}{2}\right)^u du$

$\frac{du}{dx} = 3$ $\qquad = \frac{1}{3}\left(\frac{1}{2}\right)^u \cdot \frac{1}{\ln 1/2} + C$

$\frac{du}{3} = dx$

$\ln\frac{1}{2} = -\ln 2$ $\qquad = \left(\frac{1}{2}\right)^{3x+2} \cdot \frac{1}{3\ln 1/2} + C$

$= \frac{-1}{3\ln 2}\left(\frac{1}{2}\right)^{3x+2} + C$

107. $\int_4^{12} \frac{1}{x}\, dx$

$= \ln|x| \Big]_4^{12}$

$= \ln 12 - \ln 4$

$= \ln\left(\frac{12}{4}\right)$

$= \ln 3$

109. $\int_2^4 \frac{dt}{3t+4} = \int_{10}^{16} \frac{1}{u} \frac{du}{3}$

$u = 3t + 4$

$\frac{du}{dt} = 3$

$\frac{du}{3} = dt$

$t = 2$
$u = 3(2)+4$
$u = 10$

$t = 4$
$u = 3(4)+4$
$u = 16$

$= \frac{1}{3} \ln|u| \Big|_{10}^{16}$

$= \frac{1}{3}\left[\ln(16) - \ln(10)\right]$

$= \frac{1}{3}\ln\left(\frac{16}{10}\right)$

$= \frac{1}{3}\ln\left(\frac{8}{5}\right)$

111. $\int_e^{e^2} \frac{1}{t \ln t}\, dt = \int_1^2 \frac{1}{u}\, du$

$u = \ln t$

$\frac{du}{dt} = \frac{1}{t}$

$du = \frac{1}{t} dt$

$t = e$
$u = \ln e$
$u = 1$

$t = e^2$
$u = \ln e^2$
$u = 2$

$= \ln|u| \Big]_1^2$

$= \ln(2) - \ln(1)$

$= \ln 2 - 0$

$= \ln 2.$