7.7 Pg 388

1. $\lim_{x\to 3} \frac{2x^2-5x-3}{x-4}$

$= \frac{2(3)^2-5(3)-3}{3-4}$

$= \frac{18-15-3}{-1}$

$= \frac{0}{-1}$

$= 0$ L' Hopital does not apply

3. $\lim_{x\to 4} \frac{x^3-64}{x^2+16}$

$= \frac{64-64}{16+16}$

$= \frac{0}{32}$ L'Hopital does not apply

$= 0$

5. $\lim_{x\to 9} \frac{x^{1/2}+x-6}{x^{3/2}-27}$

$= \frac{\sqrt{9}+9-6}{(\sqrt[2]{9})^3-27}$

$= \frac{3+9-6}{27+27}$

$= \frac{6}{0}$ L' Hopital does not apply

$= \infty$

7. $\lim_{x\to 0} \frac{\sin 4x}{x^2+3x+1}$

$= \frac{\sin 0}{0^2+0+1}$

$= \frac{0}{1}$ L' Hopital does not apply

$= 0$

9. $\lim_{x\to 0} \frac{\cos 2x-1}{\sin 5x}$

$= \frac{\cos 0-1}{\sin 0} = \frac{1-1}{0} = \frac{0}{0}$

indeterminate form use L' Hopital

$\lim_{x\to 0} \frac{d/dx\,(\cos 2x-1)}{d/dx\,\sin 5x}$

$= \lim_{x\to 0} \frac{2(-\sin 2x)}{5\cos 5x}$

$= \frac{-2\sin 0}{5\cos 0}$

$= \frac{-2(0)}{5(1)} = 0$

11. $\lim_{x \to \infty} \frac{9x+4}{3-2x}$

$= \frac{\infty}{-\infty}$

$\lim_{x \to \infty} \frac{d/dx(9x+4)}{d/dx(3-2x)}$

$\lim_{x \to \infty} \frac{9}{-2} = -9/2$

13. $\lim_{x \to \infty} \frac{\ln x}{x^{1/2}}$

$= \frac{\infty}{\infty}$

$\lim_{x \to \infty} \frac{d/dx(\ln x)}{d/dx(x^{1/2})}$

$\lim_{x \to \infty} \frac{1/x}{1/2\, x^{-1/2}}$

$\lim_{x \to \infty} \frac{2x^{1/2}}{x^1}$

$\lim_{x \to \infty} \frac{2}{x^{1/2}}$

$= \frac{2}{\infty}$

$= 0$

15. $\lim_{x \to -\infty} \frac{\ln(x^4+1)}{x}$

$= \frac{\ln((-\infty)^4+1)}{-\infty}$

$= \frac{\ln(\infty)}{-\infty}$

$= \frac{\infty}{-\infty}$

$\lim_{x \to -\infty} \frac{d/dx \ln(x^4+1)}{d/dx\, x}$

$\lim_{x \to -\infty} \frac{\frac{1}{x^4+1} \cdot 4x^3}{1}$

$= \lim_{x \to -\infty} \frac{d/dx\, 4x^3}{d/dx\, x^4+1}$

$= \lim_{x \to -\infty} \frac{12x^2}{4x^3}$

$= \lim_{x \to -\infty} \frac{3}{x} = 0$

17. $\lim_{x \to 1} \dfrac{\sqrt{8+x} - 3x^{\frac{1}{3}}}{x^2 - 3x + 2}$

$= \dfrac{\sqrt{8+1} - 3(1)^{1/3}}{1^2 - 3(1) + 2}$

$= \dfrac{3-3}{1-3+2} = \dfrac{0}{0}$

$\lim_{x \to 1} \dfrac{d/dx \ \sqrt{8+x} - 3x^{1/3}}{d/dx \ x^2 - 3x + 2}$

$= \lim_{x \to 1} \dfrac{\frac{1}{2}(8+x)^{-1/2} - 3(\frac{1}{3})x^{-\frac{2}{3}}}{2x - 3}$

$= \dfrac{\frac{1}{2} \cdot \frac{1}{\sqrt{9}} - (1)^{-2/3}}{2(1) - 3}$

$= \dfrac{\frac{1}{6} - 1}{-1}$

$= \dfrac{-5/6}{-1} = 5/6$

19. $\lim_{x \to -\infty} \dfrac{3x - 2}{1 - 5x}$

$= \dfrac{-\infty}{\infty}$

$\lim_{x \to -\infty} \dfrac{d/dx \ 3x - 2}{d/dx \ 1 - 5x}$

$= \lim_{x \to -\infty} \dfrac{3}{-5}$

$= -3/5$

25. $\lim_{x \to 0} \dfrac{\sin 2x}{\sin 7x}$

$= \dfrac{\sin 0}{\sin 0}$

$= \dfrac{0}{0}$

$\lim_{x \to 0} \dfrac{d/dx \ \sin 2x}{d/dx \ \sin 7x}$

$= \lim_{x \to 0} \dfrac{(\cos 2x)(2)}{(\cos 7x)(7)}$

$= \dfrac{(\cos 0)(2)}{(\cos 0)(7)}$

$= \dfrac{2}{7}$

33. $\lim_{x \to \pi/2} \dfrac{\cos x}{\sin 2x}$

$= \lim_{x \to \pi/2} \dfrac{\cos x}{2 \sin x \cos x}$

$= \lim_{x \to \pi/2} \dfrac{1}{2 \sin x}$

$= \dfrac{1}{2 \sin \pi/2}$

$= \dfrac{1}{2(1)} = \dfrac{1}{2}$

40. $\lim\limits_{x\to 1} \dfrac{e^x - e}{\ln x}$

$= \dfrac{e - e}{\ln(1)}$

$= \dfrac{0}{0}$

$\lim\limits_{x\to 1} \dfrac{d/dx\,(e^x - e)}{d/dx\,\ln x}$

$= \lim\limits_{x\to 1} \dfrac{e^x}{1/x}$

$= \dfrac{e^1}{1} = e$

46. $\lim\limits_{x\to\infty} x^{\frac{1}{x^2}}$

$= \infty^{\frac{1}{\infty}}$

$y = \lim\limits_{x\to\infty} x^{\frac{1}{x^2}}$

$\ln y = \ln\left[\lim\limits_{x\to\infty} x^{\frac{1}{x^2}}\right]$

$\ln y = \lim\limits_{x\to\infty} \ln\left(x^{\frac{1}{x^2}}\right)$

$\ln y = \lim\limits_{x\to\infty} \frac{1}{x^2} \ln x$

$\ln y = \lim\limits_{x\to\infty} \dfrac{d/dx\,\ln x}{\frac{d}{dx}\,x^2}$

$\ln y = \lim\limits_{x\to\infty} \dfrac{\frac{1}{x}}{2x}$

$\ln y = \lim\limits_{x\to\infty} \dfrac{1}{x} \cdot \dfrac{1}{2x}$

$\ln y = \lim\limits_{x\to\infty} \dfrac{1}{2x^2}$

$\ln y = 0$

$e^0 = y$

$y = 1$

49. $\lim_{x \to 0} (\cos x)^{3/x^2}$

$y = \lim_{x \to 0} (\cos x)^{3/x^2}$

$\ln y = \ln \left[\lim_{x \to 0} (\cos x)^{3/x^2} \right]$

$\ln y = \lim_{x \to 0} \ln (\cos x)^{3/x^2}$

$\ln y = \lim_{x \to 0} \frac{3}{x^2} \ln(\cos x)$

$\ln y = \lim_{x \to 0} \frac{d/dx \; 3\ln(\cos x)}{d/dx \; x^2}$

$\ln y = 3 \lim_{x \to 0} \frac{\frac{1}{\cos x} \cdot (-\sin x)}{2x}$

$\ln y = 3 \lim_{x \to 0} -\frac{\tan x}{2x}$ use l'hopital again

$\ln y = 3 \lim_{x \to 0} \frac{d/dx\,(-\tan x)}{d/dx\,(2x)}$

$\ln y = 3 \lim_{x \to 0} -\frac{\sec^2 x}{2}$

$\ln y = 3 \lim_{x \to 0} \frac{-1}{2\cos^2 x}$

$\ln y = 3\left(-\frac{1}{2}\right)$

$\ln y = -\frac{3}{2}$ $\qquad e^{-3/2} = y$