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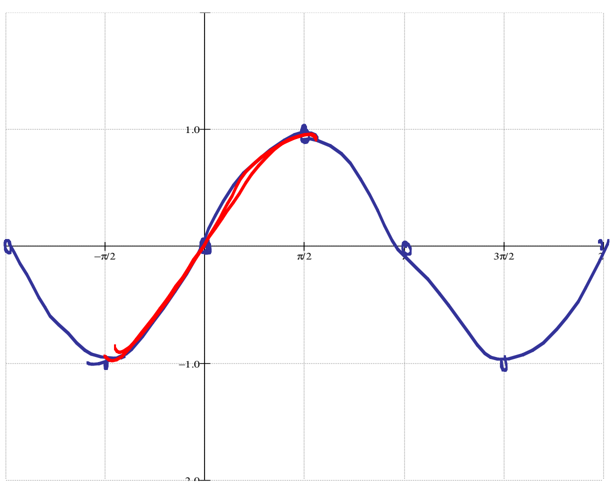
Inverse Trig Functions

The inverse of a function $f^{-1}(x)$ exists only if $f(x)$ is one to one.

<u>Consider $y = \sin x$</u>

$y = \sin x$ is a periodic function. The inverse will not be a function unless we restrict the domain.



Domain $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

Range $-1 \le x \le 1$

x	y
$-\pi/2$	-1
0	0
$\pi/2$	1

The inverse of $y = \sin x$ is

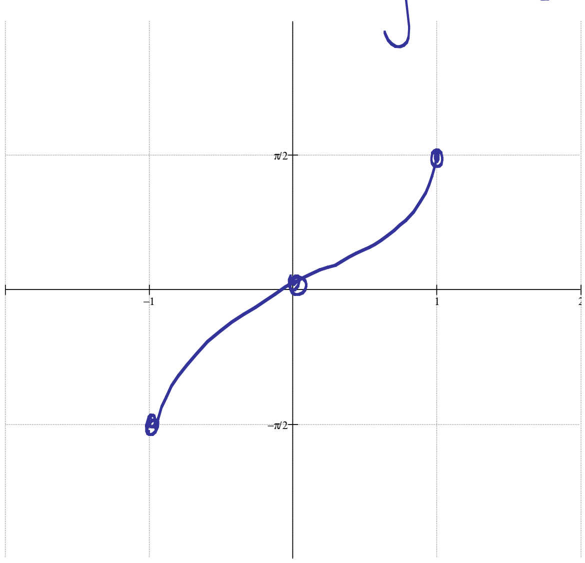
$x = \sin y$ $\quad$ $y = \sin^{-1} x$ or $y = \arcsin x$

Inverse

x	y
-1	$-\pi/2$
0	0
1	$\pi/2$

Domain
$-1 \le x \le 1$

Range
$-\pi/2 \le y \le \pi/2$



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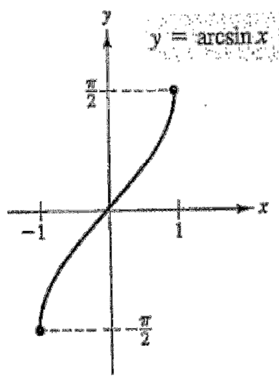
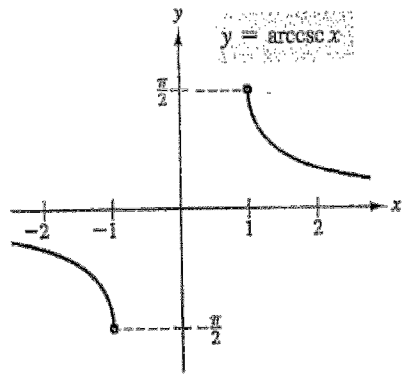
Graphs of other Inverse Trig Functions

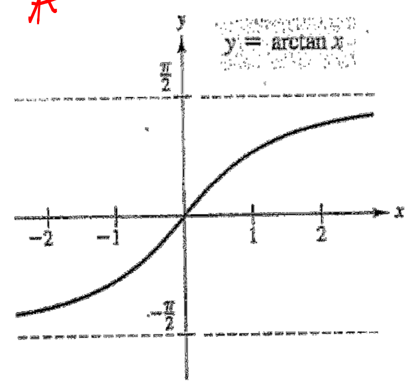
Domain: $[-1, 1]$
Range: $[-\pi/2, \pi/2]$



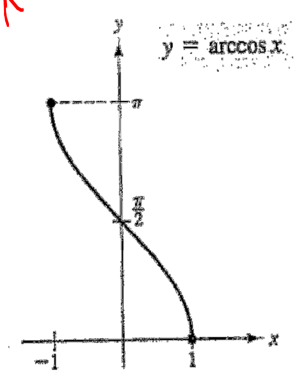
Domain: $(-\infty, -1] \cup [1, \infty)$
Range: $[-\pi/2, 0) \cup (0, \pi/2]$



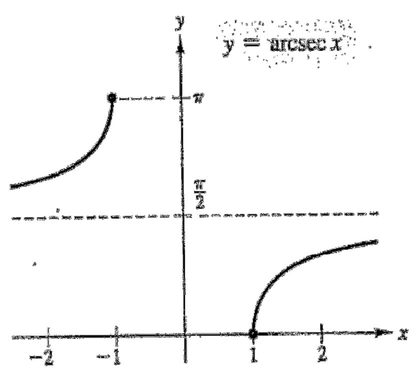
Domain: $(-\infty, \infty)$
Range: $(-\pi/2, \pi/2)$



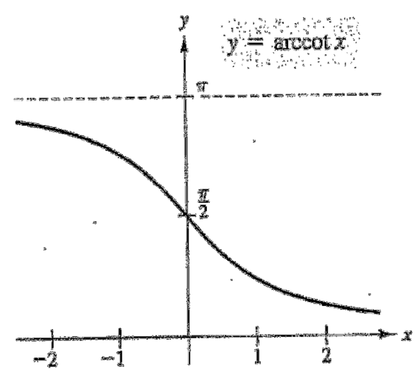
Domain: $[-1, 1]$
Range: $[0, \pi]$



Domain: $(-\infty, -1] \cup [1, \infty)$
Range: $[0, \pi/2) \cup (\pi/2, \pi]$



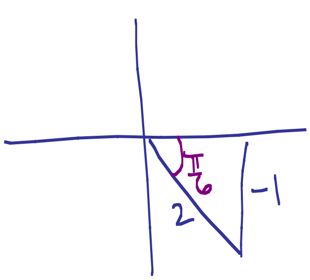
Domain: $(-\infty, \infty)$
Range: $(0, \pi)$



1. Evaluate

a) $arcsin\left(\frac{-1}{2}\right) = y \quad -\frac{\pi}{2} \le y \le \pi/2$

$\sin y = -\frac{1}{2}$

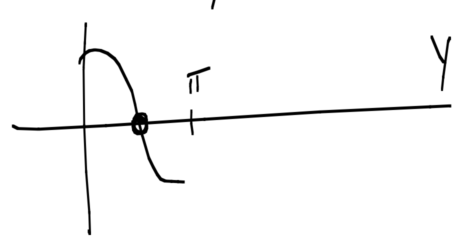


ref $\angle = \pi/6$

$y = -\frac{\pi}{6}$

b) $\arccos(0) = y \quad 0 \le y \le \pi$

$\cos y = 0$



$y = \pi/2$

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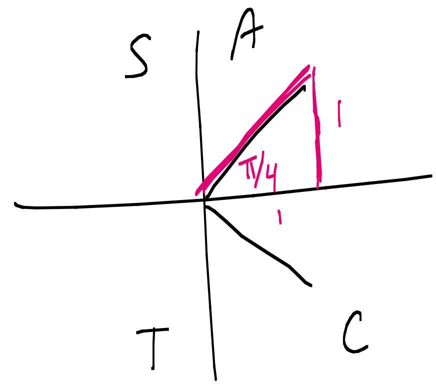
c) $\tan^{-1} 1$

$\arctan(1) = y \quad -\pi/2 \le y \le \pi/2$

$\tan y = 1$

$y = \pi/4$



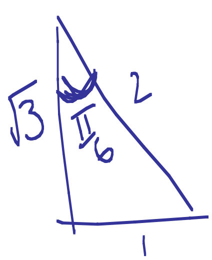
If two functions are inverses then

$f(g(x)) = x$	$g(f(y)) = y$
$-1 \le x \le 1$ $\sin(\arcsin x) = x$	$-\pi/2 \le y \le \pi/2$ $\arcsin(\sin y) = y$
$-1 \le x \le 1$ $\cos(\arccos x) = x$	$0 \le y \le \pi$ $\arccos(\cos y) = y$

2. Evaluate $\sin(arcsin\ 0.5) = 0.5$

$\sin\left(\frac{\pi}{6}\right)$

$\frac{1}{2}$



3. Solve $\arcsin(2x-3) = \frac{\pi}{3}$

$\sin(\arcsin(2x-3)) = \sin\frac{\pi}{3}$

$2x - 3 = \frac{\sqrt{3}}{2}$

$4x - 6 = \sqrt{3}$

$4x = \sqrt{3} + 6$

$x = \frac{\sqrt{3}+6}{4}$

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Derivatives of Inverse Trig Functions

Derivatives of the transcendental function $f(x) = \ln x$ is the algebraic function $f'(x) = \frac{1}{x}$
Derivative of inverse trig functions are also algebraic.

$y = arcsinx$ is the inverse of $y = sinx$

$x = \sin y$

$\frac{d}{dx}(x) = \frac{d}{dx}(\sin y)$

$1 = \cos y \frac{dy}{dx}$

$\frac{1}{\cos y} = \frac{dy}{dx}$

$\frac{1}{\sqrt{1-\sin^2 y}} = \frac{dy}{dx}$

$\frac{1}{\sqrt{1-x^2}} = \frac{dy}{dx}$

$\sin^2 y + \cos^2 y = 1$

$\cos^2 y = 1 - \sin^2 y$

$\cos y = \sqrt{1-\sin^2 y}$

$x = \sin y$

$x^2 = \sin^2 y$

$\frac{d}{dx}[\sin^{-1} u] = \frac{u'}{\sqrt{1-u^2}}$

$\frac{d}{dx}[\cos^{-1} u] = \frac{-u'}{\sqrt{1-u^2}}$

$\frac{d}{dx}[\tan^{-1} u] = \frac{u'}{1+u^2}$

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4. Find the derivative

a) $y = \sin^{-1} 2x$ $u = 2x$ $u' = 2$

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-(2x)^2}}$$

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}$$

b) $y = arccos\sqrt{x}$

$u = \sqrt{x}$
$u' = \frac{1}{2} x^{-1/2}$
$u' = \frac{1}{2\sqrt{x}}$

$$\frac{dy}{dx} = \frac{-\frac{1}{2\sqrt{x}}}{\sqrt{1-(\sqrt{x})^2}}$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{x}\sqrt{1-x}}$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{x-x^2}}$$

c) $y = arcsinx + x\sqrt{1-x^2}$

product with chain Rule

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + x \cdot \frac{1}{2}(1-x^2)^{-1/2} \cdot (-2x) + 1\sqrt{1-x^2}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} - x^2 \cdot \frac{1}{\sqrt{1-x^2}} + \sqrt{1-x^2} \cdot \frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{1-x^2+1-x^2}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{2-2x^2}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{2(1-x^2)}{\sqrt{1-x^2}} = 2\sqrt{1-x^2}$$

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Integration of Inverse Trig Functions

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$$

5. Evaluate

a) $\int \frac{dx}{\sqrt{4-x^2}}$

$$= \int \frac{dx}{\sqrt{2^2 - x^2}}$$

$a = 2 \quad u = x$

$\frac{du}{dx} = 1$

$du = dx$

$$= \arcsin \frac{x}{2} + C$$

b) $\int \frac{dx}{2+9x^2}$

$$= \int \frac{dx}{(\sqrt{2})^2 + (3x)^2}$$

$a = \sqrt{2}$

$u = 3x$

$\frac{du}{dx} = 3$

$du = 3dx$

$\frac{du}{3} = dx$

$$= \int \frac{1}{a^2 + u^2} \cdot \frac{du}{3}$$

$$= \frac{1}{3} \cdot \frac{1}{a} \arctan \frac{u}{a} + c$$

$$= \frac{1}{3} \cdot \frac{1}{\sqrt{2}} \arctan \frac{3x}{\sqrt{2}} + c$$

$$= \frac{1}{3\sqrt{2}} \arctan \frac{3x}{\sqrt{2}} + c$$