7.8 Pg 396

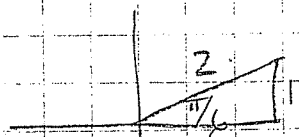
1. $\cos^{-1}(1) = \theta$

$\cos\theta = 1$

$\theta = 0$

2. $\sin^{-1}\left(\frac{1}{2}\right) = \theta$

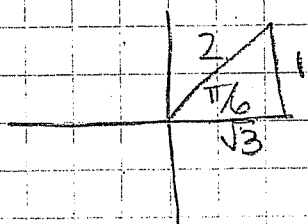
$\sin\theta = \frac{1}{2}$



$\theta = \pi/6$

4. $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \theta$

$\sec\theta = \frac{2}{\sqrt{3}}$



$\theta = \pi/6$

7. $\sin^{-1}\left(\sin\frac{\pi}{3}\right)$

$= \frac{\pi}{3}$

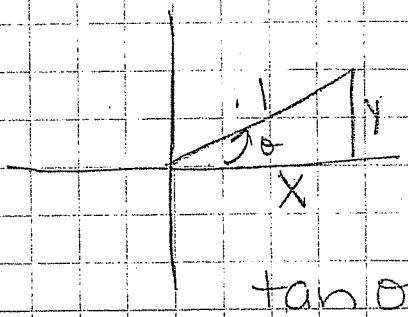
9. $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$

$= 3\pi/2$

17. $\tan(\cos^{-1}x)$

let $\theta = \cos^{-1}x$

$\cos\theta = x = \frac{x}{1}$



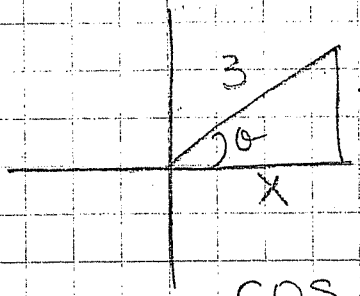
$y = \sqrt{1-x^2}$

$\tan\theta = \frac{\sqrt{1-x^2}}{x}$

21. $\cos\left(\sin^{-1}\frac{2}{3}\right)$

$\theta = \sin^{-1}\left(\frac{2}{3}\right)$

$\frac{2}{3} = \sin\theta$



$x = \sqrt{9-4}$

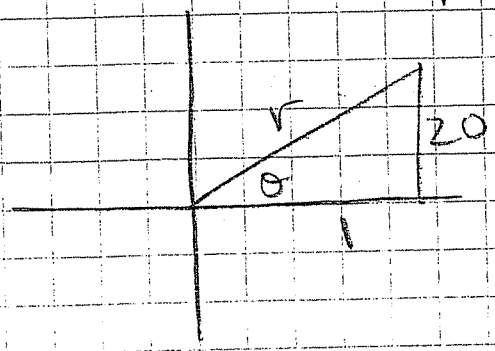
$x = \sqrt{5}$

$\cos\theta = \frac{\sqrt{5}}{3}$

27 $\cot(\tan^{-1} 20)$

$\theta = \tan^{-1}(20)$

$\tan\theta = \frac{20}{1}$



$\cot\theta = \frac{1}{20}$

29. $y = \sin^{-1} x \quad x = 3/5$

$y' = \frac{1}{\sqrt{1-x^2}}$

$y' = \frac{1}{\sqrt{1-9/25}}$

$y' = \frac{1}{\sqrt{16/25}}$

$y' = \frac{1}{4/5}$

$y' = 5/4$

33. $y = \sin^{-1}(7x)$

$y' = \frac{1}{\sqrt{1-(7x)^2}} \cdot 7$

$y' = \frac{7}{\sqrt{1-49x^2}}$

35. $y = \cos^{-1}(x^2)$

$y' = \frac{-1}{\sqrt{1-(x^2)^2}} \cdot 2x$

$y' = \frac{-2x}{\sqrt{1-x^4}}$

37. $y = x\tan^{-1} x$

$y' = 1 \cdot \tan^{-1} x + x \cdot \frac{1}{x^2+1}$

$y' = \tan^{-1} x + \frac{x}{x^2+1}$

39 $y = \arcsin(e^x)$

$y = \sin^{-1}(e^x)$

$y' = \frac{1}{\sqrt{1-(e^x)^2}} \cdot e^x$

$y' = \frac{e^x}{\sqrt{1-e^{2x}}}$

41. $y = \sqrt{1-t^2} + \sin^{-1} t$

$y' = \frac{1}{2}(1-t^2)^{-\frac{1}{2}} \cdot (-2t) + \frac{1}{\sqrt{1-t^2}}$

$y' = \frac{-t}{\sqrt{1-t^2}} + \frac{1}{\sqrt{1-t^2}}$

$y' = \frac{-t+1}{\sqrt{1-t^2}}$

43. $y = (\tan^{-1} x)^3$

$y' = 3(\tan^{-1} x)^2 \cdot \frac{1}{x^2+1}$

$y' = \frac{3(\tan^{-1} x)^2}{x^2+1}$

47. $y = \arccos(\ln x)$

$y = \cos^{-1}(\ln x)$

$y' = \frac{-1}{\sqrt{1-(\ln x)^2}} \cdot \frac{1}{x}$

$y' = \frac{-1}{x\sqrt{1-\ln^2 x}}$

53. $\int_{\tan 1}^{\tan 8} \frac{dx}{x^2+1}$

$= \tan^{-1} x \Big]_{\tan 1}^{\tan 8}$

$= \tan^{-1}(\tan 8) - \tan^{-1}(\tan 1)$

$= 8 - 1$

$= 7$

55. $\int_0^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}}$

$= \sin^{-1}(x) \Big]_0^{\frac{1}{2}}$

$= \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0)$

$= \frac{\pi}{6} - 0$

$= \frac{\pi}{6}$

$\sin^{-1}\left(\frac{1}{2}\right) = m$

$\sin(m) = \frac{1}{2}$

$m = \frac{\pi}{6}$

$\sin^{-1}(0) = n$

$\sin(n) = 0$

$n = 0$

59. $\int_0^3 \frac{dx}{x^2+3}$

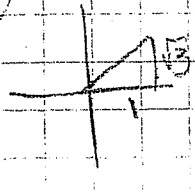
$= \int_0^3 \frac{1}{x^2+(\sqrt{3})^2}\, dx$

$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} \Big]_0^3$

$= \frac{1}{\sqrt{3}}\left[\tan^{-1}\frac{3}{\sqrt{3}} - \tan^{-1} 0\right]$

$= \frac{1}{\sqrt{3}}\left[\tan^{-1}\sqrt{3} - 0\right]$

$= \frac{1}{\sqrt{3}}\left(\frac{\pi}{3}\right) = \frac{\pi}{3\sqrt{3}}$



61. $\int \frac{dt}{\sqrt{1-16t^2}}$

$= \int \frac{dt}{\sqrt{1-(4t)^2}}$

$u = 4t$

$\frac{du}{dt} = 4t$

$\frac{du}{4} = dt$

$= \int \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{4}$

$= \frac{1}{4} \arcsin u + C$

$= \frac{1}{4} \sin^{-1}(4t) + C$

63 $\int \frac{dt}{\sqrt{5-3t^2}} = \int \frac{dt}{\sqrt{(\sqrt{5})^2+(\sqrt{3}t)^2}}$

$a = \sqrt{5}$

$u = \sqrt{3}t$

$\frac{du}{dt} = \sqrt{3}$

$\frac{du}{\sqrt{3}} = dt$

$= \int \frac{1}{\sqrt{a^2+u^2}} \cdot \frac{du}{\sqrt{3}}$

$= \frac{1}{\sqrt{3}} \arcsin \frac{\sqrt{3}t}{\sqrt{5}} + C$

$= \frac{1}{\sqrt{3}} \sin^{-1} \sqrt{\frac{3}{5}}\, t + C$

65. $\int \frac{dx}{x\sqrt{12x^2-3}}$

$\int \frac{dx}{\sqrt{3}x\sqrt{4x^2-1}}$

$\frac{1}{\sqrt{3}} \int \frac{dx}{x\sqrt{(2x)^2-1^2}}$

$u = 2x \quad a = 1$

$\frac{du}{2} = dx$

$= \frac{1}{\sqrt{3}} \int \frac{1}{\frac{u}{2}\sqrt{u^2-1}} \cdot \frac{du}{2}$

$= \frac{1}{\sqrt{3}} \int \frac{1}{u\sqrt{u^2-1}}\, du$

$= \frac{1}{\sqrt{3}} \operatorname{arcsec} \frac{u}{a} + C$

$= \frac{1}{\sqrt{3}} \cdot \operatorname{arcsec} \frac{2x}{1} + C$

67. $\int \frac{dx}{x\sqrt{x^4-1}}$

$= \int \frac{dx}{x\sqrt{(x^2)^2-1}}$ $= \int \frac{x \cdot dx}{x \cdot x\sqrt{(x^2)^2-1}}$

$u = x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x\,dx$

$= \int \frac{du/2}{u\sqrt{u^2-1}}$

$= \frac{1}{2}\operatorname{arcsec} u + C$

$= \frac{1}{2}\sec^{-1}(x^2) + C$