Pg 416 8.1

7. $\int (4x-3)e^{-x}\,dx$

$u = 4x-3 \qquad dv = e^{-x}dx$
$du = 4dx \qquad v = -e^{-x}$

$= (4x-3)\cdot(-e^{-x}) - \int -e^{-x}\cdot 4dx$
$= -e^{-x}(4x-3) + 4\int e^{-x}dx$
$= -e^{-x}(4x-3) - 4e^{-x} + C$
$= -e^{-x}[4x-3+4] + C$
$= -e^{-x}(4x+1) + C$

9. $\int x\cdot e^{5x+2}\,dx$

$u = x \qquad dv = e^{5x+2}dx$
$du = dx \qquad v = (e^{5x+2})(\frac{1}{5})$

$= x\cdot\frac{1}{5}\cdot e^{5x+2} - \int \frac{1}{5}e^{5x+2}\cdot dx$
$= \frac{1}{5}xe^{5x+2} - \frac{1}{5}\int e^{5x+2}\,dx$
$= \frac{1}{5}xe^{5x+2} - \frac{1}{5}e^{5x+2}\cdot\frac{1}{5} + C$
$= \frac{1}{5}e^{5x+2}\left[x - \frac{1}{5}\right] + C$
$= \frac{1}{25}e^{5x+2}[5x-1] + C$

11. $\int x \cos 2x \, dx$

$u = x \qquad dv = \cos 2x \, dx$
$du = dx \qquad v = \frac{1}{2}\sin 2x$

$= x \cdot \frac{1}{2}\sin 2x - \int \frac{1}{2}\sin 2x \, dx$

$= \frac{x}{2}\sin 2x - \frac{1}{2}\int \sin 2x \, dx$

$= \frac{x}{2}\sin 2x - \frac{1}{2} \cdot \frac{1}{2} \cdot (-\cos 2x) + C$

$= \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x + C$

13. $\int x^2 \sin x \, dx$

$u = x^2 \qquad dv = \sin x \, dx$
$du = 2x \, dx \qquad v = -\cos x$

$= x^2 \cdot (-\cos x) - \int -\cos x \cdot 2x \, dx$

$= -x^2 \cos x + \underbrace{\int 2x \cos x \, dx}$

use integration by parts again

$u = 2x \qquad dv = \cos x \, dx$
$du = 2dx \qquad v = \sin x$

$= -x^2 \cos x + 2x \sin x - \int \sin x \cdot 2 \, dx$

$= -x^2 \cos x + 2x \sin x - 2(-\cos x) + C$

$= -x^2 \cos x + 2x \sin x + 2\cos x + C$

19 $\int x \ln x \, dx$

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}dx \qquad v = \frac{1}{2}x^2$

$= \ln x \cdot \frac{1}{2}x^2 - \int \frac{1}{2}x^2 \cdot \frac{1}{x}dx$

$= \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x\,dx$

$= \frac{1}{2}x^2 \ln x - \frac{1}{2}\cdot\frac{1}{2}x^2 + C$

$= \frac{1}{4}x^2\left[2\ln x - 1\right] + C$

21. $\int x^2 \ln x \, dx$

$u = \ln x \qquad dv = x^2 dx$

$du = \frac{1}{x}dx \qquad v = \frac{1}{3}x^3$

$= (\ln x)\left(\frac{1}{3}x^3\right) - \int \frac{1}{3}x^3 \cdot \frac{1}{x}dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\cdot\frac{1}{3}x^3 + C$

$= \frac{1}{9}x^3\left[3\ln x - 1\right] + C$

25. $\int x\sec^2 x\,dx$

$u = x \qquad dv = \sec^2 x\,dx$

$du = dx \qquad v = \tan x$

$= x\cdot\tan x - \int \tan x\cdot dx$

$= x\tan x - \ln|\sec x| + C$

26. $\int x\tan x\sec x\,dx$

$u = x \qquad dv = \tan x\sec x\,dx$

$du = dx \qquad v = \sec x$

$= x\sec x - \int \sec x\,dx$

$= x\sec x - \ln|\sec x + \tan x| + C$

27. $\int \cos^{-1} x\,dx$

$u = \cos^{-1} x \qquad dv = 1\,dx$

$du = \frac{-1}{\sqrt{1-x^2}}\,dx \qquad v = x$

$= (\cos^{-1} x)x - \int x\cdot\frac{-1}{\sqrt{1-x^2}}\,dx$

$= x\cos^{-1} x + \underbrace{\int \frac{x}{\sqrt{1-x^2}}\,dx}$ substitution $m = 1-x^2$, $dm = -2x\,dx$, $\frac{dm}{-2} = x\,dx$

$= x\cos^{-1} x + \int m^{-1/2}\cdot\frac{dm}{-2}$

$= x\cos^{-1} x - \frac{1}{2}\left(\frac{2}{1}\right)m^{1/2} + C \qquad = x\cos^{-1} x - \sqrt{1-x^2} + C$

49 $\int_0^3 x e^{4x}\,dx$

$u = x \qquad dv = e^{4x}\,dx$

$du = dx \qquad v = \frac{1}{4}e^{4x}$

$= x \cdot \frac{1}{4}e^{4x}\Big]_0^3 - \int_0^3 \frac{1}{4}e^{4x}\,dx$

$= \frac{x}{4}e^{4x} - \frac{1}{4}\cdot\frac{1}{4}e^{4x}\Big]_0^3$

$= \frac{e^{4x}}{16}\left[4x - 1\right]\Big]_0^3$

$= \frac{e^{12}}{16}\left[12 - 1\right] - \frac{e^0}{16}\left[0 - 1\right]$

$= \frac{11}{16}e^{12} + \frac{1}{16}$

51. $\int_1^2 x \ln x\,dx$

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}dx \qquad v = \frac{1}{2}x^2$

$= \ln x \cdot \frac{x^2}{2}\Big]_1^2 - \int_1^2 \frac{1}{2}x^2 \cdot \frac{1}{x}\,dx$

$= \frac{x^2 \ln x}{2}\Big]_1^2 - \int_1^2 \frac{1}{2}x\,dx$

$= \frac{x^2 \cdot \ln x}{2} - \frac{1}{2}\cdot\frac{1}{2}x^2\Big]_1^2$

$= 2\ln 2 - \frac{1}{4}(2)^2 - \frac{\ln(1)}{2} + \frac{1}{4}(1)$

$= 2\ln 2 - 1 - 0 + \frac{1}{4} = 2\ln 2 - \frac{3}{4}$