8.1 Integration By Parts

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8.1 Integration By Parts

$$\int \frac{u'}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$$

Integration Review

1. $\int \frac{1dx}{\sqrt{16-x^2}}$

$$= \int \frac{dx}{\sqrt{4^2-x^2}} = \arcsin\frac{x}{4} + C$$

$a = 4$
$u = x$
$du = 1dx$

$$\int \frac{u'}{u^2+a^2} = \frac{1}{a}\arctan\frac{u}{a} + C$$

2. $\int \frac{4}{x^2+9}dx$

$$= \int \frac{4}{x^2+3^2}dx = 4\int \frac{1dx}{x^2+3^2}$$

$a=3$
$u = x$
$du = 1dx$

$$= 4\cdot\frac{1}{3}\arctan\frac{x}{3} + C$$

$$= \frac{4}{3}\arctan\frac{x}{3} + C$$

3. $\int \frac{x}{3x^2+2}dx$

$u = 3x^2 + 2$
$\frac{du}{dx} = 6x$
$\frac{du}{6} = xdx$

$$= \int \frac{1}{u}\cdot\frac{du}{6}$$

$$= \frac{1}{6}\int \frac{1}{u}du$$

$$= \frac{1}{6}\ln|u| + C$$

$$= \frac{1}{6}\ln|3x^2+2| + C$$

or

$$= \frac{1}{6}\ln(3x^2+2) + C$$

4. $\int^1 \frac{x+3}{\dots}dx$

$$= \int^1 \frac{x}{\sqrt{4-x^2}}dx + \int^1 \frac{3}{\sqrt{4-x^2}}dx$$

4. $\int_0^1 \frac{x+3}{\sqrt{4-x^2}}\,dx$

$u = 4 - x^2$

$\frac{du}{dx} = -2x$

$\frac{du}{-2} = x\,dx$

$x = 0$

$u = 4 - 0^2$

$u = 4$

$x = 1$

$u = 4 - 1^2$

$u = 3$

$$= \int_0^1 \frac{x}{\sqrt{4-x^2}}\,dx + \int_0^1 \frac{3}{\sqrt{4-x^2}}\,dx$$

substitution   arcsin

$$= \int_4^3 \frac{1}{\sqrt{u}} \cdot \frac{du}{-2} + 3\int_0^1 \frac{1}{\sqrt{2^2-x^2}}\,dx$$

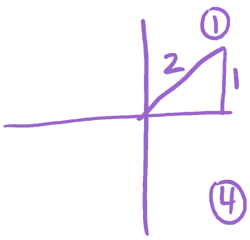
$$= \frac{1}{-2}\int_4^3 u^{-\frac{1}{2}}\,du + 3\arcsin\frac{x}{2}\Big|_0^1$$

$$= -\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{1} u^{1/2}\Big|_4^3 + 3\arcsin\frac{x}{2}\Big|_0^1$$

$$= -\sqrt{3} - (-\sqrt{4}) + 3\arcsin\frac{1}{2} - 3\arcsin 0$$

$$= -\sqrt{3} + 2 + 3\left(\frac{\pi}{6}\right) - 0$$

$$= -\sqrt{3} + 2 + \pi/2$$



$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}$

$\sin\theta = 0$

$\theta = 0$

5. $\int \sec x\,dx = \int \frac{\sec x}{1}\left(\frac{\sec x + \tan x}{\sec x + \tan x}\right) dx$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\,dx$$

$u = \sec x + \tan x$

$\frac{du}{dx} = \sec x \tan x + \sec^2 x$

$$= \int \frac{1}{u} \cdot du$$

$$= \ln|u| + C$$

$$= \ln|\sec x + \tan x| + C$$

$\int \sin u\,du = -\cos u + C$	$\int \tan u\,du = -\ln\lvert\cos u\rvert + C$
$\int \cos u\,du = \sin u + C$	$\int \cot u\,du = \ln\lvert\sin u\rvert + C$
$\int \sec^2 u\,du = \tan u + C$	$\int \sec u\,du = \ln\lvert\sec u + \tan u\rvert + C$
$\int \csc^2 u\,du = -\cot u + C$	$\int \csc u\,du = -\ln\lvert\csc u + \cot u\rvert + C$
$\int \sec u \tan u\,du = \sec u + C$	
$\int \csc u \cot u\,du = -\csc u + C$	

Integration by Parts

Using the product rule $\frac{d}{dx}[uv] = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$

Integrate both sides

$$\int \frac{d}{dx}[uv] = \int u \frac{dv}{dx} + \int v \frac{du}{dx}$$

$$uv = \int u \frac{dv}{dx} + \int v \frac{du}{dx}$$

$$uv - \int v\,du = \int u\,dv$$

$$\int u\,dv = uv - \int v\,du$$

u = function that you can find the derivative of

dv = function that you can integrate

6. $\int xe^x dx$

$u = x$

$du = dx$

$dv = e^x dx$

$v = e^x$

$\int u\,dv = uv - \int v\,du$

$\int xe^x dx = xe^x - \int e^x dx$

$= xe^x - e^x + C$

~~$u = e^x$~~

~~$du = e^x$~~

~~$dv = x\,dx$~~

~~$v = \frac{1}{2}x^2$~~

~~$= e^x \cdot \frac{1}{2}x^2 - \int \frac{1}{2}x^2 e^x dx$~~

7. $\int x^2 \ln x \; dx$

$u = \ln x$

$du = \frac{1}{x} dx$

$dv = x^2 dx$

$v = \frac{1}{3}x^3$

$\int u\,dv = uv - \int v\,du$

$\int x^2 \ln x\, dx = \ln x \cdot \frac{1}{3}x^3 - \int \frac{1}{3}x^{\cancel{3}} \cdot \frac{1}{\cancel{x}} dx$

$= \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2 dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{3} \cdot \frac{1}{3}x^3 + C$

$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$

8. $\int x\csc^2 x\, dx$

$u = x$

$du = dx$

$dv = \csc^2 x\, dx$

$v = -\cot x$

~~$\int u\,dv = uv - \int v\,du$~~

$\cancel{\int x\csc^2 x\, dx} = x(-\cot x) - \int -\cot x \cdot dx$

$= -x\cot x + \int \cot x\, dx$

$= -x\cot x + \ln|\sin x| + C$

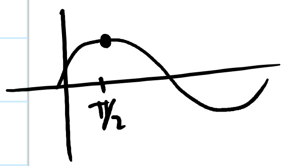
$\int u\,dv = uv - \int v\,du$

$$\int u\,dv = uv - \int v\,du$$

9. $\int_0^1 \sin^{-1} x \; dx = \int_0^1 1 \cdot \arcsin x \, dx$

$u = \arcsin x \qquad dv = 1\,dx$

$du = \frac{1}{\sqrt{1-x^2}}dx \qquad v = x$



$$= (\arcsin x)x \Big|_0^1 - \int_0^1 x \cdot \frac{1}{\sqrt{1-x^2}} dx$$

$$= 1(\arcsin 1) - 0(\arcsin 0) - \int_0^1 \frac{x}{\sqrt{1-x^2}} dx$$

$$= \frac{\pi}{2} - 0 - \int_1^0 \frac{1}{\sqrt{m}} \cdot \frac{dm}{-2}$$

$$= \frac{\pi}{2} + \frac{1}{2}\int_1^0 m^{-\frac{1}{2}} dm$$

$$= \frac{\pi}{2} + \cancel{\frac{1}{2}} \cdot \cancel{\frac{2}{1}} m^{\frac{1}{2}} \Big|_1^0$$

$$= \frac{\pi}{2} + \sqrt{0} - \sqrt{1}$$

$$= \pi/2 - 1$$

$m = 1 - x^2$

$\frac{dm}{dx} = -2x$

$\frac{dm}{-2} = x\,dx$

$x = 0$
$m = 1 - 0^2$
$m = 1$

$x = 1$
$m = 1 - 1^2$
$m = 0$

10. $\int x^2 \sin x \; dx$

$u = x^2 \qquad dv = \sin x\,dx$

$du = 2x\,dx \qquad v = -\cos x$

$$\int u\,dv = uv - \int v\,du$$

$$= x^2(-\cos x) - \int -\cos x \cdot 2x\,dx$$

$$= -x^2\cos x + \underbrace{\int 2x\cos x\,dx}_{\text{Integration by parts again}}$$

Integration by parts again

$u = 2x \qquad dv = \cos dx$

$du = 2dx \qquad v = \sin x$

$$= -x^2\cos x + 2x\sin x - \int \sin x \cdot 2dx$$

$$= -x^2\cos x + 2x\sin x - 2(-\cos x) + C$$

$$= -x^2\cos x + 2x\sin x + 2\cos x + C$$

LIATE (picking u)

Log
Inverse Trig
Algebraic
Trig
Exponential

Best
$\downarrow$
Worst.