Pre-Calculus 12

8.1 Understanding Logarithms

An exponential function $y = c^x$, has an inverse $x = c^y$. This inverse is also a function and is called a logarithmic function. A logarithmic function is written as:

$$y = \log_c x \quad \text{means} \quad x = c^y$$

base

A logarithm in base 10 is known as a common logarithm. When working with common logarithms you **do not** need to write the base.

$\log 3$ means $\log_{10} 3$ $\quad \sqrt{9} \quad \sqrt[3]{9}$

Ex. #1: Express the following in logarithmic form.

(a) $2^{-4} = \frac{1}{16}$ (b) $3^2 = 9$ (c) $4^x = n$

$\log_2 \frac{1}{16} = -4$ $\quad \log_3 9 = 2$ $\quad \log_4 n = x$

Ex. #2: Express the following in exponential form.

(a) $\log_3 27 = 3$ (b) $\log 10\,000 = 4$ (c) $\log_k t = x$

$3^3 = 27$ $\quad 10^4 = 10\,000$ $\quad k^x = t$

Ex. #3: Evaluate the following logarithms.

(a) $\log_2 64 = y$

$2^y = 64$

$2^y = 2^6$

$y = 6$

(b) $\log_4 \frac{1}{16} = w$

$4^w = \frac{1}{16}$

$4^w = 4^{-2}$

$w = -2$

(c) $\log 1000 = m$

$10^m = 1000$

$10^m = 10^3$

$m = 3$

(d) $\log_5 \sqrt{125} = t$

$5^t = \sqrt[2]{125}$

$5^t = (125)^{1/2}$

$5^t = (5^3)^{1/2}$

$5^t = 5^{3/2}$ $\quad t = 3/2$

Ex. #4: Use a calculator to evaluate the following.

(a) $\log 0.036$

-1.44

(b) $\log 0$

X

(c) $\log (-5)$

X

Therefore,

$\log_c x$ is defined if $x > 0$

Ex. #5: Determine the value of x.

(a) $\log_x 125 = \frac{3}{4}$

$x^{3/4} = 125$

$\left[x^{3/4}\right]^{4/3} = 125^{4/3}$

$x = \left(\sqrt[3]{125}\right)^4$

$x = 5^4 \quad x = 625$

(b) $\log_x 81 = \frac{4}{3}$

$x^{4/3} = 81$

$\left[x^{4/3}\right]^{\frac{3}{4}} = 81^{3/4}$

$x = \left(\sqrt[4]{81}\right)^3$

$x = 3^3$

$x = 27$

(c) $\log_4 x = -2$

$4^{-2} = x$

$\frac{1}{16} = x$

(d) $\log_{16} x = -\frac{1}{4}$

$16^{-\frac{1}{4}} = x$

$\sqrt[4]{\frac{1}{16}} = x$

$\frac{1}{2} = x$

Properties of logarithms

- $\log_c 1 =$ 0
- $\log_c c =$ 1
- $\log_c c^x =$ x
- $c^{\log_c x} =$ x

$\log_{10} 1 = 0$

$\log_{10} 10 = 1$

$\log_{10} 10^2 = 2$

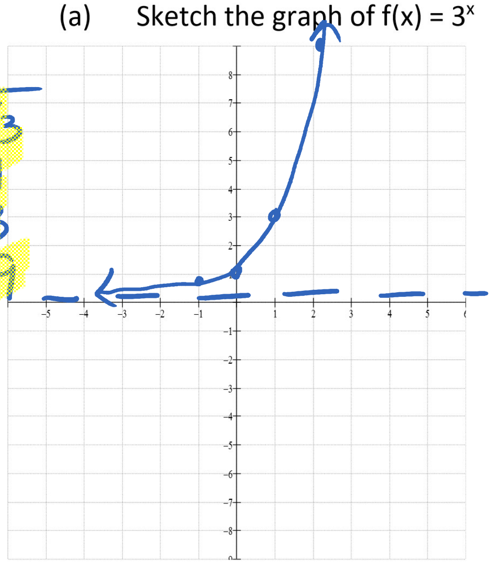
$\log_{10} 100 = 2$

$10^{\log_{10} 2} = 2$

Ex. #6: Answer the following:

(a) Sketch the graph of $f(x) = 3^x$

x	y
-1	1/3
0	1
1	3
2	9



(b) State the domain, range, equation of the asymptote, and any intercepts.

$\{x \mid x \in \mathbb{R}\}$

$\{y \mid y > 0,\ y \in \mathbb{R}\}$

$y = 0$

x-int: None

y-int: $(0,1)$

pick values for y

(c) ~~What is the inverse of~~ f(y) = 3^x?

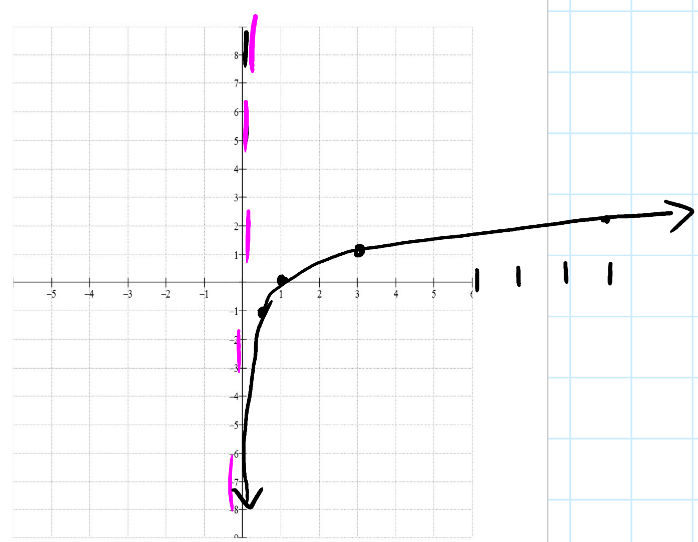
$x = 3^y$

$\boxed{\log_3 x = y}$

same

x	y
1/3	-1
1	0
3	1
9	2

(d) Sketch the graph of the inverse.



(e) State the domain, range, equation of the asymptote, and any intercepts for the inverse.

$\{x \mid x > 0,\ x \in \mathbb{R}\}$

$\{y \mid y \in \mathbb{R}\}$

$x = 0$ asymptote

x-int: $(1,0)$

y-int: None