8.8 Pg 472

1. $\int_0^2 x^2\,dx \quad N=4 \qquad \Delta x = \frac{2-0}{4} = \frac{1}{2}$

Trapezoid

$x_0 = 0$

$x_1 = 0 + \frac{1}{2} = \frac{1}{2}$

$x_2 = 0 + \frac{1}{2}(2) = 1$

$x_3 = 0 + \frac{1}{2}(3) = 3/2$

$x_4 = 0 + \frac{1}{2}(4) = 2$

$$\int_0^2 x^2\,dx \approx \frac{1}{2}\cdot\frac{1}{2}\left[f(0) + 2f(\tfrac{1}{2}) + 2f(1) + 2f(3/2) + f(2)\right]$$

$$\approx \frac{1}{4}\left[0 + 2\frac{1}{4} + 2\cdot 1 + 2\frac{9}{4} + 4\right]$$

$$\approx \frac{1}{4}\left[0 + \frac{1}{2} + \frac{4}{2} + \frac{9}{2} + \frac{8}{2}\right]$$

$$\approx \frac{1}{4}\left(\frac{22}{2}\right) = \frac{22}{8} = \frac{11}{4}$$

Midpoint

$c_1 = 0 + (1-\frac{1}{2})\cdot\frac{1}{2} = \frac{1}{4} \qquad \Delta x = \frac{2-0}{4} = \frac{1}{2}$

$c_2 = 0 + (2-\frac{1}{2})\cdot\frac{1}{2} = 3/4$

$c_3 = 0 + (3-\frac{1}{2})\cdot\frac{1}{2} = 5/4$

$c_4 = 0 + (4-\frac{1}{2})\cdot 1/2 = 7/4$

$$\int_0^2 x^2\,dx \approx \frac{1}{2}\left[f(\tfrac{1}{4}) + f(3/4) + f(\tfrac{5}{4}) + f(\tfrac{7}{4})\right]$$

$$\approx \frac{1}{2}\left[\frac{1}{16} + \frac{9}{16} + \frac{25}{16} + \frac{49}{16}\right]$$

$$\frac{1}{2}\left[\frac{84}{16}\right]$$

$$= \frac{21}{8}$$

3. $\int_1^4 x^3\,dx \qquad \Delta x = \frac{4-1}{6} = \frac{3}{6} = \frac{1}{2}$

Trapezoid

$x_0 = 1$

$x_1 = 1 + \frac{1}{2} = 3/2$

$x_2 = 1 + \frac{1}{2}(2) = 4/2 = 2$

$x_3 = 1 + \frac{1}{2}(3) = 5/2$

$x_4 = 1 + 1/2(4) = 6/2 = 3$

$x_5 = 1 + \frac{1}{2}(5) = 7/2$

$x_6 = 1 + \frac{1}{2}(6) = 8/2 = 4$

$$\int_1^4 x^3 dx = \frac{1}{2}\cdot\frac{1}{2}\left[f(1) + 2f\left(\frac{3}{2}\right) + 2f(2) + 2f\left(\frac{5}{2}\right) + 2f(3) + 2f\left(\frac{7}{2}\right) + f(4)\right]$$

$$= \frac{1}{4}\left[1 + 2\left(\frac{27}{8}\right) + 2(8) + 2\left(\frac{125}{8}\right) + 2(27) + 2\left(\frac{343}{8}\right) + 64\right]$$

$$= \frac{1}{4}\left[135 + \frac{990}{8}\right]$$

$$= \frac{1}{4}\left[\frac{1080}{8} + \frac{990}{8}\right] = \frac{2070}{32} = \frac{1035}{16}$$

Midpoint

$\Delta x = \frac{4-1}{6} = \frac{1}{2}$

$c_1 = 1 + \left(1 - \frac{1}{2}\right)\frac{1}{2} = 5/4$

$c_2 = 1 + \left(2 - \frac{1}{2}\right)\frac{1}{2} = 7/4$

$c_3 = 1 + \left(3 - \frac{1}{2}\right)\frac{1}{2} = 9/4$

$c_4 = 1 + \left(4 - \frac{1}{2}\right)\frac{1}{2} = 11/4$

$c_5 = 1 + \left(5 - \frac{1}{2}\right)\frac{1}{2} = 13/4$

$c_6 = 1 + \left(6 - 1/2\right)\frac{1}{4} = 15/4$

$$\int_1^4 x^3 dx = \frac{1}{2}\left[f(5/4) + f(7/4) + f(9/4) + f(11/4) + f\left(\frac{13}{4}\right) + f\left(\frac{15}{4}\right)\right]$$

$$= \frac{1}{2}\left[\frac{125}{64} + \frac{343}{64} + \frac{729}{64} + \frac{1331}{64} + \frac{2197}{64} + \frac{3375}{64}\right]$$

$$= \frac{8100}{128}$$

$$= \frac{2025}{32}$$

5. $\int_1^4 \frac{1}{x}dx \quad N=6 \quad \Delta x = \frac{4-1}{6} = \frac{1}{2}$

Trapezoid

$x_0 = 1$
$x_1 = 1 + \frac{1}{2} = 3/2$
$x_2 = 1 + \frac{1}{2}(2) = 2$
$x_3 = 1 + \frac{1}{2}(3) = 5/2$
$x_4 = 1 + 1/2(4) = 3$
$x_5 = 1 + 1/2(5) = 7/2$
$x_6 = 1 + 1/2(6) = 4$

$$\int_1^4 \frac{1}{x}dx = \frac{1}{2}\cdot\frac{1}{2}\left[f(1)+2f(3/2)+2f(2)+2f(5/2)+2f(3)+2f(\tfrac{7}{2})+f(4)\right]$$

$$= \frac{1}{4}\left[1+2\left(\frac{2}{3}\right)+2\left(\frac{1}{2}\right)+2\left(\frac{2}{5}\right)+2\left(\frac{1}{3}\right)+2\left(\frac{2}{7}\right)+\frac{1}{4}\right]$$

$$= \frac{1}{4}\left[1+\frac{4}{3}+1+\frac{4}{5}+\frac{2}{3}+\frac{4}{7}+\frac{1}{4}\right]$$

$$= \frac{1}{4}\left[2+\frac{6}{3}+\frac{4}{5}+\frac{4}{7}+\frac{1}{4}\right]$$

$$= \frac{1}{4}\left[4+\frac{112}{140}+\frac{80}{140}+\frac{35}{140}\right]$$

$$\frac{1}{4}\left[4+\frac{227}{140}\right] = \frac{787}{560}$$

Midpoint

$c_1 = 1 + (1-\frac{1}{2})\frac{1}{2} = 5/4$
$c_2 = 1 + (2-\frac{1}{2})\frac{1}{2} = 7/4$
$c_3 = 1 + (3-\frac{1}{2})\frac{1}{2} = 9/4$
$c_4 = 1 + (4-1/2)\frac{1}{2} = 11/4$
$c_5 = 1 + (5-1/2)\frac{1}{2} = 13/4$
$c_6 = 1 + (6-1/2)\frac{1}{2} = 15/4$

$$\int_1^4 \frac{1}{x}dx = \frac{1}{2}\left[f(5/4)+f(7/4)+f(9/4)+f(11/4)+f(13/4)+f(15/4)\right]$$

$$= \frac{1}{2}\left[\frac{4}{5}+\frac{4}{7}+\frac{4}{9}+\frac{4}{11}+\frac{4}{13}+\frac{4}{15}\right]$$

$$= 1.3769$$

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96. $\int_2^5 f(x)\,dx$

$\underline{T_2}$ $\Delta x = \frac{5-2}{2} = \frac{3}{2}$

x	2	2.5	3	3.5	4	4.5	5
$f(x)$	$1/2$	2	1	0	$-3/2$	-4	-2

$X_0 = 2$

$X_1 = 2 + 1\left(\frac{3}{2}\right) = \frac{7}{2} = 3.5$

$X_2 = 2 + 2\left(\frac{3}{2}\right) = 5$

$T_2 = \frac{3}{2} \cdot \frac{1}{2}\left[f(2) + 2f(3.5) + f(5)\right]$

$= \frac{3}{4}\left[\frac{1}{2} + 2(0) + (-2)\right]$

$= \frac{3}{4}\left[\frac{1}{2} - 2\right]$

$= \frac{3}{4}\left[\frac{-3}{2}\right]$

$= \frac{-9}{8}$

$\underline{M_3}$ $\quad \Delta x = \frac{5-2}{3} = \frac{3}{3} = 1$

$C_1 = 2 + (1-\frac{1}{2})(1) = 2 + \frac{1}{2} = 2.5$

$C_2 = 2 + (2-\frac{1}{2})(1) = 2 + \frac{3}{2} = 3.5$

$C_3 = 2 + (3-\frac{1}{2})(1) = 2 + \frac{5}{2} = 4.5$

$M_3 = 1\left[f(2.5) + f(3.5) + f(4.5)\right]$

$= 1\left[2 + 0 + (-4)\right]$

$= 1(-2)$

$= -2$