8.2 Transformations of Logarithmic Functions

The graph of $y = a\log_c b(x-h)+k$ can be obtained by transforming the graph of $y=\log_c x$. Use mapping notation to show the changes to the point (x, y)

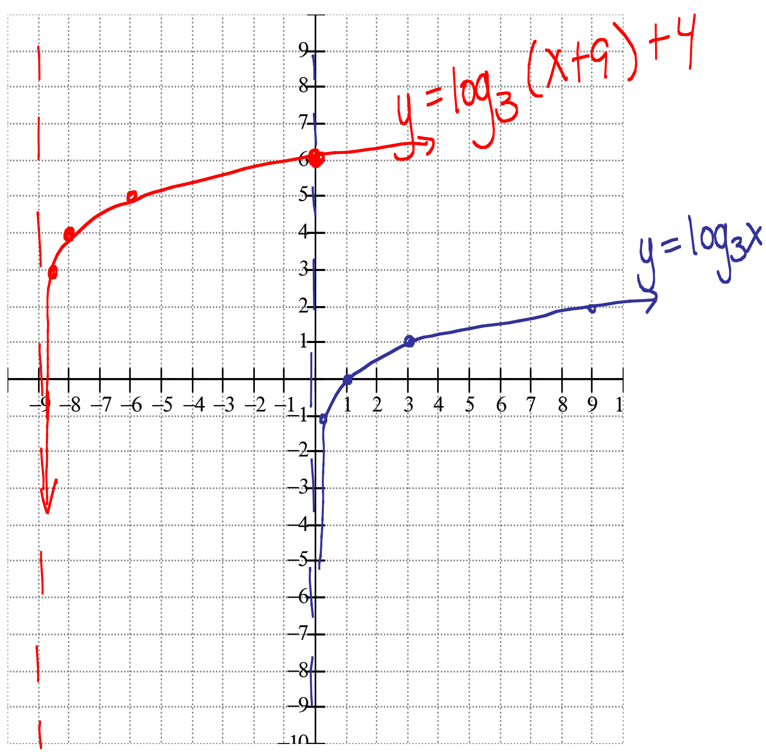
Parameter	Transformation
a	$(x, y) \rightarrow (x, ay)$
b	$(x, y) \rightarrow (\frac{x}{b}, y)$
h	$(x, y) \rightarrow (x+h, y)$
k	$(x, y) \rightarrow (x, y+k)$

Ex. #1: Graph the base function $y=\log_3 x$ and the transformed function $y = \log_3(x+9)+4$ on the same grid. For the transformed function state the equation of the asymptote and find the intercepts.

$y = \log_3 x$ $\qquad 3^y = x$

x	y
1/3	-1
1	0
3	1
9	2

Translate left 9 and up 4



Equation of asymptote: $x = -9$

x-intercept

make $y=0$

$0 = \log_3(x+9)+4$

$-4 = \log_3(x+9)$

$3^{-4} = x+9$

$\frac{1}{81} = x+9$

$x = \frac{1}{81} - 9$

$x = \frac{1}{81} - \frac{729}{81} = -\frac{728}{81}$

y-intercept

make $x=0$

$y = \log_3(0+9)+4$

$y = \log_3 9 + 4$

$y = 2 + 4$

$y = 6$

$\log_3 9 = m$

$3^m = 9$

$m = 2$

Change of Base:

The change of base formula is often used to evaluate logarithms with a calculator when the base is not 10. $m = any\ number\ of\ your\ choice$, however your calculator will require $m = 10$

$$\boxed{\log_c x = \frac{\log_m x}{\log_m c}}$$ *

Ex. #2: Graph the base function $y = \log_2 x$ and the transformed function $y = -\log_2(2x+6)$ on the same grid. State the equation of asymptote and find the intercepts of the transformed function accurate to 2 decimal places.

$y = \log_2 x$

$2^y = x$

½	-1
1	0
2	1
4	2
8	3
16	4

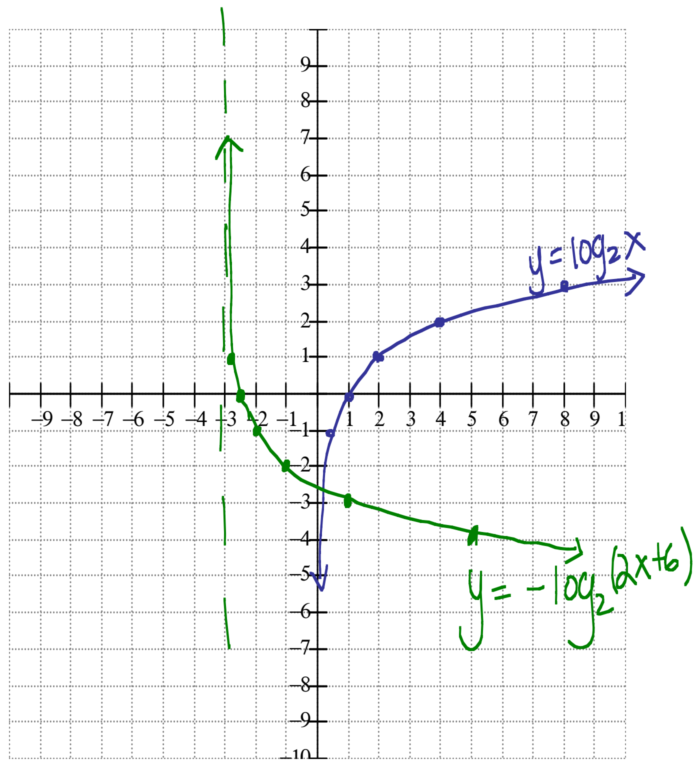
$y = -\log_2 2(x+3)$

$a = -1$ $b = 2$

divide x's by 2

translate left 3

$\frac{1}{4}$	$\cancel{\frac{1}{2}}$	$\cancel{-1}$	1
$\frac{1}{2}$	$\cancel{1}$	$\cancel{0}$	0
1	$\cancel{2}$	$\cancel{1}$	-1
2	$\cancel{4}$	$\cancel{2}$	-2
4	$\cancel{8}$	$\cancel{3}$	-3
8	$\cancel{16}$	$\cancel{4}$	-4



Equation of Asymptote : $x = -3$

x-intercept

Make $y = 0$

$0 = -\log_2(2x+6)$

$0 = \log_2(2x+6)$

$2^0 = 2x+6$

$1 = 2x+6$

$-5 = 2x$ $\quad x = -5/2$

y-intercept

Make $x = 0$

$y = -\log_2(2(0)+6)$

$y = -\log_2(6)$

$y = -\left[\frac{\log_{10} 6}{\log_{10} 2}\right]$

$y = -2.58$

Change of base
base 10

Ex. #3: The Shannon-Hartley theorem is used to determine the highest possible rate for transmitting information. The formula is $\boxed{C = B\log_2(r+1)}$, where r is the signal-to-noise ratio, B is the bandwidth in hertz, and C is the rate in bits per second.

a) Describe the transformations in the theorem, compared to the graph of $C = \log_2 r$.

Vertical stretch factor of "B"

Translate left 1

b) If the bandwidth is 10 000 Hz and the signal-to noise ratio is 3.1, determine the transmission rate.

$C = ?$

$B = 10\,000$

$r = 3.1$

$C = 10\,000\log_2(3.1+1)$

$C = 10\,000\log_2(4.1)$

$C = 10\,000\left[\frac{\log_{10}4.1}{\log_{10}2}\right]$ Change of base

$C = 20356.24$ bits/sec

Ex. #4: Write an equation for the following transformations if the base equation is $f(x) = \log_2 x$

- Vertical stretch by a factor of 3 $a = 3$
- Reflection over the y axis b is negative
- Horizontal stretch by a factor of 5 $b = -\frac{1}{5}$
- Horizontal translation left 1 $h = -1$ $(x+1)$
- Vertical translation down 8 $k = -8$

$f(x) = 3\log_2\left(-\frac{1}{5}(x+1)\right) - 8$

If the point (8,3) was on *f(x)*, what point will be on the transformed function?

$(8,3) \longrightarrow \underset{\text{reflection}}{(-8,3)} \longrightarrow \underset{\text{Stretches}}{(-8(5), 3(3))} \longrightarrow \underset{\text{translations}}{(-40-1, 9-8)}$

$= \boxed{(-41, 1)}$