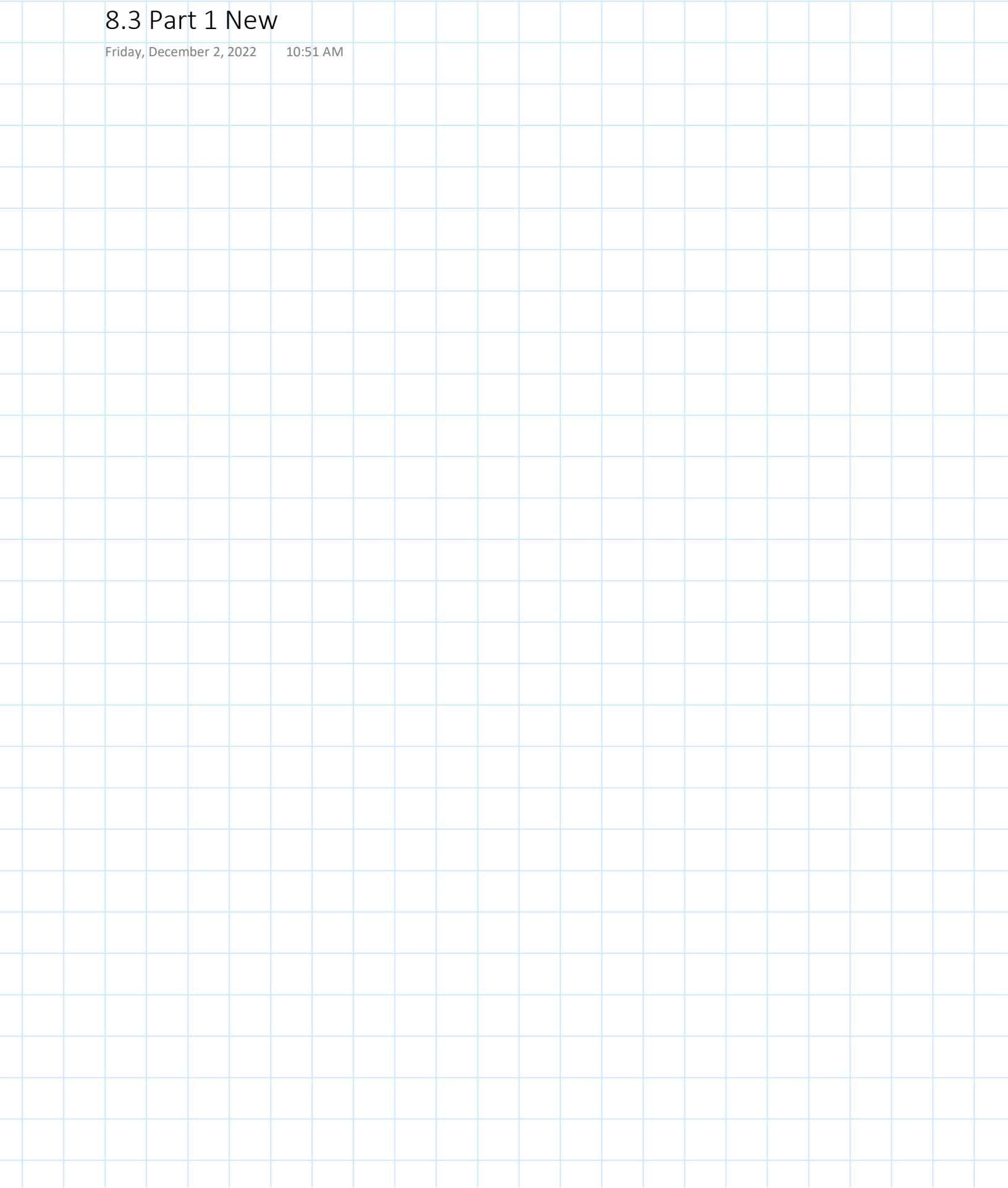


8.3 Part 1 New

Friday, December 2, 2022 10:51 AM



8.3 Laws of Logarithms Part 1

Product Law for Logs

$$\log_c MN = \log_c M + \log_c N$$

Quotient Law for Logs

$$\log_c \frac{M}{N} = \log_c M - \log_c N$$

Power Law for Logs

$$\log_c M^P = P \log_c M$$

Ex. #1: Prove the product law for logs

let

$$\log_c M = x$$

$$C^x = M$$

$$\log_c N = y$$

$$C^y = N$$

$$M \cdot N = C^x \cdot C^y$$

$$MN = C^{x+y}$$

$$\log_c MN = x + y$$

$$\log_c(MN) = \log_c M + \log_c N$$

Ex. #2: Use log laws to expand into individual logs

$$\begin{aligned}
 \text{a) } & \log_3 \frac{x^2 n^4}{y} \\
 & = \log_3 (x^2 n^4) - \log_3 y \\
 & = \log_3 x^2 + \log_3 n^4 - \log_3 y \\
 & = 2 \log_3 x + 4 \log_3 n - \log_3 y
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & \log \frac{a}{b^3 c} \\
 & = \log a - \log (b^3 c) \\
 & = \log a - (\log b^3 + \log c) \\
 & = \log a - \log b^3 - \log c \\
 & = \log a - \log b^3 - \frac{1}{3} \log c
 \end{aligned}$$

Ex. #3: Use log laws to evaluate. (do not use a calculator)

$$\begin{aligned}
 \text{a) } & \log_{12} 18 + \log_{12} 8 \\
 & = \log_{12} (18 \cdot 8) \\
 & = \log_{12} 144 \\
 & = \log_{12} 12^2 \\
 & = 2
 \end{aligned}$$

$$\begin{array}{r}
 6 \\
 \times 18 \\
 \hline
 144
 \end{array}$$

$$\log_c C^x = x$$

$$\begin{aligned}
 \text{b) } & 2\log_3 6 - \frac{1}{2}\log_3 64 + \log_3 2 \\
 &= \log_3 6^2 - \log_3 64^{\frac{1}{2}} + \log_3 2 \\
 &= \log_3 36 - \log_3 8 + \log_3 2 \\
 &= \log_3 \frac{36(2)}{8} \quad \leftarrow \log_3 \frac{36}{8} + \log_3 2 \\
 & \qquad \qquad \qquad \log_3 \frac{36}{8}(2) \\
 &= \log_3 9 \\
 &= \log_3 3^2 \\
 &= 2
 \end{aligned}$$

Ex. #4: If $\log_8 3 = x$ and $\log_8 7 = y$ find an expression for $\log_8 441$

$$\begin{array}{c}
 441 \\
 \swarrow \quad \searrow \\
 3 \quad 147 \\
 \quad \swarrow \quad \searrow \\
 \quad 3 \quad 49 \\
 \quad \quad \swarrow \quad \searrow \\
 \quad \quad 7 \quad 7
 \end{array}$$

$$\begin{aligned}
 & \log_8 441 \\
 &= \log_8 (3^2 \cdot 7^2) \\
 &= \log_8 3^2 + \log_8 7^2 \\
 &= 2\log_8 3 + 2\log_8 7 \\
 &= 2x + 2y
 \end{aligned}$$

$$2x + 2y$$

Ex. #5: If $\log_2 5 = x$ and $\log_2 7 = y$ find an expression for $\log_2 \frac{\sqrt[3]{5}}{28}$

$$\log_2 \sqrt[3]{5} - \log_2 28$$

$$\log_2 5^{1/3} - \log_2 (7 \cdot 4)$$

$$\log_2 5^{1/3} - (\log_2 7 + \log_2 4)$$

$$\log_2 5^{1/3} - (\log_2 7 + \log_2 2^2)$$

$$\frac{1}{3} \log_2 5 - (\log_2 7 + 2 \log_2 2)$$

$$\frac{1}{3} \log_2 5 - \log_2 7 - 2 \log_2 2$$

$$\frac{1}{3}x - y - 2(1)$$

$$\frac{1}{3}x - y - 2$$