Pre-Calculus 12

8.3 Laws of Logarithms Part 2

Ex. #1: Find any restrictions. Write the expression as a single log in simplest form.

a) $4\log_3 x - \frac{1}{2}(\log_3 x + 5\log_3 x)$

$x > 0$

$4\log_3 x - \frac{1}{2}\log_3 x - \frac{5}{2}\log_3 x$

$\log_3 x^4 - \log_3 x^{1/2} - \log_3 x^{5/2}$

$\log_3 \frac{x^4}{x^{1/2} \cdot x^{5/2}}$

$\log_3 \frac{x^4}{x^3}$

$\log_3 x$

$4\log_3 x - \frac{1}{2}(6\log_3 x)$

$4\log_3 x - 3\log_3 x$

$\log_3 x$

b) $\log_2(x^2 - 9) - \log_2(x^2 - x - 6)$

$\log_2 \frac{x^2 - 9}{x^2 - x - 6}$

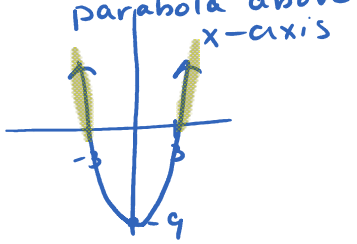
$\log_2 \frac{(x-3)(x+3)}{(x-3)(x+2)}$

$\log_2 \frac{x+3}{x+2}$

$x^2 - 9 > 0$

$(x-3)(x+3) > 0$

parabola above x-axis

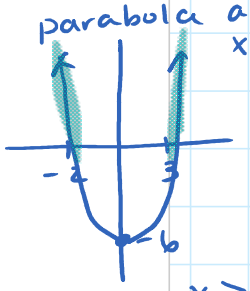


$x < -3 \quad x > 3$

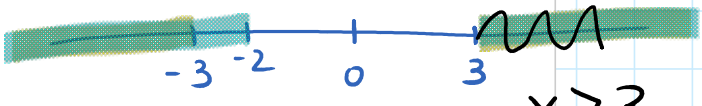
$x^2 - x - 6 > 0$

$(x-3)(x+2) > 0$

parabola above x-axis



$x < -2 \quad x > 3$



$x > 3$

$x < -3$

Restrictions are found from the original function.

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The pH scale measures the acidity and alkalinity of a solution

$$pH = -\log[H^+]$$

$[H^+]$ = *hydrogen ion concentration in moles per litre*

Ex. #2: Find the pH of a solution has a hydrogen ion concentration of 2.63×10^{-6} moles/litre.

$pH = -\log[H^+]$

$pH = -\log(2.63 \times 10^{-6})$

$pH = 5.6$

Ex. #3: Find the hydrogen ion concentration of a solution with a pH of 7.8.

$pH = -\log[H^+]$

$\frac{7.8}{-1} = \frac{-\log[H^+]}{-1}$

$-7.8 = \log_{10}[H^+]$

$10^{-7.8} = [H^+]$

$[H^+] = 1.58 \times 10^{-8}$ moles/L

Ex. #4: Cola has a pH of 2.5 and milk has a pH of 6.6. How many more times acidic is cola than milk?

Cola

$pH = 2.5$

$pH = -\log[H^+]$

$2.5 = -\log[H^+]$

$-2.5 = \log_{10}[H^+]$

$10^{-2.5} = [H^+]$

Milk

$pH = 6.6$

$pH = -\log[H^+]$

$6.6 = -\log[H^+]$

$-6.6 = \log_{10}[H^+]$

$10^{-6.6} = [H^+]$

$\frac{[H^+]_{cola}}{[H^+]_{milk}}$

$\frac{10^{-2.5}}{10^{-6.6}}$

$10^{4.1}$

$-2.5 - (-6.6)$

$-2.5 + 6.6$

4.1

12509.25 times more acidic

Ex. #5: An apple is 5 times more acidic than a pear. The pH of a pear is 3.8. What is the pH of the apple?

Apple
$pH = x$
$x = -\log[H^+]$
$-x = \log[H^+]$
$10^{-x} = [H^+]$

Pear
$pH = 3.8$
$3.8 = -\log[H^+]$
$-3.8 = \log[H^+]$
$10^{-3.8} = [H^+]$

More acidic on top

$$\frac{[H^+]_{apple}}{[H^+]_{pear}} = 5$$

$$\frac{10^{-x}}{10^{-3.8}} = 5$$

Exponential Form: $10^{-x+3.8} = 5$

Logarithmic Form:
$\log_{10} 5 = -x + 3.8$
$\log 5 - 3.8 = -x$
$-\log 5 + 3.8 = x$
$x = 3.1$