

8.4 Part 2 New

Monday, December 5, 2022 2:06 PM

$$\log_c X = Y$$

$$c^Y = X$$

$$8.1$$

$$\log_c 1 = 0$$

$$\log_c c = 1$$

$$\log_c c^X = X$$

$$c^{\log_c X} = X$$

Proof

8.4 Part 2 Solving Logarithmic Equations

$$c, L, R > 0 \quad c \neq 1$$

$$\log_c L = \log_c R$$

$$\text{Then } L = R$$

$$\log_c L = \log_c R$$

$$c^{\log_c R} = L$$

$$R = L$$

Ex. #1: Solve $\log_5(3x - 2) = \log_5 8$

$$3x - 2 = 8$$

$$3x = 10$$

$$x = \frac{10}{3}$$

$$\log_c X \quad X > 0$$

$$3x - 2 > 0$$

$$3x > 2$$

$$x > \frac{2}{3}$$

$$\frac{10}{3} > \frac{2}{3} \checkmark$$

$$\log_c c = 1$$

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$$\log_{10} 10 = 1$$

$$\begin{aligned} 2x + 1 &> 0 \\ 2x &> -1 \\ x &> -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} x - 2 &> 0 \\ x &> 2 \end{aligned}$$

Ex. #2: Solve $\log(2x + 1) - \log(x - 2) = 1$
Quotient Rule

$$\log \frac{2x+1}{x-2} = \log 10$$

$$\cancel{(x-2)} \left(\frac{2x+1}{\cancel{x-2}} \right) = 10(x-2)$$

$$2x + 1 = 10(x - 2)$$

$$2x + 1 = 10x - 20$$

$$21 = 8x$$

$$\boxed{\frac{21}{8} = x}$$

$$\frac{21}{8} > 2 \quad \checkmark$$

Ex. #3: Solve $\log_2 x + \log_2(x - 2) = 3$

$$\log_2(x(x-2)) = 3$$

$$\log_2(x(x-2)) = \log_2 2^3$$

$$x(x-2) = 2^3$$

$$x^2 - 2x = 8$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$\boxed{x=4}$$

$$\cancel{x=-2}$$

Extraneous root

$$\begin{aligned} \log_c c^x &= x \\ \log_2 2^3 &= 3x > 0 \end{aligned}$$

$$\begin{aligned} x - 2 &> 0 \\ x &> 2 \end{aligned}$$

$$\begin{aligned} -x &= -8 \\ -4 + 2 &= -2 \end{aligned}$$

Ex. #4: $\log_{\sqrt{2}}(x+1) + \log_{\sqrt{2}}(x-2) = 4$

$$\log_{\sqrt{2}}(x+1) + \log_{\sqrt{2}}(x-2) = \log_{\sqrt{2}}(\sqrt{2})^4$$

$$\log_{\sqrt{2}}((x+1)(x-2)) = \log_{\sqrt{2}} 4$$

$$(x+1)(x-2) = 4$$

$$x^2 - 2x + x - 2 = 4$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x = 3$$

~~$x = -2$~~ extraneous root

FOIL

$$\begin{aligned} -x &= -6 \\ -3 + 2 &= -1 \end{aligned}$$

$$x+1 > 0$$

$$x > -1$$

$$x-2 > 0$$

$$x > 2$$

$$\log_c c^x = x$$

$$\log_{\sqrt{2}} \sqrt{2}^4 = 4$$

$$\sqrt{2} \cdot \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{2}$$

$$2 \cdot 2$$

$$4$$