8.5 Natural Logs and e

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8.5 Natural Logs and e^x

Rational numbers can be written as a fraction.

Eg. $\frac{1}{2}$, $3 = \frac{6}{2}$, -7, $0 = \frac{0}{1}$ $\quad .\overline{6} = \frac{2}{3}$

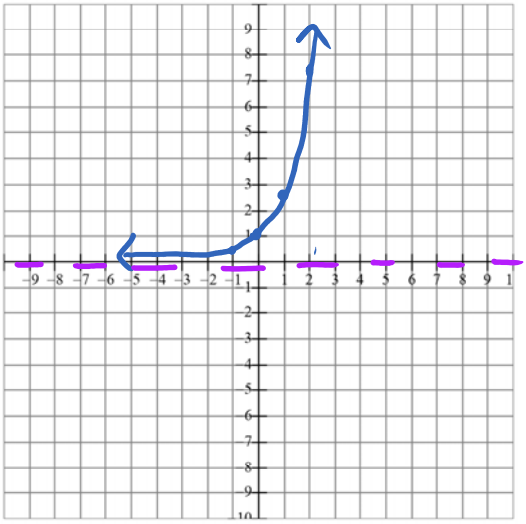
$\sqrt{4} = 2$

Irrational numbers can not be written as a fraction.

Eg. $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi$, $e \approx 2.718$

Ex. #1: Graph $y = e^x$ on the axes. State the domain, range, and equation of the asymptote for the function.

x	y
-1	$e^{-1} = .37$
0	$e^0 = 1$
1	$e^1 = 2.718$
2	$e^2 = 7.39$
3	$e^3 = 20.09$



Domain: $\{x : x \in \mathbb{R}\}$

Range: $\{y : y > 0, y \in \mathbb{R}\}$

Asymptote: $y = 0$

x-intercept: None

y-intercept: $(0, 1)$

Ex. #2: Find the equation of the inverse of $y = e^x$

Switch x and y $\quad x = e^y \quad \log_e x = y \quad y = \ln x$

Logarithms in base e are called natural logarithms.

$$\log_e x = \ln x$$

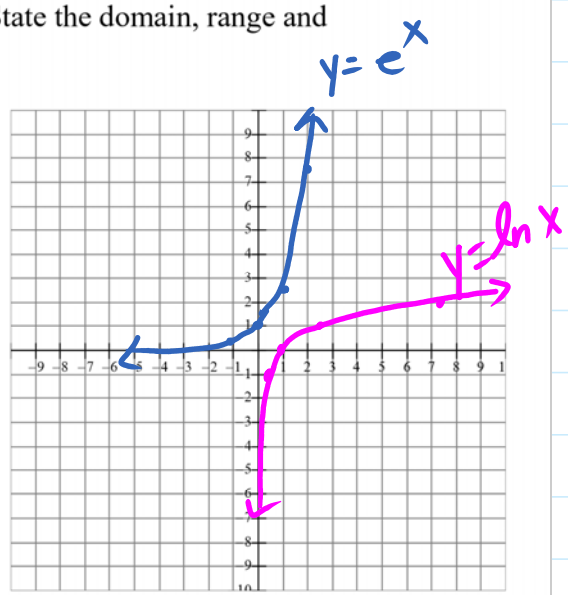
Ex. #3: Graph $y = e^x$ and $y = \ln x$ on the grid. State the domain, range and equation of the asymptote for $y = \ln x$

$y = e^x$

x	y
-1	.37
0	1
1	2.72
2	7.39

$y = \ln x$

x	y
.37	-1
1	0
2.72	1
7.39	2



for $y = \ln x$

Domain: $\{x : x > 0, x \in \mathbb{R}\}$

Range: $\{y : y \in \mathbb{R}\}$

Asymptote: $x = 0$

x-intercept: $(1, 0)$

y-intercept: None

Ex. #4: Evaluate using a calculator.

a) $\ln 2.5 = .916$

b) $\ln 0 =$ error

c) $\ln(-3) =$ error

d) $\ln e = 1$

$\ln e = \log_e e$

All the log rules and laws are the same for the natural log

Ex. #5: Write as a single logarithm.

a) $\ln 5 + 2\ln 3$

$= \ln 5 + \ln 3^2$

$= \ln 5 + \ln 9$

$= \ln(5 \cdot 9)$

$= \ln 45$

b) $2(\ln 21 - \ln 7) + \frac{1}{2}\ln 16$

$= 2\left(\ln\left(\frac{21}{7}\right)\right) + \frac{1}{2}\ln 16$

$= 2\ln 3 + \frac{1}{2}\ln 16$

$= \ln 3^2 + \ln 16^{1/2}$

$= \ln 9 + \ln 4$

$= \ln(9 \cdot 4)$

$= \ln(36)$

Pre-Calculus 12

$\ln q = \log_e q$

Ex. #6: Solve the following equations. (3 decimal places)

a) $\frac{85}{5} = \frac{5e^{3x}}{5}$

$17 = e^{3x}$

$\ln 17 = \ln e^{3x}$

$\ln 17 = 3x \ln e$

$\ln 17 = 3x(1)$

$\frac{\ln 17}{3} = x$

$x = .944$

b) $\frac{2\ln(2x+3)}{2} = \frac{8}{2}$

$\ln(2x+3) = 4$

$e^4 = 2x + 3$

$e^4 - 3 = 2x$

$\frac{e^4 - 3}{2} = x$

$x = 25.799$

$2x + 3 > 0$

$2(25.799) + 3 > 0$

> 0

Ex. #7: In 1986 the population of Canada was 25.5 million and was growing at the rate of approximately 1.2% per annum. $P = 25.5e^{0.012t}$ where P is population in millions and t is years after 1986.

a) Use the formula to predict the population for the year 2000.

year 2000

$t = 14$

$P = 25.5\, e^{0.012(14)}$

$P = 30.2$ million

b) Assuming that the growth rate remains constant, determine the year in which the population will reach 40 million.

$P = 40$ find t $\quad P = 25.5\, e^{0.012t}$

$40 = 25.5\, e^{0.012t}$

$\frac{40}{25.5} = e^{0.012t}$

$\ln\left(\frac{40}{25.5}\right) = \ln e^{0.012t}$

$\ln\left(\frac{40}{25.5}\right) = 0.012t \ln e$

$\frac{\ln\left(\frac{40}{25.5}\right)}{0.012} = t$

$t = 37.52$

year $= 1986 + 37.52$

year $= 2023.52$

during year 2023

1. Write each expression as a single logarithm

a) $\ln 24 + \ln 4 - 2\ln 3$

Pre-Calculus 12

1. Write each expression as a single logarithm
   a) $\ln 24 + \ln 4 - 2\ln 3$
   b) $\frac{1}{2}(\ln 45 - \ln 5)$
   c) $3\ln 2 - 2\ln 3 + \ln 27 - \frac{1}{3}\ln 125$
   d) $\frac{1}{3}\ln x + \frac{2}{5}\ln x^2 - \frac{2}{3}\ln x$

2. Solve each equation, giving answers rounded to three decimal places.
   a) $65 = e^{7n}$
   b) $1500 = 5e^{0.045x}$
   c) $2 = 4e^{5k}$
   d) $45 = e^{y+2}$
   e) $\ln 625 = 2.5x$
   f) $\ln(3t + 2) = 4$
   g) $2\ln t = 0.24$
   h) $3\ln(2x + 1) = 6$

3. Temperature, T, in degrees Celsius, of a cup of coffee t minutes after it is poured is given by $T = 95e^{-0.05t}$.
   a) How hot was the coffee when it was first poured?
   b) Find the temperature of the coffee 10 min later.
   c) When is the temperature of the coffee 45°C?

4. If two languages have evolved separately from a common ancestral language, the number of years since the split, *T(w)* is given by $T(w) = -5000\ln w$ where *w* is the percent of words that are still common to both languages. If two languages split 2000 years ago, what percent of the words would you expect to find in both languages today?

5. When Uranium238 decays, the percent P remaining after t years is given by the equation $P = 100e^{(-1.53\times10^{-1})t}$
   a) What percent remains after 10 million years
   b) Determine the half-life of Uranium-238

Answers for #1	Answers for #2	Answers for #3	Answer for #4	Answers for #5
a) $\ln\frac{32}{3}$ b) $\ln 3$ c) $\ln\frac{24}{5}$ d) $\ln x^{\frac{7}{15}}$	a) .596 b) 126.752 c) -.139 d) 1.807 e) 2.575 f) 17.533 g) 1.127 h) 3.195	a) 95° b) 56.7° c) 15.57 mins	67%	a) 99.85% a) 4,530,373,729