9.1 Part 1 Rational Functions Using Transformations

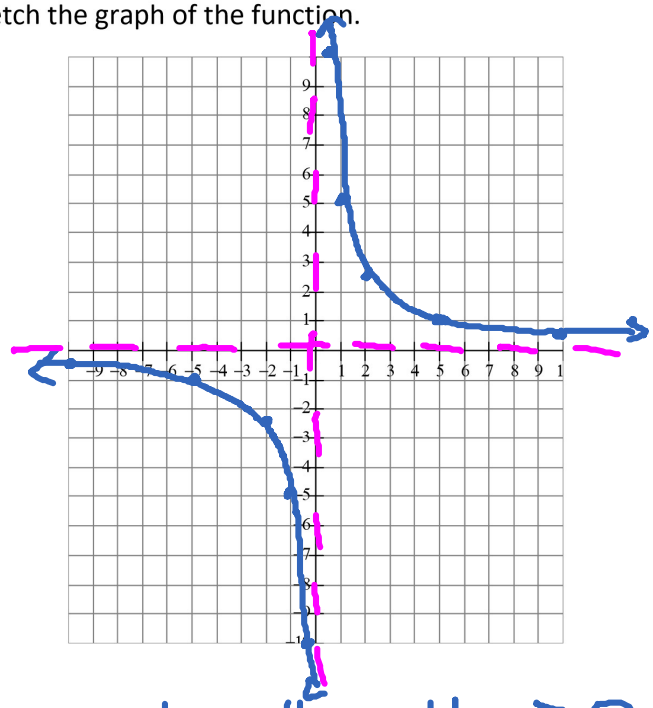
A **Rational Function** is a function that can be written in the form $\frac{P(x)}{q(x)}$

where p(x) and q(x) are polynomials $q(x) \neq 0$

Ex. #1: Use the table of values to sketch the graph of the function.

$y = \frac{5}{x}$

x	y
-10	-.5
-5	-1
-2	-2.5
-1	-5
-.5	-10
[illegible]	[illegible]
0	undefined
[illegible]	[illegible]
.5	10
1	5
2	2.5
5	1
10	.5



As the values of $|x|$ get larger the values of y get smaller $y \to 0$

As the values of $|x|$ get closer to zero the values of y get larger $y \to \infty$ or $y \to -\infty$

Non-Permissible Values: $x \neq 0$

Asymptotes: Vertical asymptote $x = 0$ horizontal $y = 0$

Domain: $\{x : x \neq 0, x \in \mathbb{R}\}$

Range: $\{y \mid y \neq 0, y \in \mathbb{R}\}$

Graphing Rational Functions using Transformations:

$$y = \frac{a}{x-h} + k$$

- a vertical stretch
- h horizontal translation
- k vertical translation

$y = \frac{1}{\frac{1}{2}(x-1)}$

$y = \frac{1}{\frac{1}{2}} \cdot \frac{1}{x-1}$

$y = 1 \cdot \frac{2}{1} \cdot \frac{1}{x-1}$

Ex. #2: Sketch the graph of $y = \frac{1}{x}$ and the graph of the function $y = \frac{2}{x-4} + 3$. State the equation of the asymptotes for the transformed function and the domain and range.

$y = \frac{2}{x-1}$

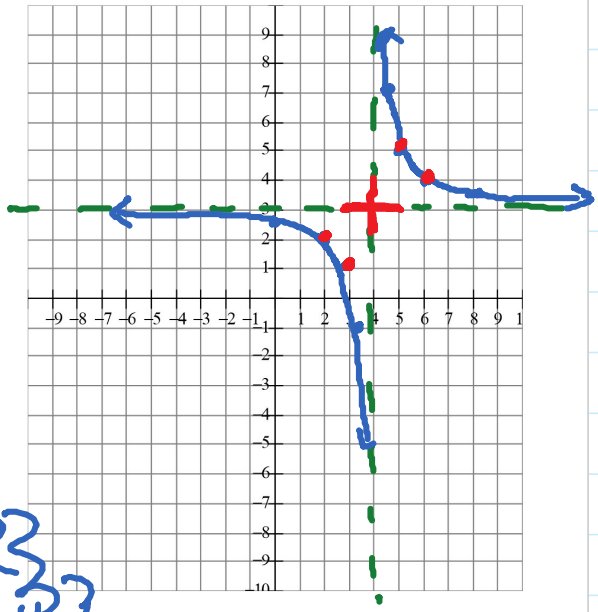
→ 4 ↑ 3

$y = \frac{1}{x}$

x	y
½	2
1	1
2	½
4	¼

$y = \frac{2}{x}$ mult y's by 2

x	y
½	4
1	2
2	1
4	½



Vertical Asymptote: $x = 4$

Horizontal Asymptote: $y = 3$

Domain : $\{x \mid x \neq 4, x \in \mathbb{R}\}$

Range : $\{y \mid y \neq 3, y \in \mathbb{R}\}$

How are the points related to the symmetry in the graph? Before the translation if (x,y) is on the graph then (y,x) (-x,-y) and (-y,-x) are also on the graph.

Ex. #3: Manipulate the function $y = \frac{3x+1}{x+1}$ into the form $y = \frac{a}{x-h} + k$ to reveal the

1-3

1-3

Ex. #3: Manipulate the function $y = \frac{3x+1}{x+1}$ into the form $y = \frac{a}{x-h} + k$ to reveal the asymptotes then sketch the graph. Find the intercepts of the function.

text book

Change the numerator to a factor of the denominator

$$y = \frac{3(x+1-1)+1}{x+1}$$

$$y = \frac{3(x+1)-3+1}{x+1}$$

$$y = \frac{3(x+1)}{x+1} + \frac{-2}{x+1}$$

$$y = \frac{-2}{x+1} + 3$$

Long Division

$$x+1 \overline{)\,3x+1}$$

$$3$$

$$3x+3$$

$$-2$$

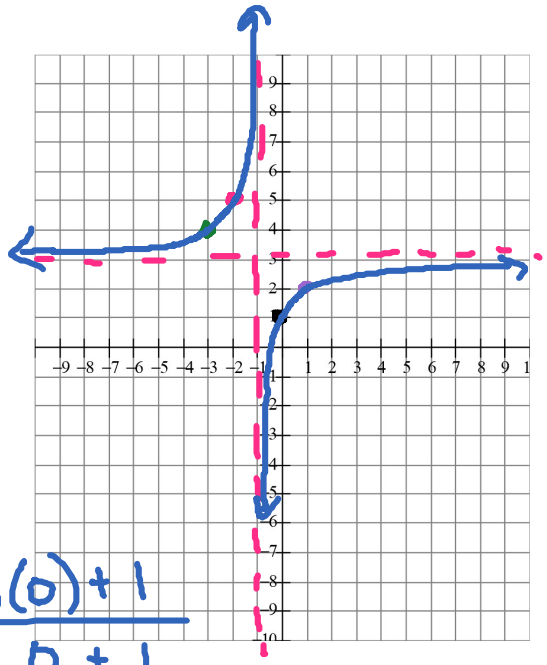
$$y = \frac{-2}{x+1} + 3$$

Vertical Asymptote: $x = -1$

Horizontal Asymptote: $y = 3$

$y = \frac{1}{x}$	
1	1

$y = \frac{-2}{x}$	
1	−2
−2	1
−1	2
2	−1



x-Intercept

$y = 0$

$$(x+1)0 = \frac{3x+1}{x+1}(x+1)$$

$$0 = 3x+1$$

$$-1 = 3x$$

$$-\frac{1}{3} = x$$

y-Intercept

$x = 0$

$$y = \frac{3(0)+1}{0+1}$$

$$y = \frac{1}{1}$$

$$y = 1$$

$-1-(-4)$

Ex. #4: Sketch the graph of the function $y = \frac{2x-1}{x-2}$.

$$x-2 \overline{)\,2x-1}$$
$$2x-4$$
$$3$$

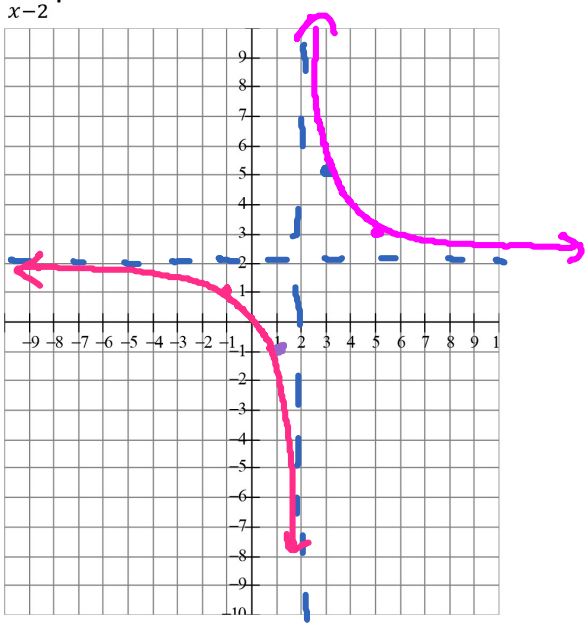
$$y = \frac{3}{x-2} + 2$$

$y = \frac{1}{x}$

1	1

$y = \frac{3}{x}$

1	3
3	1
-1	-3
-3	-1



Ex. #5: Find the asymptotes and intercepts of the function $y = \frac{5x+7}{x-4}$.

Vertical Asymptote:

When the denominator = 0

$x - 4 = 0$

$x = 4$

Horizontal Asymptote:

Translation up or down

Rewrite using long division

$$x-4 \overline{)\,5x+7}$$
$$5x-20$$
$$27$$

$$y = \frac{27}{x-4} + 5$$

asymptote $y = 5$

x-intercept

$y = 0$

$0 = \frac{5x+7}{x-4}$

$0 = 5x + 7$

$-7 = 5x$

$-\frac{7}{5} = x$

y-intercept

$x = 0$

$y = \frac{5(0)+7}{0-4}$

$y = \frac{7}{-4}$

$y = -\frac{7}{4}$

$y = \frac{27}{0-4} + 5$

$y = \frac{27}{-4} + \frac{20}{4}$

$y = -\frac{27}{4} + \frac{20}{4}$

$y = -\frac{7}{4}$