9.2 New

Sunday, May 24, 2020 10:25 AM

9.2 Analyzing Rational Functions

Graphs of rational functions may have a variety of shapes and features

A vertical asymptote corresponds to non-permissible value in the denominator

Non-permissible value can result in a point of discontinuity

Point of Discontinuity: point where the graph is not continuous (Hole)

Ex. #1: Find the point of discontinuity and sketch the graph of the function.

$y = \frac{x^2+x-12}{x-3}$

$x \neq 3$

$y = \frac{(x+4)(x-3)}{x-3}$

Since the factors cancel there is a point of discontinuity

$y = x+4$

$m = 1$

$b = 4$

$y = mx + b$

m = slope

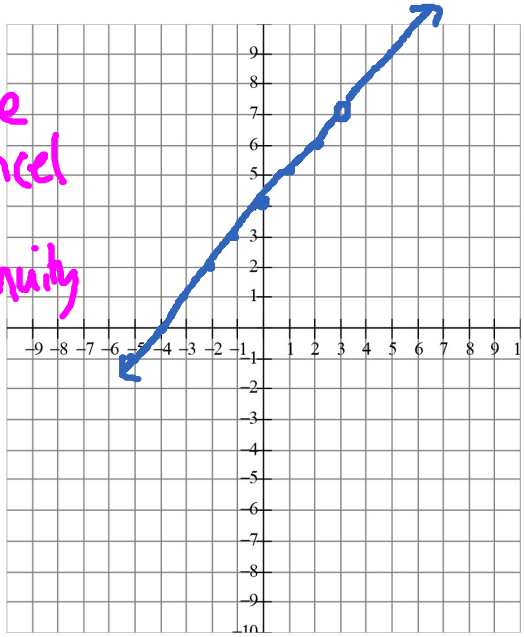
b = y-int

Find the y-value

$y = x+4 \quad x=3$

$y = 3+4$

$y = 7$

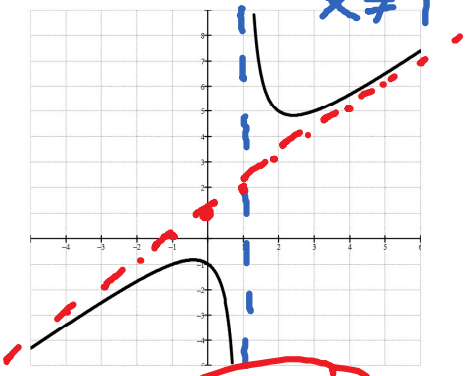


Point of Discontinuity: $(3,7)$

Domain: $\{x \mid x \neq 3, x \in \mathbb{R}\}$ Range: $\{y \mid y \neq 7, y \in \mathbb{R}\}$

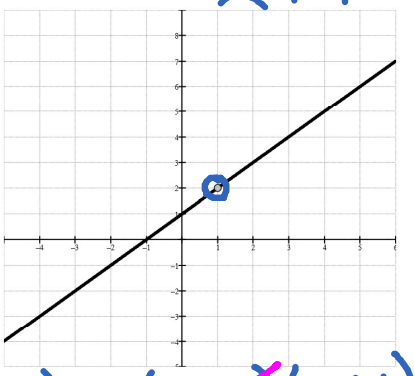
Ex. #2: Graph the two functions and compare their behavior near nay non-permissible values. Explain the differences.

$$f(x) = \frac{x^2 + 1}{x - 1}$$



$x \neq 1$

$$g(x) = \frac{x^2 - 1}{x - 1}$$



$x \neq 1$

$$x - 1 \overline{)\, x^2 + 0x + 1}$$

$x + 1$

$x^2 - x$

$1x + 1$

$1x - 1$

2

$$y = \frac{2}{x - 1} + (x + 1)$$

slant asymptote

$y = x + 1$

$$g(x) = \frac{(x-1)(x+1)}{x-1}$$

$g(x) = x + 1$

$x = 1$

$g(1) = 1 + 1$

$g(1) = 2$

Characteristic	$f(x) = \frac{x^2 + 1}{x - 1}$	$g(x) = \frac{x^2 - 1}{x - 1}$
Non-Permissible Value	$x \neq 1$	$x \neq 1$
Feature at Non-Permissible Value	vertical asymptote	point of discontinuity
Behaviour near Non-Permissible Value	$x \to 1$ $y \to \infty$ or $y \to -\infty$ $\lvert y \rvert \to \infty$	$x \to 1$ $y \to 2$

y-values

Ex. #3: Match the equation of the rational function to the graph. Use the location of any asymptotes, discontinuities, and intercepts for each function to make the appropriate choice.

$$f(x) = \frac{x^2 + 3x}{x^2 - 9} \qquad g(x) = \frac{3x + 1}{x^2 + 1} \qquad h(x) = \frac{3x}{x^2 - 9}$$

$f(x) = \frac{x(x+3)}{(x-3)(x+3)}$ $y = \frac{0}{0-3}$ $y = \frac{0}{-3}$ $y = 0$

$(x^2-9)\,0 = \left(\frac{x^2+3x}{x^2-9}\right)(x^2-9)$ $0 = x^2 + 3x$ $0 = x(x+3)$ $x = 0$ $x = -3$

$x^2 + 1 = 0$ $x^2 = -1$

$0 = \frac{3x+1}{x^2+1}$ $0 = 3x + 1$ $-1 = 3x$ $-\frac{1}{3} = x$

$y = \frac{3(0)+1}{0^2+1}$ $y = \frac{1}{1}$ $y = 1$

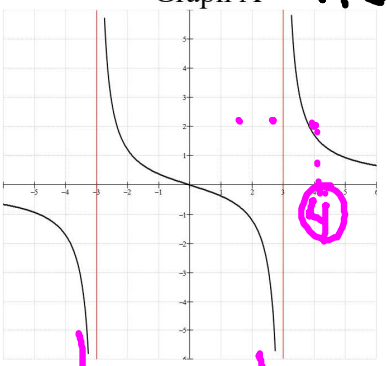
$h(x) = \frac{3x}{(x-3)(x+3)}$

$0 = \frac{3x}{(x-3)(x+3)}$ $0 = 3x$ $0 = x$

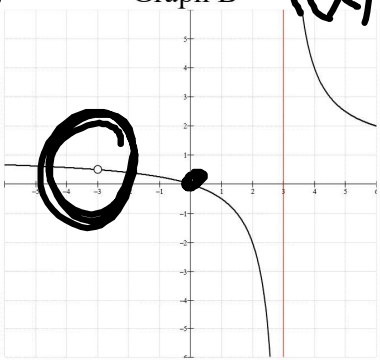
$y = \frac{3(0)}{(0-3)(0+3)}$ $y = \frac{0}{-9}$ $y = 0$

	$f(x)$	$g(x)$	$h(x)$
Vertical Asymptotes	$x = 3$	X	$x = 3$, $x = -3$
Discontinuity	$x = -3$	X	X
Intercepts	$(0,0)$	$(-\frac{1}{3}, 0)$, $(0,1)$	$(0,0)$

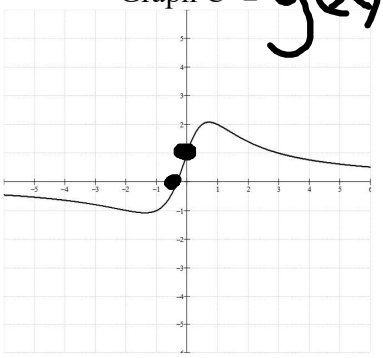
Graph A = $h(x)$

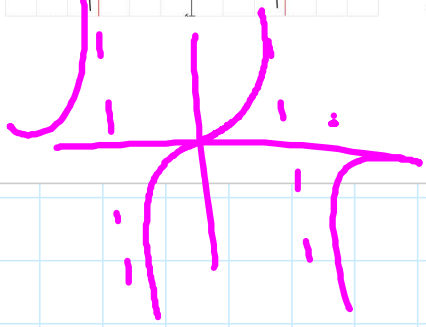


Graph B = $f(x)$



Graph C = $g(x)$





Name: _____

1. A
2. B
3. C
4.

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3. C
4.