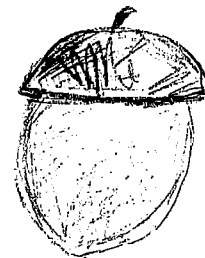


# Calculus AB – Acorn, 2008



## Calculus AB: Section I

### Part A Sample Multiple-Choice Questions

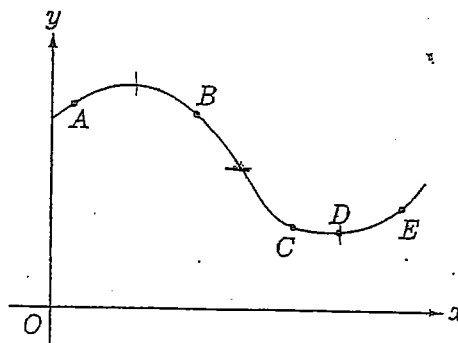
A calculator may not be used on this part of the exam.

1. What is  $\lim_{h \rightarrow 0} \frac{\cos\left(\frac{3\pi}{2} + h\right) - \cos\left(\frac{3\pi}{2}\right)}{h}$ ?
- (A) 1  
(B)  $\frac{\sqrt{2}}{2}$   
(C) 0  
(D) -1  
(E) The limit does not exist.

2. At which of the five points on the graph in the figure

at the right are  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  both negative?

- (A) A  
(B) B  
(C) C  
(D) D  
(E) E

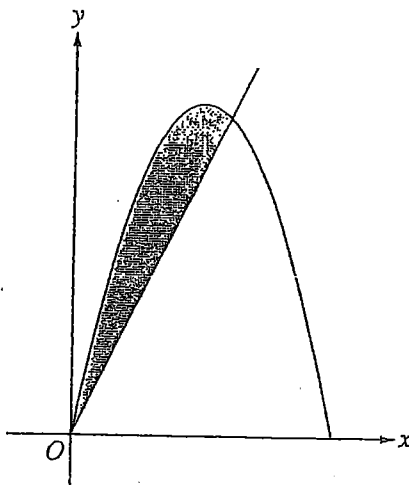


3. The slope of the tangent to the curve  $y^3x + y^2x^2 = 6$  at  $(2, 1)$  is
- (A)  $-\frac{3}{2}$   
(B) -1  
(C)  $-\frac{5}{14}$   
(D)  $-\frac{3}{14}$   
(E) 0

4. Let  $S$  be the region enclosed by the graphs of  $y = 2x$  and  $y = 2x^2$  for  $0 \leq x \leq 1$ . What is the volume of the solid generated when  $S$  is revolved about the line  $y = 3$ ?
- (A)  $\pi \int_0^1 \left( (3 - 2x^2)^2 - (3 - 2x)^2 \right) dx$
- (B)  $\pi \int_0^1 \left( (3 - 2x)^2 - (3 - 2x^2)^2 \right) dx$
- (C)  $\pi \int_0^1 (4x^2 - 4x^4) dx$
- (D)  $\pi \int_0^2 \left( \left( 3 - \frac{y}{2} \right)^2 - \left( 3 - \sqrt{\frac{y}{2}} \right)^2 \right) dy$
- (E)  $\pi \int_0^2 \left( \left( 3 - \sqrt{\frac{y}{2}} \right)^2 - \left( 3 - \frac{y}{2} \right)^2 \right) dy$
5. Which of the following statements about the function given by  $f(x) = x^4 - 2x^3$  is true?
- (A) The function has no relative extremum.
- (B) The graph of the function has one point of inflection and the function has two relative extrema.
- (C) The graph of the function has two points of inflection and the function has one relative extremum.
- (D) The graph of the function has two points of inflection and the function has two relative extrema.
- (E) The graph of the function has two points of inflection and the function has three relative extrema.
6. If  $f(x) = \sin^2(3 - x)$ , then  $f'(0) =$
- (A)  $-2 \cos 3$
- (B)  $-2 \sin 3 \cos 3$
- (C)  $6 \cos 3$
- (D)  $2 \sin 3 \cos 3$
- (E)  $6 \sin 3 \cos 3$
7. Which of the following is the solution to the differential equation  $\frac{dy}{dx} = \frac{4x}{y}$ , where  $y(2) = -2$ ?
- (A)  $y = 2x$  for  $x > 0$
- (B)  $y = 2x - 6$  for  $x \neq 3$
- (C)  $y = -\sqrt{4x^2 - 12}$  for  $x > \sqrt{3}$
- (D)  $y = \sqrt{4x^2 - 12}$  for  $x > \sqrt{3}$
- (E)  $y = -\sqrt{4x^2 - 6}$  for  $x > \sqrt{1.5}$

Sample Questions for Calculus AB: Section I

8. What is the average rate of change of the function  $f$  given by  $f(x) = x^4 - 5x$  on the closed interval  $[0, 3]$ ?
- (A) 8.5  
(B) 8.7  
(C) 22  
(D) 33  
(E) 66
9. The position of a particle moving along a line is given by  $s(t) = 2t^3 - 24t^2 + 90t + 7$  for  $t \geq 0$ . For what values of  $t$  is the speed of the particle increasing?
- (A)  $3 < t < 4$  only  
(B)  $t > 4$  only  
(C)  $t > 5$  only  
(D)  $0 < t < 3$  and  $t > 5$   
(E)  $3 < t < 4$  and  $t > 5$
10.  $\int (x-1)\sqrt{x} \, dx =$
- (A)  $\frac{3}{2}\sqrt{x} - \frac{1}{\sqrt{x}} + C$   
(B)  $\frac{2}{3}x^{3/2} + \frac{1}{2}x^{1/2} + C$   
(C)  $\frac{1}{2}x^2 - x + C$   
(D)  $\frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2} + C$   
(E)  $\frac{1}{2}x^2 + 2x^{3/2} - x + C$
11. What is  $\lim_{x \rightarrow \infty} \frac{x^2 - 4}{2 + x - 4x^2}$ ?
- (A) -2  
(B)  $-\frac{1}{4}$   
(C)  $\frac{1}{2}$   
(D) 1  
(E) The limit does not exist.



12. The figure above shows the graph of  $y = 5x - x^2$  and the graph of the line  $y = 2x$ . What is the area of the shaded region?

- (A)  $\frac{25}{6}$   
 (B)  $\frac{9}{2}$   
 (C) 9  
 (D)  $\frac{27}{2}$   
 (E)  $\frac{45}{2}$

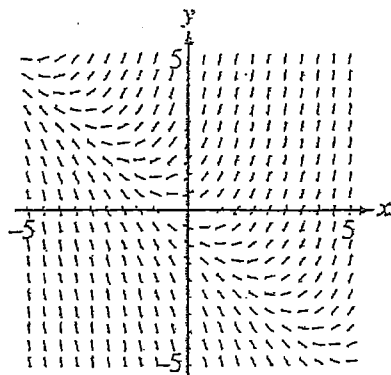
13. If  $y = 5 + \int_2^{2x} e^{-t^2} dt$ , which of the following is true?

- (A)  $\frac{dy}{dx} = e^{-x^2}$  and  $y(0) = 5$   
 (B)  $\frac{dy}{dx} = e^{-x^2}$  and  $y(1) = 5$   
 (C)  $\frac{dy}{dx} = e^{-4x^2}$  and  $y(1) = 5$   
 (D)  $\frac{dy}{dx} = 2e^{-4x^2}$  and  $y(0) = 5$   
 (E)  $\frac{dy}{dx} = 2e^{-4x^2}$  and  $y(1) = 5$

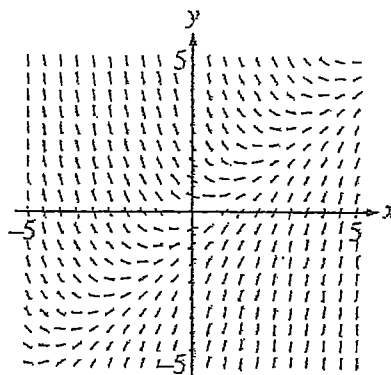
Sample Questions for Calculus AB: Section I

14. Which of the following is a slope field for the differential equation  $\frac{dy}{dx} = \frac{x}{y}$ ?

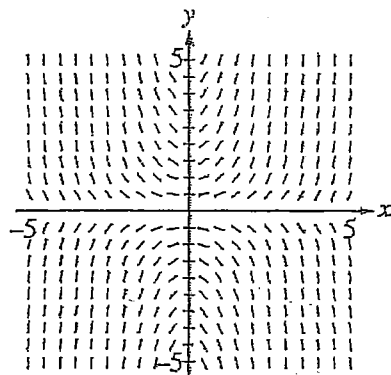
(A)



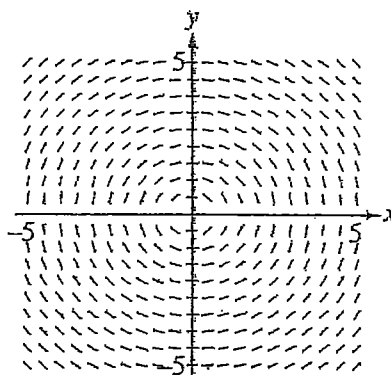
(B)



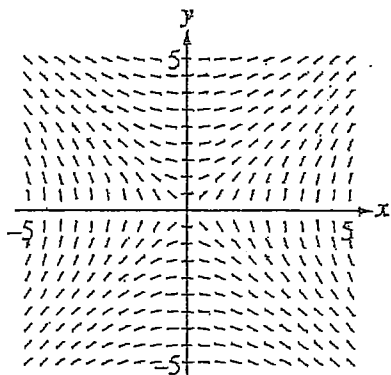
(C)



(D)



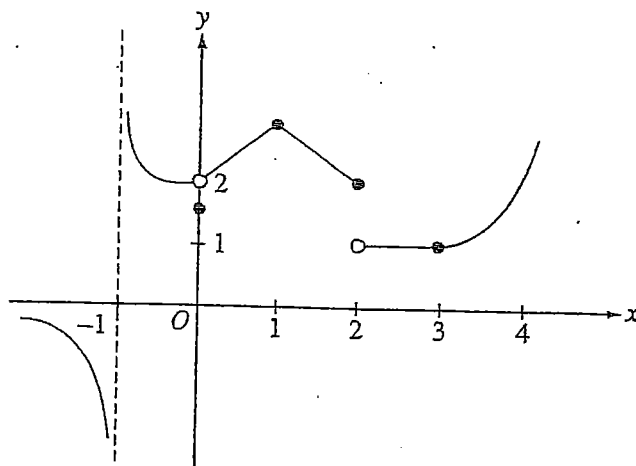
(E)



## Part B Sample Multiple-Choice Questions

A graphing calculator is required for some questions on this part of the exam.

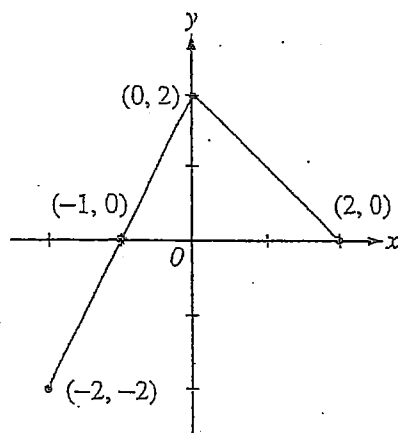
15. A particle travels along a straight line with a velocity of  $v(t) = 3e^{(-t/2)}\sin(2t)$  meters per second. What is the total distance, in meters, traveled by the particle during the time interval  $0 \leq t \leq 2$  seconds?
- (A) 0.835  
 (B) 1.850  
 (C) 2.055  
 (D) 2.261  
 (E) 7.025
16. A city is built around a circular lake that has a radius of 1 mile. The population density of the city is  $f(r)$  people per square mile, where  $r$  is the distance from the center of the lake, in miles. Which of the following expressions gives the number of people who live within 1 mile of the lake?
- (A)  $2\pi \int_0^1 r f(r) dr$   
 (B)  $2\pi \int_0^1 r(1 + f(r)) dr$   
 (C)  $2\pi \int_0^2 r(1 + f(r)) dr$   
 (D)  $2\pi \int_1^2 r f(r) dr$   
 (E)  $2\pi \int_1^2 r(1 + f(r)) dr$



17. The graph of a function  $f$  is shown above. If  $\lim_{x \rightarrow b} f(x)$  exists and  $f$  is not continuous at  $b$ , then  $b =$
- (A) -1  
 (B) 0  
 (C) 1  
 (D) 2  
 (E) 3

$x$	1.1	1.2	1.3	1.4
$f(x)$	4.18	4.38	4.56	4.73

18. Let  $f$  be a function such that  $f''(x) < 0$  for all  $x$  in the closed interval  $[1, 2]$ . Selected values of  $f$  are shown in the table above. Which of the following must be true about  $f'(1.2)$ ?
- (A)  $f'(1.2) < 0$   
 (B)  $0 < f'(1.2) < 1.6$   
 (C)  $1.6 < f'(1.2) < 1.8$   
 (D)  $1.8 < f'(1.2) < 2.0$   
 (E)  $f'(1.2) > 2.0$
19. Two particles start at the origin and move along the  $x$ -axis. For  $0 \leq t \leq 10$ , their respective position functions are given by  $x_1 = \sin t$  and  $x_2 = e^{-2t} - 1$ . For how many values of  $t$  do the particles have the same velocity?
- (A) None  
 (B) One  
 (C) Two  
 (D) Three  
 (E) Four

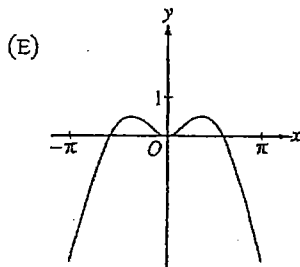
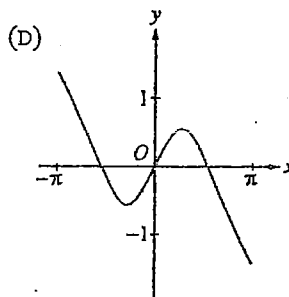
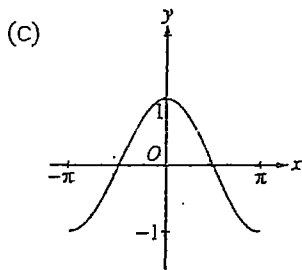
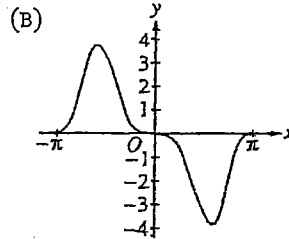
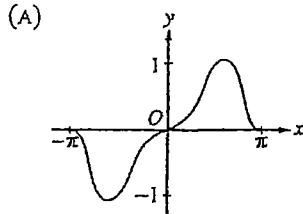


Graph of  $f$

20. The graph of the function  $f$  shown above consists of two line segments. If  $g$  is the function defined by  $g(x) = \int_0^x f(t) dt$ , then  $g(-1) =$
- (A)  $-2$   
 (B)  $-1$   
 (C)  $0$   
 (D)  $1$   
 (E)  $2$

Sample Questions for Calculus AB: Section I

21. The graphs of five functions are shown below. Which function has a nonzero average value over the closed interval  $[-\pi, \pi]$ ?



22. A differentiable function  $f$  has the property that  $f(5) = 3$  and  $f'(5) = 4$ . What is the estimate for  $f(4.8)$  using the local linear approximation for  $f$  at  $x = 5$ ?

- (A) 2.2  
 (B) 2.8  
 (C) 3.4  
 (D) 3.8  
 (E) 4.6



Sample Questions for Calculus AB: Section I

23. Oil is leaking from a tanker at the rate of  $R(t) = 2,000e^{-0.2t}$  gallons per hour, where  $t$  is measured in hours. How much oil leaks out of the tanker from time  $t = 0$  to  $t = 10$ ?
- (A) 54 gallons  
 (B) 271 gallons  
 (C) 865 gallons  
 (D) 8,647 gallons  
 (E) 14,778 gallons
24. If  $f'(x) = \sin\left(\frac{\pi e^x}{2}\right)$  and  $f(0) = 1$ , then  $f(2) =$
- (A) -1.819  
 (B) -0.843  
 (C) -0.819  
 (D) 0.157  
 (E) 1.157

Answers to Calculus AB Multiple-Choice Questions

<i>Part A</i>	<i>Part B</i>
1. A	15.* D
2. B	16. D
3. C	17. B
4. A	18. D
5. C	19.* D
6. B	20. B
†7. C	21. E
8. C	22. A
9. E	23.* D
10. D	24.* E
11. B	
12. B	
13. E	
14. E	

\*Indicates a graphing calculator-active question.

†For resources on differential equations, see the Home Pages for Calculus AB and Calculus BC at AP Central.