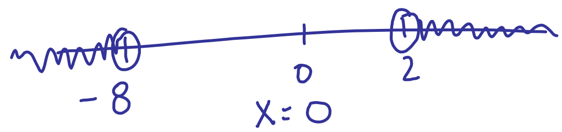
<u>Pre-requisites for AP Calculus</u>

Interval on the Real number line

- Bounded Open Interval $\{x: a < x < b\}$ (a, b)
- Bounded Closed Interval $\{x: a \leq x \leq b\}$ $[a, b]$
- Unbounded Open Interval $\{x: x < b\}$ $(-\infty, b)$
  $\{x: x > a\}$ (a, ∞)
- Entire Real Line $\{x: x \epsilon R\}$ $(-\infty, \infty)$

Solving Inequalities

a) *Solve* $|x+3| > 5$



$x + 3 > 5$
$x > 2$

$-(x+3) > 5$
$-x - 3 > 5$
$-x > 8$
$x < -8$

$x = 0$
$|0+3| > 5$
$|3| > 5$
$3 > 5$
X

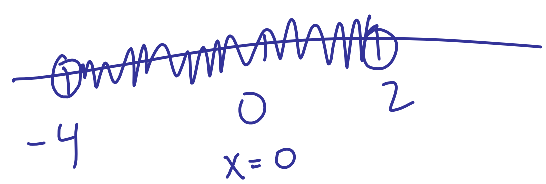
$$\boxed{x > 2, \; x < -8}$$

b) Quadratic *Solve* $x^2 < -2x + 8$

$x^2 + 2x - 8 < 0$
$(x+4)(x-2) < 0$

$x + 4 < 0$
$x < -4$

$x - 2 < 0$
$x < 2$



$x = 0$
$0^2 < 0 + 8$
$0 < 8$
✓

$-4 < x < 2$

Coordinate Geometry

- Distance Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Midpoint Formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
- Standard Form of a Circle $(x-h)^2 + (y-k)^2 = r^2$

center (h,k)
radius $= r$

- Standard Form of a Parabola $y = a(x-p)^2 + q$

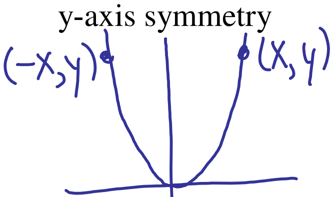
Rewrite $y = 2x^2 - 12x + 7$ *in standard form*

$$y = 2(x^2 - 6x) + 7$$
$$y = 2(x^2 - 6x + \boxed{9}) + 7 - 2\boxed{9}$$
$$y = 2(x-3)^2 + 7 - 18$$
$$y = 2(x-3)^2 - 11$$

$$\left(-\frac{6}{2}\right)^2 = (-3)^2 = 9$$

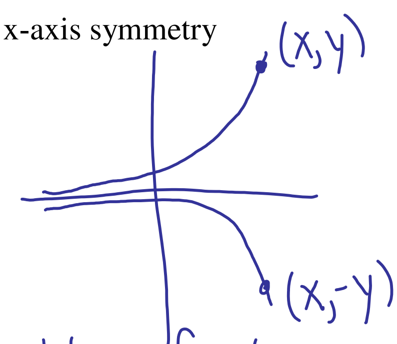
Symmetry

y-axis symmetry



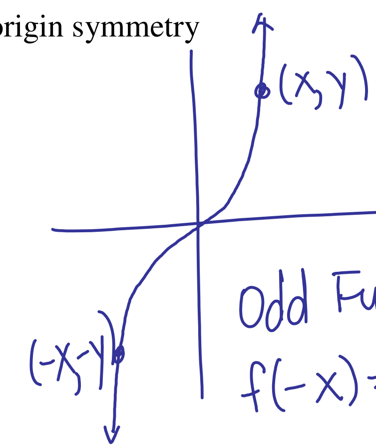
Even function
$f(-x) = f(x)$

x-axis symmetry



Not a function

origin symmetry



Odd Function
$f(-x) = -f(x)$

Determine if the function is even or odd.

$f(x) = x^3 - x$

$$f(-x) = (-x)^3 - (-x)$$
$$= -x^3 + x$$
$$= -(x^3 - x)$$
$$= -f(x)$$

Odd

$f(x) = x^2 + 1$

$$f(-x) = (-x)^2 + 1$$
$$= x^2 + 1$$
$$= f(x)$$

Even

Intercepts

Find the x and y intercepts of $y = x^3 - 4x$

x-intercept
$y = 0$

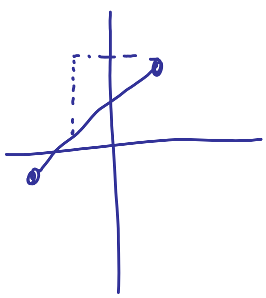
$0 = x^3 - 4x$
$0 = x(x^2 - 4)$
$0 = x(x+2)(x-2)$
$x = 0 \quad x = -2 \quad x = 2$

y-intercept
$x = 0$
$y = 0^3 - 4(0)$
$y = 0$

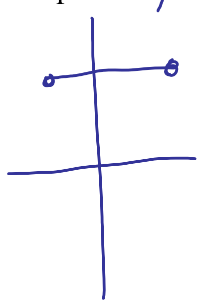
Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

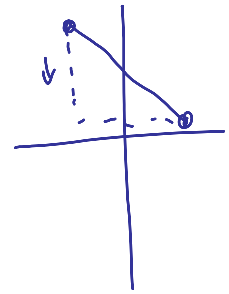
Positive Slope



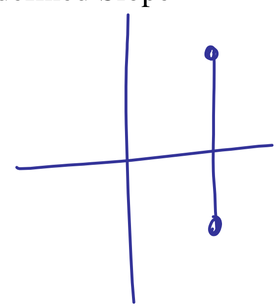
Zero Slope $= \frac{0}{7}$



Negative Slope



No Slope or Undefined Slope $= \frac{7}{0}$



Equations of Lines

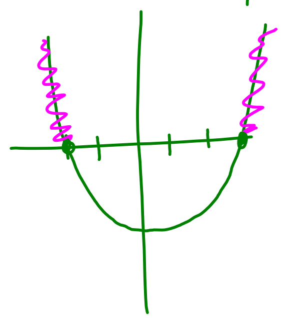
- General Form $Ax + By + C = 0$
- Vertical $x = a$
- Horizontal $y = b$
- Point Slope $y - y_1 = m(x - x_1)$
- Slope Intercept $y = mx + b$

Parallel Lines: same slope $m_1 = m_2$

Perpendicular Lines: negative reciprocal slopes

$m_1 = -\frac{1}{m_2}$ $\quad m_1 = 3 \quad m_2 = -\frac{1}{3}$

$y = x^2 - x - 6$



Domain and Range

$x \geq 3 \quad x \leq -2$

$[3, \infty) \cup (-\infty, -2]$

Domain: Set of all x-values
Range : Set of all y-values

Find the domain of $(x) = \sqrt{x^2 - x - 6}$

$x^2 - x - 6 \geq 0$

$(x-3)(x+2) \geq 0$

$x \geq 3 \quad x \geq -2$

Transformations

Vertical shift		**Horizontal shift**	
• $y = f(x) - c$	down	• $y = f(x - c)$	Right
• $y = f(x) + c$	up	• $y = f(x + c)$	Left
Reflections		**Stretches**	
• $y = -f(x)$	Reflect over x-axis	• $y = af(x)$	Vertical stretch factor $\lvert a \rvert$
• $y = f(-x)$	Reflect over y-axis	• $y = f(bx)$	Horizontal stretch factor $\lvert \frac{1}{b} \rvert$

Composition of Functions

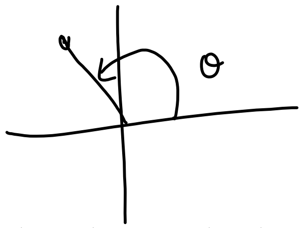
$(f \circ g)(x) = f(g(x))$

Find $(f \circ g)(x)$ *if* $f(x) = 2x + 3$ *and* $g(x) = x^2 + 1$

$$(f \circ g)(x) = f(g(x))$$
$$= f(x^2 + 1)$$
$$= 2(x^2 + 1) + 3$$
$$= 2x^2 + 2 + 3 \quad = 2x^2 + 5$$

Trigonometry

Angles in Standard Position: Measured from the positive x-axis in a counterclockwise direction.



Coterminal Angles: Angles that have the same terminal arm.

$\theta \pm 360^\circ$ or $\theta \pm 2\pi$

$$\text{coterminal} = \theta \pm 360^\circ n$$
$$= \theta \pm 2\pi n$$
$$n \in I$$

Radian Measure: $180^\circ = \pi\ radians$

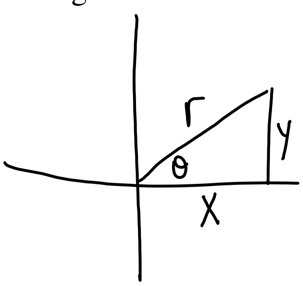
Convert 40° *into radians.*

$$40^\circ \cdot \left(\frac{\pi}{180^\circ}\right) = \frac{2\pi}{9}$$

Convert $\frac{\pi}{2}$ *into degrees*

$$\frac{\pi}{2} \cdot \left(\frac{180^\circ}{\pi}\right) = 90^\circ$$

Trig Functions:



$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

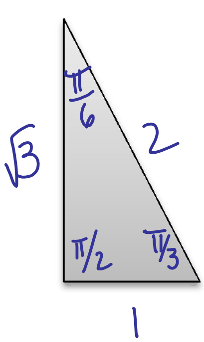
$\tan\theta = \frac{y}{x}$

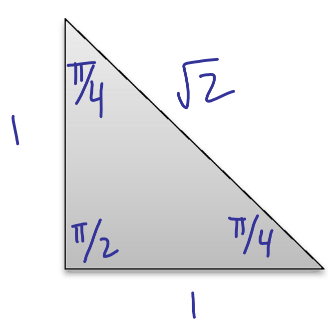
$\csc\theta = \frac{r}{y}$

$\sec\theta = \frac{r}{x}$

$\cot\theta = \frac{x}{y}$

Exact Values and Special Triangles:





$\cos 2\theta = 1 - 2\sin^2\theta$

Trig Equations:

Solve $\cos 2\theta = 2 - 3\sin\theta$ *where* $0 \leq \theta < 2\pi$

$1 - 2\sin^2\theta = 2 - 3\sin\theta$

$0 = 2\sin^2\theta - 3\sin\theta + 1$

$\_\, x \,\_ = 2$

$-2 + -1 = -3$

$0 = \underbrace{2\sin^2\theta - 2\sin\theta} \underbrace{- 1\sin\theta + 1}$

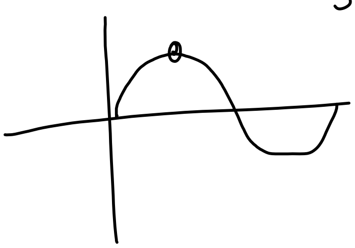
$0 = 2\sin\theta(\sin\theta - 1) - 1(\sin\theta - 1)$

$0 = (\sin\theta - 1)(2\sin\theta - 1)$

$\sin\theta - 1 = 0$

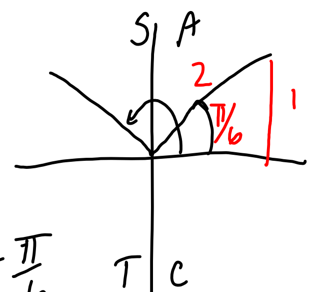
$\sin\theta = 1$

$\theta = \pi/2$



$2\sin\theta - 1 = 0$

$\sin\theta = \frac{1}{2} \quad \frac{y}{r}$

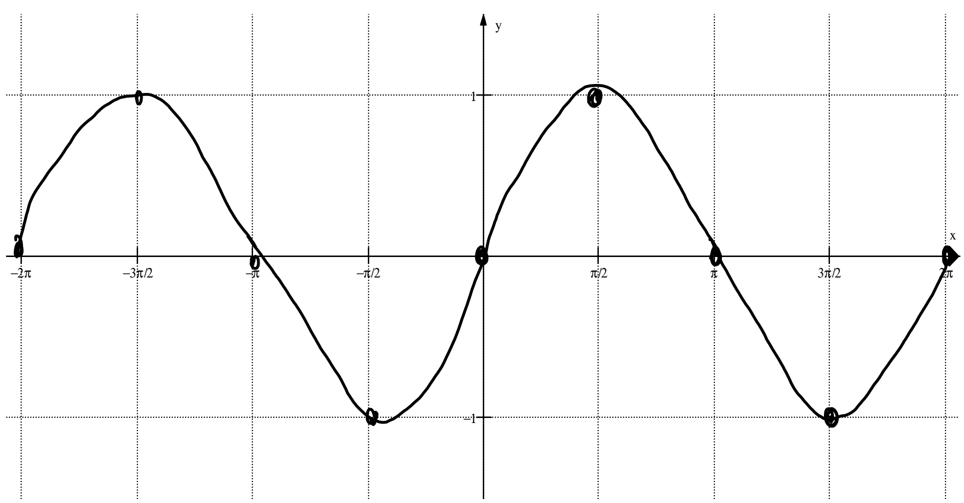


$\theta = \frac{\pi}{6}$ $\quad \theta = \pi - \frac{\pi}{6}$

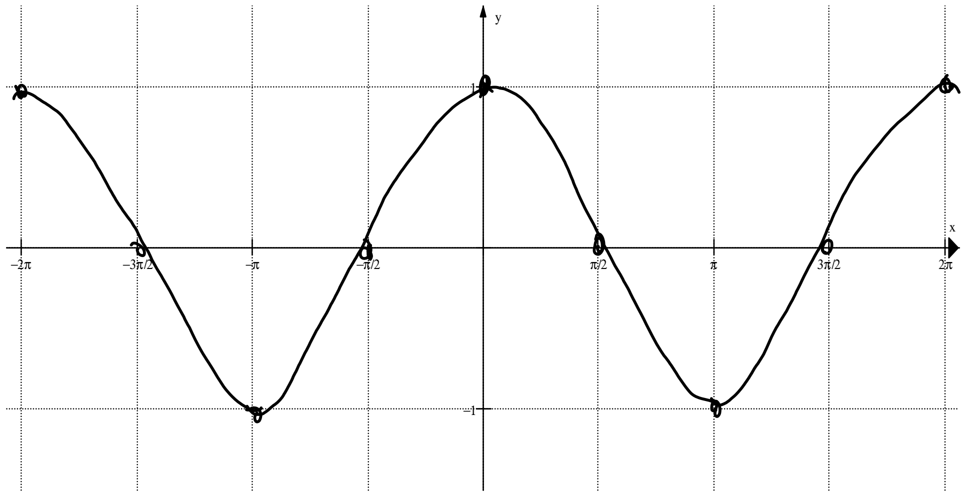
$\theta = 5\pi/6$

Graphs of Trig Functions:

$y = \sin x$



$y = \cos x$



$y = \tan x$