Pg 409

#10 $g'(8) = ?$ $\qquad f(-1) = 8$

$g(x) = f^{-1}(x)$

$(-1, 8)$ is on $f(x)$

$(8, -1)$ is on $g(x)$

$$g'(8) = \frac{1}{f'(-1)} \qquad f'(-1) = 12$$

$$= \frac{1}{12}.$$

13) $f(x) = 9e^{-4x}$

$$f'(x) = 9e^{-4x} \cdot (-4)$$

$$= -36e^{-4x}$$

15) $f(x) = \frac{e^{-x}}{x}$ $\qquad$ Quotient Rule.

$$f'(x) = \frac{xe^{-x}(-1) - (1)e^{-x}}{x^2}$$

$$= \frac{-xe^{-x} - e^{-x}}{x^2}$$

$$= \frac{-e^{-x}(x+1)}{x^2}.$$

17. $G(s) = (\ln(s))^2$ chain Rule.

$G'(s) = 2[\ln(s)]^1 \cdot \frac{1}{s}$

$= \frac{2\ln s}{s}$

$= \frac{\ln s^2}{s}$

19. $g(t) = e^{4t-t^2}$

$g'(t) = e^{4t-t^2} \cdot (4-2t)$

$g'(t) = (4-2t)\, e^{4t-t^2}$

21. $f(\theta) = \ln(\sin\theta)$ chain Rule

$f'(\theta) = \frac{1}{\sin\theta} \cdot \cos\theta$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$

23. $f(x) = \ln(e^x - 4x)$

$f'(x) = \frac{1}{e^x - 4x} \cdot (e^x - 4)$

$= \frac{e^x - 4}{e^x - 4x}$

25. $f(x) = e^{x+\ln x}$

$f'(x) = e^{x+\ln x} \cdot \left(1 + \frac{1}{x}\right)$ ← text stops here

$= e^x \cdot e^{\ln x} \left(\frac{x}{x} + \frac{1}{x} \right)$

$= e^x \cdot x \left(\frac{x+1}{x} \right)$

$= \frac{e^x x (x+1)}{x}$

$= e^x (x+1)$

27. $h(y) = 2^{1-y}$

$h'(x) = 2^{1-y} \cdot \ln(2) \cdot (-1)$

$= -(\ln 2) 2^{1-y}$

29. $f(x) = 7^{-2x}$

$f'(x) = 7^{-2x} \cdot \ln 7 \cdot (-2)$

$= -2(\ln 7) 7^{-2x}$

35. $R(s) = s^{\ln s}$

$\ln R = \ln\left(s^{\ln s}\right)$

$\ln R = (\ln s)(\ln s)$

$\ln R = (\ln s)^2$

$\frac{1}{R}\frac{dR}{ds} = 2\ln s \cdot \frac{1}{s}$

$\frac{dR}{ds} = R \cdot \frac{\ln s^2}{s}$

$\frac{dR}{ds} = s^{\ln s} \cdot \frac{\ln s^2}{s}$

43. $y=f(x)$ at $x=4$ tangent line $y=-2x+12$

$y=-2(4)+12$ on $f(x)$ $(4,4)$

$y=4$

on inverse $(4,4)$

Slope of tangent to $f(x)$:

$m=-2$

slope of tangent to inverse

$m=\frac{-1}{2}$

reciprocals

$y-4=\frac{-1}{2}(x-4)$ Equation of line

$y=\frac{-x}{2}+2+4$

$y=\frac{-1}{2}x+6$

53. $y = \dfrac{(x+1)^3}{(4x-2)^2}$

$$\ln y = \ln\left(\frac{(x+1)^3}{(4x-2)^2}\right)$$

$$\ln y = \ln(x+1)^3 - \ln(4x-2)^2$$

$$\ln y = 3\ln(x+1) - 2\ln(4x-2)$$

$$\frac{1}{y}\frac{dy}{dx} = 3\left(\frac{1}{x+1}\right) - 2\left(\frac{1}{4x-2}\right)(4)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{3}{x+1} - \frac{8}{4x-2}$$

$$\frac{dy}{dx} = y\left[\frac{3}{x+1} - \frac{8}{4x-2}\right]$$

$$\frac{dy}{dx} = \frac{(x+1)^3}{(4x-2)^2}\left[\frac{3}{x+1} - \frac{8}{4x-2}\right]$$

67. $\int e^{9-2x}\,dx$

$u = 9-2x$

$\frac{du}{dx} = -2$

$\frac{du}{-2} = dx$

$\int e^{u} \cdot \frac{du}{-2}$

$= -\frac{1}{2} e^{u} + C$

$= -\frac{1}{2} e^{9-2x} + C$

69. $\int e^{-2x} \sin\left(e^{-2x}\right) dx$

$u = e^{-2x}$

$\frac{du}{dx} = -2e^{-2x}$

$\frac{du}{-2} = e^{-2x}\,dx$

$\int \sin u \cdot \frac{du}{-2}$

$= -\frac{1}{2}\left(-\cos u\right) + C$

$= \frac{\cos e^{-2x}}{2} + C$

71. $\int_{1}^{3} e^{4x-3}\,dx$

$u = 4x-3$

$\frac{du}{dx} = 4$

$\frac{du}{4} = dx$

$x = 1$

$u = 4(1) - 3$

$u = 1$

$x = 3$

$u = 4(3) - 3$

$u = 9$

$\int_{1}^{9} e^{u} \frac{du}{4}$

$= \frac{1}{4} e^{u} \Big|_{1}^{9}$

$= \frac{1}{4}\left[e^{9} - e^{1}\right]$

73. $\int_1^e \frac{\ln x}{x}\,dx = \int_1^e \ln x \cdot \frac{1}{x}\,dx$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x}\,dx$

$x = 1$
$u = \ln 1$
$u = 0$

$x = e$
$u = \ln e$
$u = 1$

$= \int_0^1 u\,du$

$= \frac{1}{2}u^2 \Big|_0^1$

$= \frac{1}{2}(1^2 - 0^2)$

$= \frac{1}{2}$

75. $\int_{\frac{1}{3}}^{2/3} \frac{dx}{\sqrt{1-x^2}}$

$= \arcsin x \Big]_{\frac{1}{3}}^{2/3}$

$= \sin^{-1}(\frac{2}{3}) - \sin^{-1}(\frac{1}{3})$

79. $\int_0^3 \frac{x\,dx}{x^2+9}$

$= \int_9^{18} \frac{1}{u} \cdot \frac{du}{2}$

$u = x^2+9$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x\,dx$

$x = 0$
$u = 0+9$
$u = 9$

$x = 3$
$u = 3^2+9$
$u = 18$

$= \frac{1}{2} \ln u \Big]_9^{18}$

$= \frac{1}{2}\left[\ln 18 - \ln 9\right]$

$= \frac{1}{2}\left[\ln \frac{18}{9}\right]$

$= \frac{1}{2} \ln 2$

81. $\int \frac{x\,dx}{\sqrt{1-x^4}}$

$\int \frac{1}{\sqrt{1^2-(x^2)^2}} \cdot x\,dx$

$a = 1$

$u = x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x\,dx$

$\int \frac{1}{\sqrt{a^2-u^2}} \cdot \frac{du}{2}$

$\frac{1}{2} \arcsin \frac{u}{a} + C$

$\frac{1}{2} \arcsin x^2 + C$

93. a) $y' = Ky$

$y = P_0 e^{Kt}$

$\frac{1}{2} = 1 e^{K(24.5)}$

$\ln \frac{1}{2} = \ln e^{24.5K}$

$\ln \frac{1}{2} = 24.5K \ln e$

$\frac{\ln \frac{1}{2}}{24.5} = K$

$y = P_0 e^{\frac{\ln \frac{1}{2}}{24.5} t}$

$y = P_0 \left(e^{\ln \frac{1}{2}}\right)^{t/24.5}$

$y = P_0 \left(\frac{1}{2}\right)^{t/24.5}$

b) $P_0 = 2$ $\qquad$ $t =$ days

$y = 2\left(\frac{1}{2}\right)^{t/24.5}$

$y = 2\left(\frac{1}{2}\right)^{365/24.5}$

$y = 6.55 \times 10^{-5}$

$y = 0.0000655$ Kg

$y = 0.0655$ g

109. $\lim\limits_{x \to 3} \dfrac{4x-12}{x^2-5x+6}$ direct sub $\dfrac{12-12}{9-15+6} = \dfrac{0}{0}$

$\lim\limits_{x \to 3} \dfrac{4(x-3)}{(x-3)(x-2)}$ or $\lim\limits_{x \to 3} \dfrac{d/dx\,(4x-12)}{d/dx\,(x^2-5x+6)}$

$= \lim\limits_{x \to 3} \dfrac{4}{x-2}$

$= \dfrac{4}{3-2}$

$= 4$

$= \lim\limits_{x \to 3} \dfrac{4}{2x-5}$

$= \dfrac{4}{2(3)-5}$

$= 4$

111. $\lim\limits_{x \to 0^+} x^{\frac{1}{2}} \ln x$ direct sub $0(-\infty)$

$\lim\limits_{x \to 0^+} \dfrac{\ln x}{x^{-1/2}}$

$\lim\limits_{x \to 0^+} \dfrac{d/dx\, \ln x}{d/dx\, x^{-1/2}}$

$\lim\limits_{x \to 0^+} \dfrac{\frac{1}{x}}{-\frac{1}{2}x^{-3/2}}$

$\lim\limits_{x \to 0^+} -2x^{-1} \cdot x^{3/2}$

$\lim\limits_{x \to 0^+} -2x^{\frac{1}{2}}$

$= 0$

123 $\lim_{n\to\infty} \left(1+\frac{4}{n}\right)^n$

$$e^x = \lim_{n\to\infty} \left(1+\frac{x}{n}\right)^n$$

$$\therefore \lim_{n\to\infty} \left(1+\frac{4}{n}\right)^n = e^4$$

124 $\lim_{n\to\infty} \left(1+\frac{4}{n}\right)^{3n}$

$$= \left[\lim_{n\to\infty} \left(1+\frac{4}{n}\right)^n\right]^3$$

$$= (e^4)^3$$

$$= e^{12}$$