Chapter 3

Check Your Understanding 3.1

Practise

1. Determine whether each function is a polynomial function. Justify your answers.

   a) $f(x) = 2x^4 - 3x + 2$

   The degree is ______, which is an ____________ number.

   $f(x)$ ______ a polynomial function.
   (*is* or *is not*)

   b) $y = 3^x + 5$

   The term 3^x means this is an ____________ function.

   This function ______ a polynomial function.
   (*is* or *is not*)

   c) $g(x) = 9$

   $g(x)$ has degree ______.

   This function ______ a polynomial function.
   (*is* or *is not*)

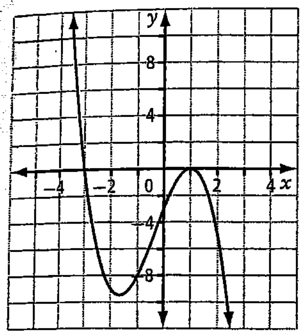
   d) $y = x^{-2} + 7x^3 + 1$

2. Complete the table for each polynomial function.

Polynomial Function	Degree	Type	Leading Coefficient	Constant Term
a) $f(x) = 6x^3 - 5x^2 + 2x - 8$	3			
b) $y = -2x^5 + 5x^3 + x^2 + 1$		Quintic		
c) $g(x) = x^3 - 7x^4$				0
d) $p(x) = 10x - 9$				
e) $y = -0.5x^2 + 4x + 3$				
f) $h(x) = 3x^4 - 8x^3 + x^2 + 2$			3	
g) $y = -5$		Constant		

For each graph of a polynomial function,
- determine whether the function has odd or even degree
- determine whether the leading coefficient is positive or negative
- state the number of x-intercepts
- state the domain and range

a) $f(x) = -x^3 - x^2 + 5x - 3$



The graph extends from quadrant ______ to quadrant ______.

The function has ______ degree.

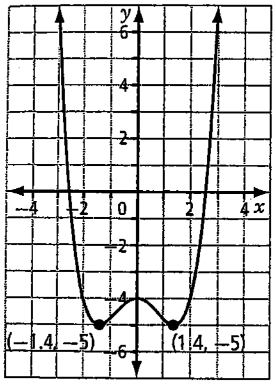
The leading coefficient is ______.

There are ______ x-intercepts.

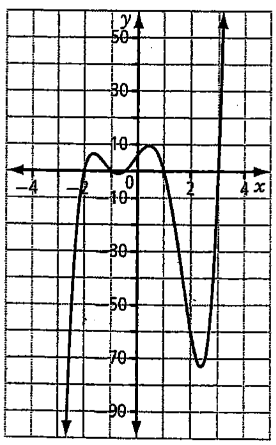
Domain: ____________

Range: ____________

b)



c)



4. For each function, use the degree and the sign of the leading coefficient to describe the end behaviour of its graph. State the possible number of x-intercepts and the value of the y-intercept.

a) $g(x) = 4x^5 - x^3 + 3x^2 - 6x + 2$

The degree is ______ with a __________ leading coefficient.

The graph extends from quadrant ______ to quadrant ______.

There are a maximum of ______ x-intercepts. The y-intercept is ______.

b) $y = -x^4 - 2x^5 + x^3 - 3x^2 + x$

c) $h(x) = x - 7x^3 - 6$

d) $y = 3x^5 - 2x^4 + 5x^3 - x^2 + x + 3$

e) $p(x) = 5x^4 - 6x - 1$

Check Your Understanding 3.2

Practise

1. a) Use long division to divide $x^3 + 3x^2 - 2x + 5$ by $x + 1$. Express the result in the form $\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$.

   b) Identify any restrictions on the variable.

   c) Write the corresponding statement that can be used to check the division. Then, verify your answer.

2. Divide using long division. Then, verify your answer using synthetic division.

   a) $(2x^2 - x + 5) \div (x + 3)$

   b) $(x^3 - x - 10) \div (x + 4)$

c) $(3x^4 + 2x^3 - 6x + 1) \div x$

d) $(-4x^4 + 11x - 7) \div (x - 3)$

3. Express each result in #2 above in the form $\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$.

   Identify any restrictions on the variable.

   a) $\frac{2x^2 - x + 5}{x + 3}$

   b)

   c)

   d)

4. Determine the remainder when each polynomial function is divided by $x - 2$. Use the remainder theorem.

   a) $P(x) = 2x^3 + 3x^2 - 17x - 30$

   b) $P(x) = x^3 + x^2 - 4x + 4$

5. Determine each remainder.

   a) $(6x^2 - x + 15) \div (x + 1)$

   b) $(x^3 - x^2 - 2x - 1) \div (x + 2)$

   c) $(2x^3 - 5x^2 - 13x + 2) \div (x - 4)$

   d) $(x^4 - 3x^2 - 5x + 2) \div (x - 2)$

Apply

6. For each dividend, determine the value of k if the remainder is -2.

a) $(2x^3 - 5x^2 - 4x + k) \div (x + 1)$

b) $(x^3 - 4x^2 + kx + 10) \div (x - 3)$

c) $(3x^3 + kx^2 - 13x + 4) \div (x + 2)$

d) $(kx^3 - 4x^2 - 5x + 8) \div (x - 2)$

7. For what value of m will the polynomial $P(x) = x^3 + 6x^2 + mx - 4$ have the same remainder when it is divided by $x - 1$ and $x + 2$?

Since the remainder is the same, determine the value of m by solving $P(1) = P(\_\_\_\_\_\_)$.

8. You can model the volume, in cubic centimetres, of a rectangular box by the polynomial function $V(x) = 3x^3 + x^2 - 12x - 4$. Determine expressions for the other dimensions of the box if the height is $x + 2$.

Check Your Understanding 3.3

Practise

1. What is the corresponding binomial factor of a polynomial, $P(x)$, given the value of the zero?

a) $P(2) = 0$

b) $P(-4) = 0$

c) $P(b) = 0$

d) $P(-d) = 0$

2. Determine whether $x + 1$ is a factor of each polynomial.

a) $x^3 + x^2 - x - 1$

b) $x^4 - 3x^3 - 4x^2 + x + 1$

c) $2x^3 - x^2 - 3x - 1$

d) $4x^4 + 7x + 3$

3. State whether each polynomial has $x + 3$ as a factor.

a) $x^3 + x^2 - x + 6$

b) $2x^3 + 9x^2 + 10x + 3$

c) $x^3 + 27$

d) $x^4 - 9x^2 + 2x + 6$

4. What are the possible integral zeros of each polynomial?

a) $x^3 - 3x^2 + 4x - 16$

b) $x^3 + 2x^2 + 8x + 12$

c) $x^3 - 3x^2 + 10x - 32$

d) $x^4 + 8x^3 - 9x^2 + 2x + 18$

5. Factor fully.

a) $x^3 - x^2 - 4x + 4$

b) $x^3 - 2x^2 - 4x + 8$

c) $x^3 + 3x^2 + 3x + 1$

d) $x^4 + 2x^3 - x - 2$

6. Factor fully.

a) $x^3 + 2x^2 - 9x - 18$

b) $4x^3 - 8x^2 + x + 3$

c) $6x^3 + x^2 - 31x + 10$

d) $x^4 + x^3 - 13x^2 - 25x - 12$

Apply

7. Determine the value(s) of k so that the binomial is a factor of the polynomial.

a) $P(x) = x^3 + 5x^2 + kx + 6$ $\quad x + 2$

If $x + 2$ is a factor, then $P(\_\_\_\_\_\_) = \_\_\_\_\_\_$.

b) $P(x) = kx^3 - 10x^2 + 2x + 3$ $\quad x - 3$

Check Your Understanding 3.4

Practise

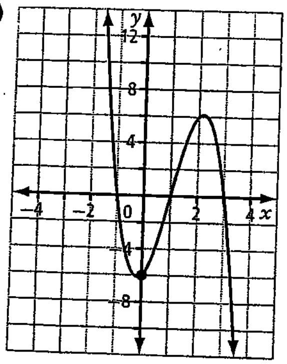
1. Solve.

a) $4x^3(x + 2)(2x - 1) = 0$

b) $(x + 1)^2(x - 3)(x - 5) = 0$

c) $x^3 - 8 = 0$

2. Use the graph of the given function to write the corresponding polynomial equation. State the roots of the equation. The roots are all integral values.

a) 

The graph of the function has ________ x-intercepts. It crosses the x-axis at each of the x-intercepts. All the x-intercepts are of ______________ multiplicity.
(*even* or *odd*)

The least possible multiplicity of each x-intercept is ________, so the least possible degree is ________.

The graph extends up into quadrant ________ and down into quadrant ________, so the leading coefficient is ______________.
(*positive* or *negative*)

The y-intercept is ________; this is the ______________ term in the equation of the function.

The zeros, or x-intercepts, are ________, ________, and ________. The product of the roots is ______________.

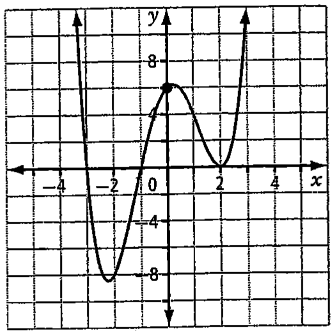
Compare the product of the roots to the y-intercept to determine the vertical stretch, a.

$a =$ ________

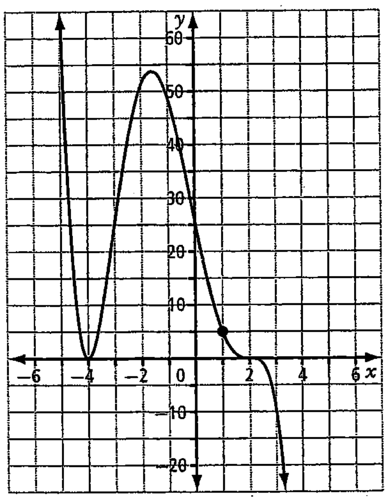
The equation of the polynomial function is

$f(x) =$ ________(________)(________)(________).

b)



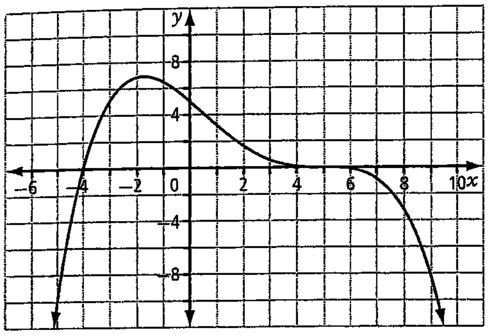
c)



In this case, use the coordinates of the point (1, 5) to solve for *a*.

3. For each graph,
   - state the x-intercepts
   - state the intervals where the function is positive and the intervals where the function is negative
   - explain whether the graph might represent a polynomial that has zero(s) of multiplicity 1, 2, or 3.

a)



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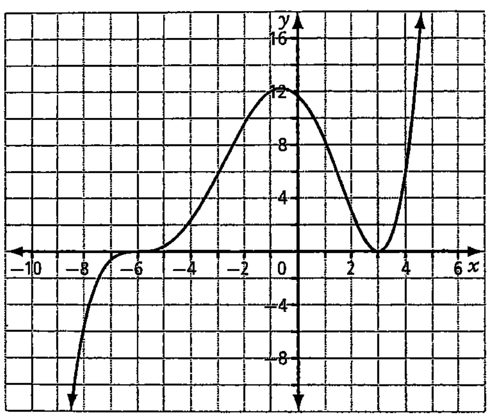

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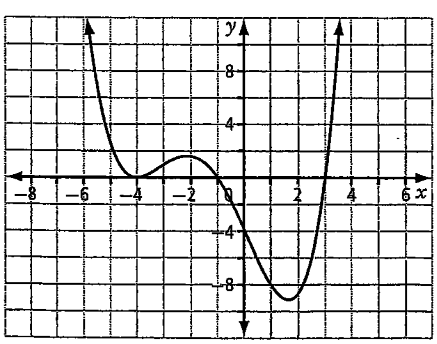
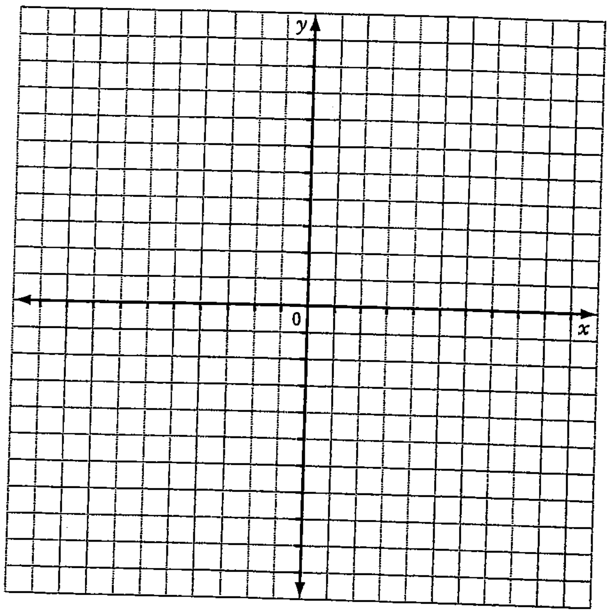
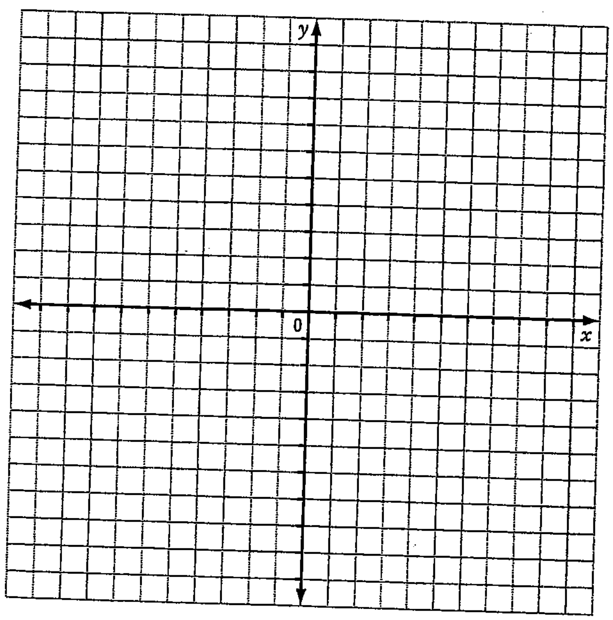


4. Without using technology, sketch the graph of each function. Label all intercepts. (Hint: Factor.)

a) $f(x) = -2x^3 + 3x^2 + 11x - 6$



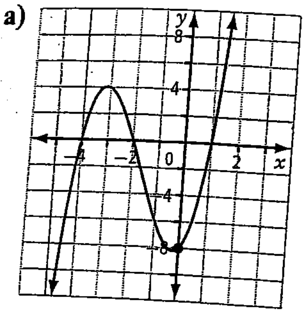
b) $g(x) = x^4 + 5x^3 + 6x^2 - 4x - 8$



Apply

5. Determine the equation for each graph of a polynomial function shown.

a)



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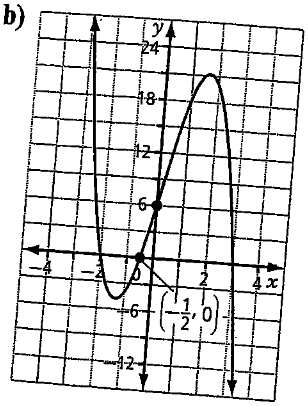
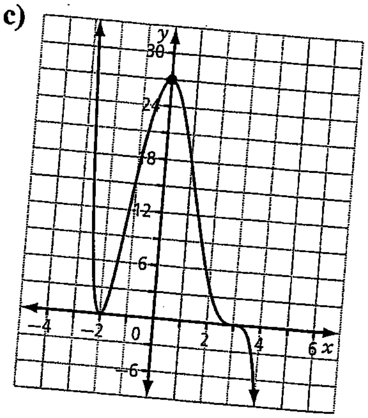


c)



7. Determine the equation with least degree for each polynomial function.

a) quartic function with zeros 2 (multiplicity 3) and -5, and y-intercept 30

b) quintic function with zeros -1 (multiplicity 2), 3 (multiplicity 1), and -2 (multiplicity 2), and constant term -12

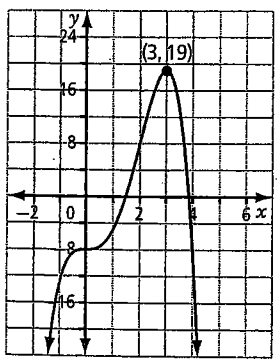
Chapter 3 Review

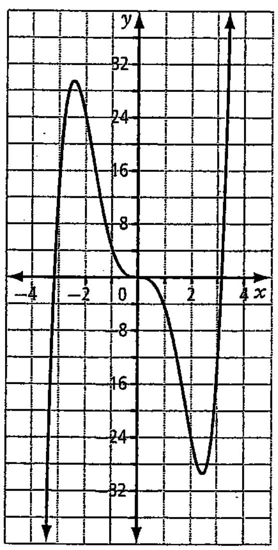
3.1 Characteristics of Polynomial Functions, pages 66–77

1. Complete the chart for each polynomial function.

Polynomial Function	Degree	Type	Leading Coefficient	Constant Term
a) $f(x) = -2x^4 - x^3 + 3x - 7$				
b) $y = 3x^5 + 2x^4 - x^3 + 3$				
c) $g(x) = 0.5x^3 - 8x^2$				
d) $p(x) = 10$				

2. For each of the following,
   - determine whether the graph represents on odd-degree or an even-degree polynomial function
   - determine whether the leading coefficient of the corresponding function is positive or negative
   - state the number of x-intercepts
   - state the domain and range

a) 

b) 

3. The distance, d, in metres, travelled by a boat from the moment it leaves shore can be modelled by the function $d(t) = 0.002t^3 + 0.05t^2 + 0.3t$, where t is the time, in seconds.

a) What is the degree of the function $d(t)$?

b) What are the leading coefficient and constant of this function? What does the constant represent?

c) Describe the end behaviour of the graph of this function.

d) What are the restrictions on the domain of this function? Explain why you selected those restrictions.

e) What distance has the boat travelled after 15 s?

3.2 The Remainder Theorem, pages 78–83

4. **a)** Use long division to divide $5x^3 - 7x^2 - x + 6$ by $x - 1$.

Express the result in the form $\frac{P(x)}{x - a} = Q(x) + \frac{R}{x - a}$.

b) Identify any restrictions on the variable.

c) Write the corresponding statement that can be used to check the division. Then, verify your answer.

5. Determine the remainder resulting from each division.

a) $(x^3 + 2x^2 - 3x + 9) \div (x + 3)$

b) $(2x^3 + 7x^2 - x + 1) \div (x + 2)$

c) $(x^3 + 2x^2 - 3x + 5) \div (x - 3)$

d) $(2x^4 + 7x^2 - 8x + 3) \div (x - 4)$

6. a) Determine the value of m such that when $f(x) = x^4 - mx^3 + 7x - 6$ is divided by $x - 2$, the remainder is -8.

b) Use the value of m from part a) to determine the remainder when $f(x)$ is divided by $x + 2$.

7. When a polynomial $P(x)$ is divided by $x - 2$, the quotient is $x^2 + 4x - 7$ and the remainder is -4. What is the polynomial?

3.3 The Factor Theorem, pages 84–90

8. What is the corresponding binomial factor of a polynomial, $P(x)$, given the value of the zero?

a) $P(7) = 0$

b) $P(-6) = 0$

c) $P(c) = 0$

9. Determine whether $x + 2$ is a factor of each polynomial.

a) $x^3 + 2x^2 - x - 2$

b) $x^4 + 2x^3 - 4x^2 + x + 10$

10. What are the possible integral zeros of each polynomial?

a) $x^3 - 5x^2 + 3x - 27$

b) $x^3 + 6x^2 + 2x + 36$

11. Factor fully.

a) $x^3 - 4x^2 + x + 6$

b) $3x^3 - 5x^2 - 26x - 8$

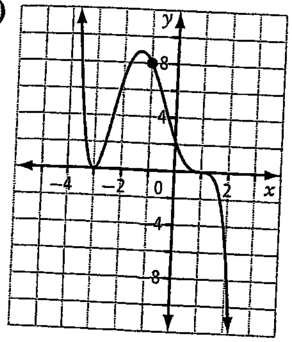
c) $5x^4 + 12x^3 - 101x^2 + 48x + 36$

d) $2x^4 + 5x^3 - 8x^2 - 20x$

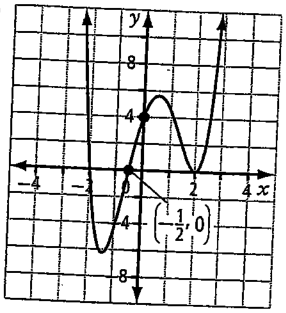
3.4 Equations and Graphs of Polynomial Functions, pages 91–102

13. For each graph of a polynomial function, determine
- the least possible degree
- the sign of the leading coefficient
- the x-intercepts and their multiplicity
- the intervals where the function is positive and the intervals where it is negative
- the equation for the polynomial function

a)



b)



15. Determine the equation with least degree for a cubic function with zeros −2 (multiplicity 2) and 3 (multiplicity 1), and y-intercept 36.

Chapter 3

3.1 Characteristics of Polynomial Functions, pages 66–77

1. a) Yes; polynomial function of degree 4
   b) No; exponential function
   c) Yes; polynomial function of degree 0
   d) No; function has a variable with a negative exponent

2.

Polynomial Function	Degree	Type	Leading Coefficient	Constant Term
a) $f(x) = 6x^3 - 5x^2 + 2x - 8$	3	Cubic	6	−8
b) $y = -2x^5 + 5x^3 + x^2 + 1$	5	Quintic	−2	1
c) $g(x) = x^3 - 7x^4$	4	Quartic	−7	0
d) $p(x) = 10x - 9$	1	Linear	10	−9
e) $y = -0.5x^2 + 4x + 3$	2	Quadratic	−0.5	3
f) $h(x) = 3x^4 - 8x^3 + x^2 + 2$	4	Quartic	3	2
g) $y = -5$	0	Constant	0	−5

3. a) odd degree; negative leading coefficient; 2 x-intercepts; domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \in \mathrm{R}\}$
   b) even degree; positive leading coefficient; 2 x-intercepts; domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \geq -5, y \in \mathrm{R}\}$
   c) odd degree; positive leading coefficient; 5 x-intercepts; domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \in \mathrm{R}\}$

4. a) degree 5; positive leading coefficient; extends from quadrant III to I; maximum of 5 x-intercepts; y-intercept of 2
   b) degree 5; negative leading coefficient; extends from quadrant II to IV; maximum of 5 x-intercepts; y-intercept of 0
   c) degree 3; negative leading coefficient; extends from quadrant II to IV; maximum of 3 x-intercepts; y-intercept of −6
   d) degree 5; positive leading coefficient; extends from quadrant III to I; maximum of 5 x-intercepts; y-intercept of 3
   e) degree 4; positive leading coefficient; opens upward; maximum of 4 x-intercepts; y-intercept of −1

3.2 The Remainder Theorem, pages 78–83

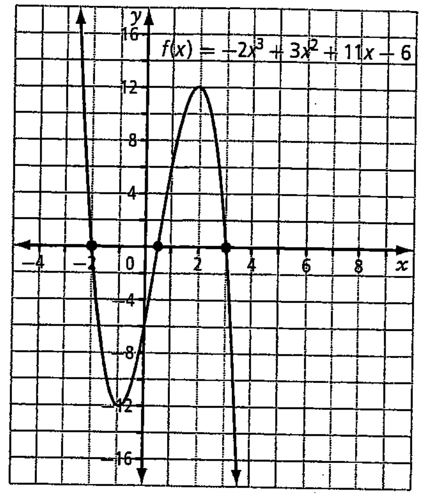
1. a) $\frac{x^3 + 3x^2 - 2x + 5}{x + 1} = x^2 + 2x - 4 + \frac{9}{x + 1}$
   b) $x \neq -1$
   c) $(x + 1)(x^2 + 2x - 4) + 9 = x^3 + 3x^2 - 2x + 5$
2. a) $Q(x) = 2x - 7, R = 26$
   b) $Q(x) = x^2 - 4x + 15, R = -70$
   c) $Q(x) = 3x^3 + 2x^2 - 6, R = 1$
   d) $Q(x) = -4x^3 - 12x^2 - 36x - 97, R = -298$
3. a) $\frac{2x^2 - x + 5}{x + 3} = 2x - 7 + \frac{26}{x + 3}, x \neq -3$
   b) $\frac{x^3 - x - 10}{x + 4} = x^2 - 4x + 15 - \frac{70}{x + 4}, x \neq -4$
   c) $\frac{3x^4 + 2x^3 - 6x + 1}{x} = 3x^3 + 2x - 6 + \frac{1}{x}, x \neq 0$
   d) $\frac{-4x^4 + 11x - 7}{x - 3} = -4x^3 - 12x^2 - 36x - 97 - \frac{298}{x - 3},$ $x \neq 3$
4. a) −36 b) 8
5. a) 22 b) −9 c) −2 d) −4
6. a) $k = 1$ b) $k = -1$ c) $k = -2$ d) $k = 2$
7. $m = 3$
8. $x - 2, 3x + 1$

3.3 The Factor Theorem, pages 84–90

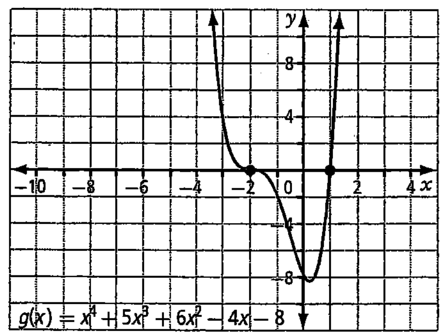
1. a) $x-2$ b) $x+4$ c) $x-b$ d) $x+d$
2. a) Yes b) Yes c) No d) Yes
3. a) No b) Yes c) Yes d) Yes
4. a) $\pm1, \pm2, \pm4, \pm8, \pm16$
   b) $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$
   c) $\pm1, \pm2, \pm4, \pm8, \pm16, \pm32$
   d) $\pm1, \pm2, \pm3, \pm6, \pm9, \pm18$
5. a) $(x-2)(x+2)(x-1)$
   b) $(x-2)^2(x+2)$
   c) $(x+1)^3$
   d) $(x-1)(x+2)(x^2+x+1)$
6. a) $(x+2)(x-3)(x+3)$
   b) $(2x+1)(x-1)(2x-3)$
   c) $(x-2)(2x+5)(3x-1)$
   d) $(x+1)^2(x+3)(x-4)$
7. a) $k=9$ b) $k=3$

3.4 Equations and Graphs of Polynomials Functions, pages 91–102

1. a) $x = 0, -2, \frac{1}{2}$
   b) $x = -1, 3, 5$
   c) $x = 2$
2. a) $f(x) = -2(x-1)(x+1)(x-3); -1, 1, 3$
   b) $f(x) = 0.5(x-2)^2(x+1)(x+3); -1, -3, 2$
   c) $f(x) = -0.2(x-2)^3(x+4)^2; -4, 2$
3. a) -4 and 5; positive for $-4 < x < 5$; negative for $x < -4$ and $x > 5$; -4 (multiplicity 1) and 5 (multiplicity 3); the function changes sign at both, but is flatter at $x = 5$
   b) -6 and 3; positive for $-6 < x < 3$ and $x > 3$; negative for $x < -6$; -6 (multiplicity 3) and 3 (multiplicity 2); the function changes sign at $x = -6$, but not at $x = 3$
   c) $-4, -1$, and 3; positive for $x < -4$, $-4 < x < -1$, and $x > 3$; negative for $-1 < x < 3$; -4 (multiplicity 2), -1 (multiplicity 1), and 3 (multiplicity 1); the function changes sign at $x = -1$ and at $x = 3$, but not at $x = -4$
4. a) x-intercepts: -2, 0.5, 3 (all of multiplicity 1); y-intercept: -6



   b) x-intercepts: -2 (multiplicity 3) and 1 (multiplicity 1); y-intercept: -8



5. a) $f(x) = (x+4)(x-1)(x+2)$
   b) $f(x) = -(2x+1)(x-3)(x+2)$
   c) $f(x) = -0.25\,(x+2)^2(x-3)^3$
7. a) $y = -\frac{3}{4}(x-2)^3(x+5)$
   b) $y = (x+1)^2(x-3)(x+2)^2$

Chapter 3 Review, pages 103–107

1.

Polynomial Function	Degree	Type	Leading Coefficient	Constant Term
a) $f(x) = -2x^4 - x^3 + 3x - 7$	4	Quartic	-2	-7
b) $y = 3x^5 + 2x^4 - x^3 + 3$	5	Quintic	3	3
c) $g(x) = 0.5x^3 - 8x^2$	3	Cubic	0.5	0
d) $p(x) = 10$	0	Constant	0	10

2. a) even degree; negative leading coefficient; 2 x-intercepts; domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \le 19, y \in \mathrm{R}\}$
   b) odd degree; positive leading coefficient; 3 x-intercepts; domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \in \mathrm{R}\}$

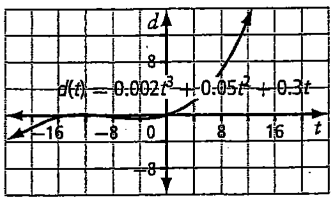
3. a) degree 3

b) leading coefficient: 0.002; constant: 0; The constant represents the distance that the boat is from the shore at time 0 s (the initial position of the boat).

c) degree: 3; positive leading coefficient; extends from quadrant III to I

d) domain: $\{t \mid t \geq 0, t \in R\}$; it is impossible to have negative time

e) When $t = 15$, $d(15) = 22.5$. After 15 s, the boat is 22.5 m from the shore.

f) 

4. a) $\frac{5x^3 - 7x^2 - x + 6}{x - 1} = 5x^2 - 2x - 3 + \frac{3}{x - 1}$

b) $x \neq 1$

c) $(x - 1)(5x^2 - 2x - 3) + 3 = 5x^3 - 7x^2 - x + 6$

5. a) $R = 9$ b) $R = 15$

c) $R = 41$ d) $R = 595$

6. a) $m = 4$ b) 28

7. $P(x) = x^3 + 2x^2 - 15x + 10$

8. a) $x - 7$ b) $x + 6$ c) $x - c$

9. a) Yes b) No

10. a) $\pm 1, \pm 3, \pm 9, \pm 27$

b) $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36$

11. a) $(x - 3)(x - 2)(x + 1)$

b) $(x - 4)(x + 2)(3x + 1)$

c) $(x - 3)(x - 1)(x + 6)(5x + 2)$

d) $x(x - 2)(x + 2)(2x + 5)$

12. $x - 1$, $x + 2$, and $5x + 2$

13. a) degree 5; negative leading coefficient; −3 (multiplicity 2) and 1 (multiplicity 3); the function changes sign at $x = 1$, but not at $x = -3$; positive for $x < -3$ and $-3 < x < 1$; negative for $x > 1$; $f(x) = -0.25(x + 3)^2(x - 1)^3$

b) degree 4; positive leading coefficient; −2 (multiplicity 1), −0.5 (multiplicity 1), and 2 (multiplicity 2); the function changes sign at $x = -2$ and at $x = -0.5$, but not at $x = 2$; positive for $x < -2$, $-0.5 < x < 2$, and $x > 2$; negative for $-2 < x < -0.5$; $f(x) = 0.5(x + 2)(2x + 1)(x - 2)^2$

14. a) $a = -2$; vertical stretch by a factor of 2 and reflection in the x-axis

$b = \frac{1}{3}$; horizontal stretch by a factor of 3

$h = 1$; translation of 1 unit to the right

$k = 4$; translation of 4 units up

b) domain: $\{x \mid x \in R\}$; range: $\{y \mid y \in R\}$

15. $y = -3(x + 2)^2(x - 3)$