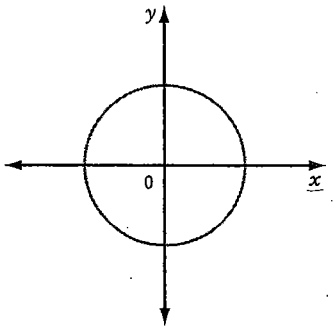
Chapter 4

Check Your Understanding 4.1

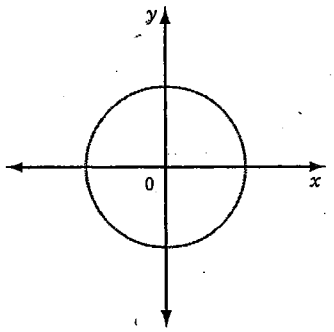
Practise

1. Sketch each angle in standard position. Change each degree measure to radian measure. Express your answer as an exact value (in terms of fractions of π).

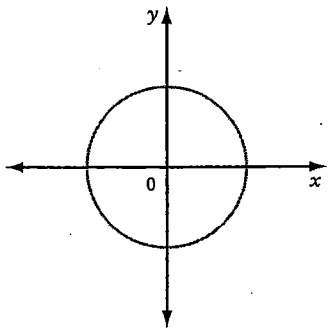
a) 60°



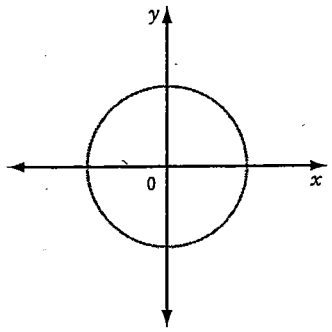
b) 315°



c) −210°

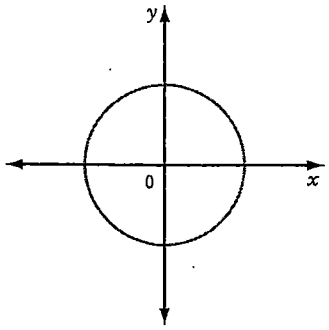


d) 600°

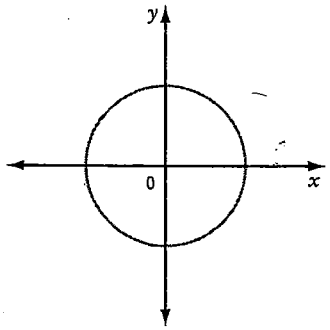


2. Draw each angle in standard position. Change each degree measure to radian measure. Express your answer as a decimal rounded to two decimal places.

a) 101°



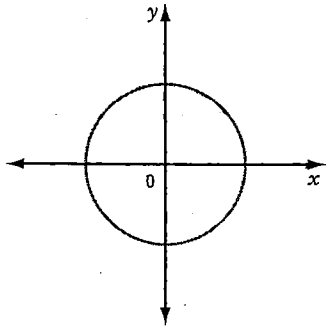
b) 57.3°



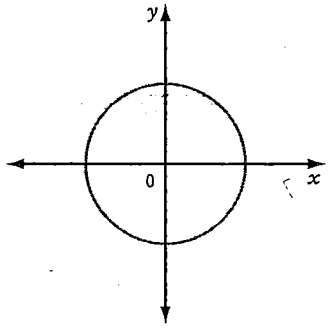
For additional practice, see #3 on page 175 of *Pre-Calculus 12*.

3. Sketch each angle in standard position. Change each radian measure to degree measure. If necessary, round your answer to two decimal places.

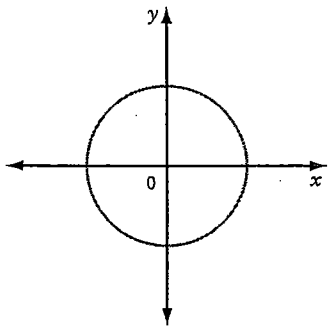
a) $\frac{\pi}{2}$



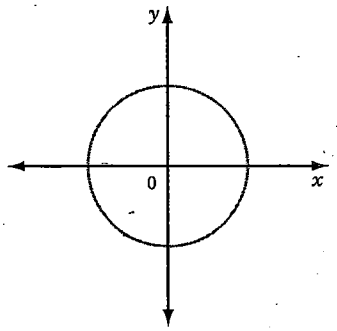
b) $\frac{4\pi}{3}$



c) $-\frac{2\pi}{9}$



d) 2



4. Determine one positive and one negative angle coterminal with each angle given.

a) 349°

b) −487°

c) $\frac{2\pi}{3}$

d) $\frac{9\pi}{4}$

5. For each angle θ, determine all coterminal angles within the given domain. Write an expression for all angles coterminal with θ in general form.

a) $\theta = 255°$ within the domain $-720° \leq \theta < 720°$

$\theta - 2(360°)$	$\theta - 360°$	$\theta + 360°$	$\theta + 2(360°)$

For n rotations, the general form for angles coterminal with 255° is

_____, $n \in \mathrm{N}$.

b) $\theta = \pi$ within the domain $-4\pi \leq \theta < 4\pi$

$\theta - 4\pi$	$\theta - 2\pi$	$\theta + 2\pi$	$\theta + 4\pi$

For n rotations, the general form for angles coterminal with π is

_____, $n \in \mathrm{N}$.

c) $\theta = \frac{5\pi}{6}$ within the domain $-2\pi \leq \theta < 6\pi$

For n rotations, the general form for angles coterminal with $\frac{5\pi}{6}$ is

_____, $n \in \mathrm{N}$.

Also try #11 on page 176 of *Pre-Calculus 12*.

6. Determine the arc length subtended by each central angle. Give answers to the nearest hundredth of a unit.

a) radius 20 cm, central angle $\frac{2\pi}{3}$

Check Your Understanding 4.2

Practise

1. Determine the equation of a circle centred at (0, 0) with each radius.

   a) 25 units

   b) 1.1 units

2. Is each point on the unit circle? Give evidence to support your answer.

   a) (0.65, −0.76)

   The equation of the unit circle is _____.

Left Side	Right Side

   Conclusion: _____

   b) $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

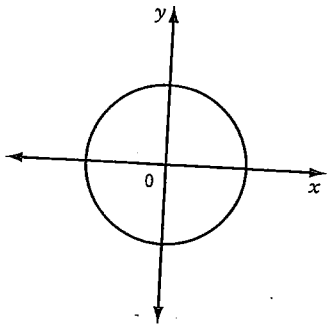
   c) $\left(\frac{\sqrt{7}}{2}, -\frac{1}{7}\right)$

What information could you use to answer part c) other than a left side/right side proof?

3. Determine the missing coordinate for each point on the unit circle. Draw a diagram to support your answer.

a) point $A\left(x, \frac{5}{13}\right)$ in quadrant II

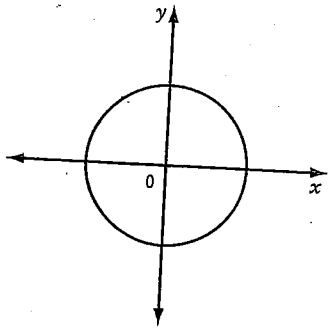
$x^2 + y^2 = 1$



Since quadrant _____ is specified, take only the _____ root.
(*positive* or *negative*)

Therefore, $x =$ _____.

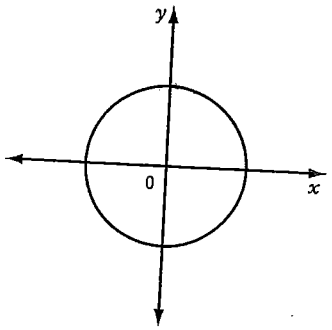
b) point $B\left(\frac{1}{6}, y\right)$ in quadrant IV



Since quadrant _____ is specified, take only the _____ root.
(*positive* or *negative*)

Therefore, $y =$ _____.

c) point $C\left(x, -\frac{1}{2}\right)$ in quadrant III

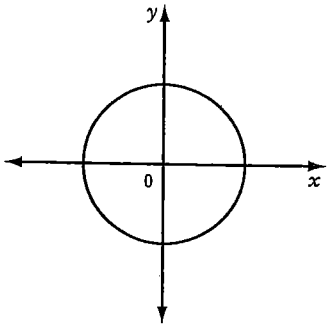


Do you recognize this pair of values? What angle is associated with it?

4. If $P(\theta)$ is the point at which the terminal arm of angle θ in standard position intersects the unit circle, determine the exact coordinates of each of the following.

The word *exact* in the question is a clue to use special triangles.

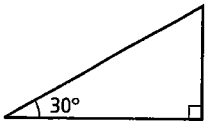
a) $P\left(\frac{\pi}{2}\right)$



b) $P(2\pi)$

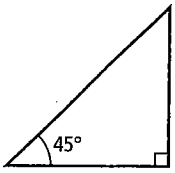
c) $P\left(\frac{2\pi}{3}\right)$

d) $P\left(\frac{5\pi}{4}\right)$



e) $P\left(-\frac{\pi}{4}\right)$

f) $P\left(\frac{23\pi}{6}\right)$



5. Determine the value of angle θ in standard position, $0 \leq \theta < 2\pi$, given the coordinates of $P(\theta)$, the point at which the terminal arm intersects the unit circle.

The domain is given in radians, so your answers should also be in radians.

a) $P(\theta) = (-1, 0)$ $\theta =$ _____

b) $P(\theta) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $\theta =$ _____

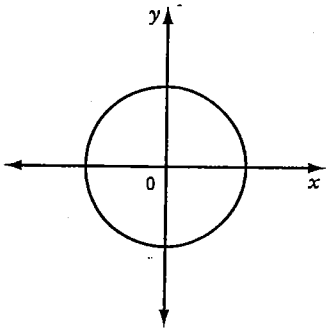
c) $P(\theta) = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ $\theta =$ _____

d) $P(\theta) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ $\theta =$ _____

This question should help you complete #5 and #6 on page 187 of *Pre-Calculus 12*.

6. Determine the arc length on the unit circle from (1, 0) to each point.

a) $P\left(\frac{\pi}{2}\right)$

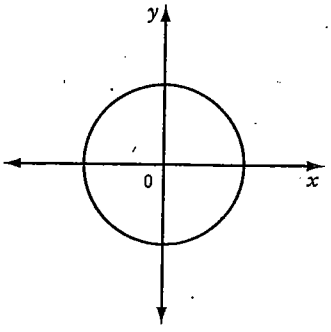


$\theta =$ _____

On the unit circle, $r = 1$.

So, $a =$ _____.

b) $P(\theta)\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$



$\theta =$ _____

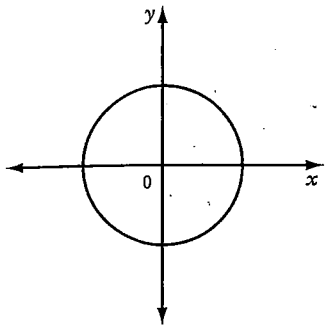
On the unit circle, $r = 1$.

So, $a =$ _____.

Check Your Understanding 4.3

Practise

1. Point P $\left(\frac{7}{25}, -\frac{24}{25}\right)$ is on the unit circle and on the terminal arm of an angle θ in standard position. Determine the values of the six trigonometric ratios for angle θ.

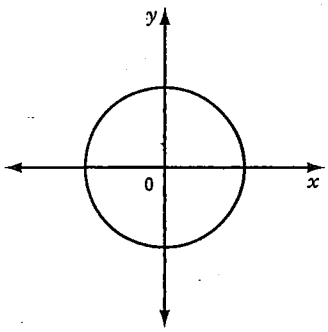


$\sin\theta =$ _____ $\cos\theta =$ _____ $\tan\theta =$ _____

$\csc\theta =$ _____ $\sec\theta =$ _____ $\cot\theta =$ _____

2. Without using a calculator, determine the sign (+ or −) of each of the following.

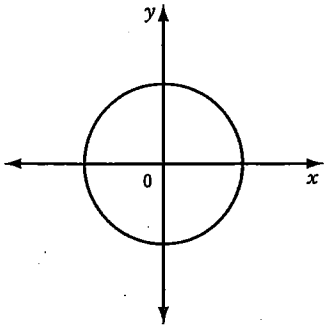
a) sin 580°



quadrant _____

sign is _____

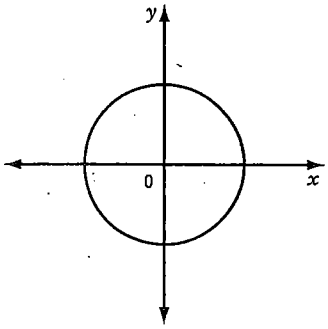
b) tan 1



quadrant _____

sign is _____

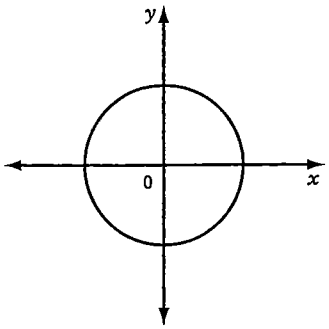
c) $\csc\theta = \frac{2\pi}{3}$



quadrant _____

sign is _____

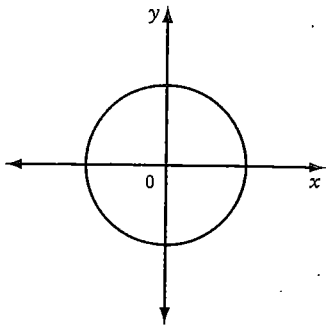
d) $\sec\theta = \frac{5\pi}{4}$



quadrant _____

sign is _____

3. In which quadrant(s) is (are) the terminal arm(s) of angle θ given the following conditions?



a) $\cot\theta$ is positive _____

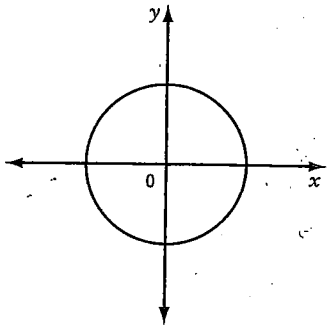
b) $\cot\theta$ is positive and $\sin\theta$ is negative _____

c) $\csc\theta = 1.2$ _____

d) $\csc\theta = 1.2$ and $\cos\theta = -0.574$ _____

4. What is the exact value for each trigonometric ratio?

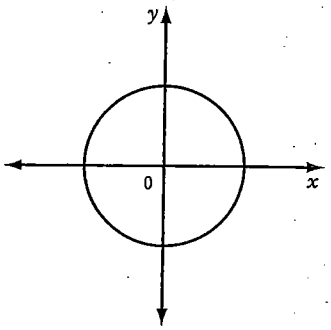
a) $\cos\frac{\pi}{3}$



$P\left(\frac{\pi}{3}\right)$ is in quadrant _____.

$\theta_R =$ _____

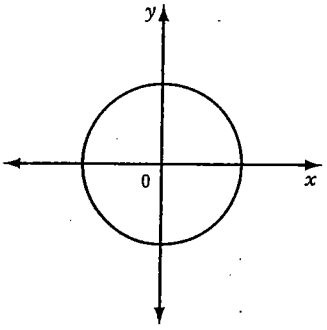
b) $\sin\frac{\pi}{4}$



$P\left(\frac{\pi}{4}\right)$ is in quadrant _____.

$\theta_R =$ _____

c) $\tan 3\pi$

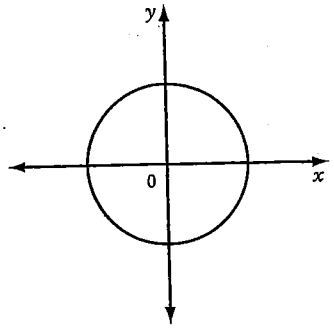


$P(3\pi)$ is a quadrantal angle.

d) $\cot\left(-\frac{2\pi}{3}\right)$

$P\left(-\frac{2\pi}{3}\right)$ is in quadrant _____.

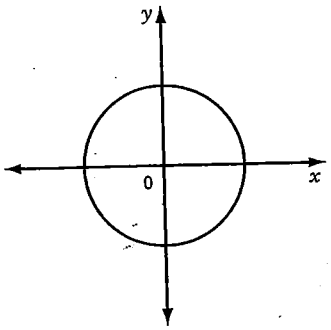
$\theta_R =$ _____



e) $\sec\frac{5\pi}{6}$

$P\left(\frac{5\pi}{6}\right)$ is in quadrant _____.

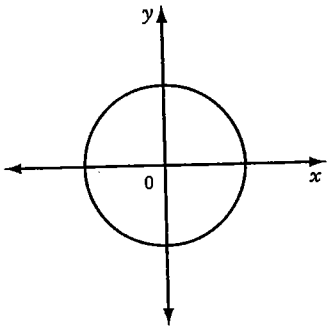
$\theta_R =$ _____



f) $\csc\left(-\frac{9\pi}{4}\right)$

$P\left(-\frac{9\pi}{4}\right)$ is in quadrant _____.

$\theta_R =$ _____



5. Determine the approximate value for each trigonometric ratio, to three decimal places.

a) sec 74°

sec θ is the reciprocal of _____

quadrant: _____

sign (+ or −): _____

b) cot 104°

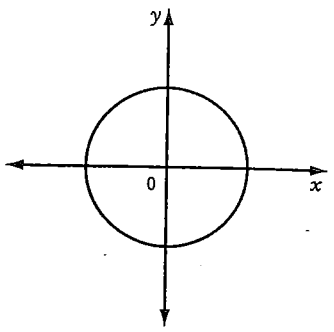
c) csc 2.8

d) $\sec\left(-\frac{7\pi}{10}\right)$

These questions are similar to #1 and #2 on page 201 of *Pre-Calculus 12*

Apply

6. Determine the measure of all angles that satisfy the following conditions. Round your answers to the nearest degree.



Which other coterminal angles fall within the domain?

a) $\tan \theta = -3.078$ in the domain $0° \leq \theta < 720°$

$\theta_R = \tan^{-1}(+3.078)$

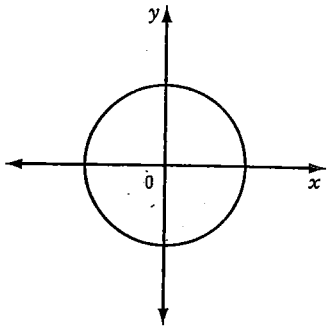
$\approx$ _____

Tangent is negative in quadrants _____ and _____.

Therefore, $\tan \theta = -3.078$ when $\theta \approx$ _____,

_____, _____, and _____, $0° \leq \theta < 720°$.

b) $\sec \theta = -1.046$ in the domain $-360° \leq \theta < 360°$



$\sec \theta$ is the reciprocal of _____.

$\theta_R \approx$ _____

Secant is negative in quadrants _____ and _____.

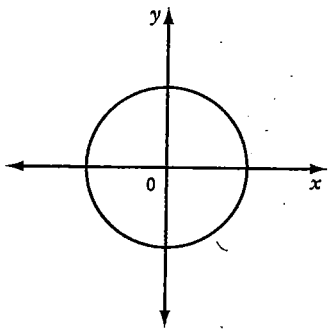
Therefore, $\sec \theta = -1.046$ when $\theta \approx$ _____,

_____, _____, and _____, $-360° \leq \theta < 360°$.

7. Determine the measure of all angles that satisfy the following conditions. Give exact answers.

Drawing the special triangle may help.

a) $\sin \theta = -\frac{\sqrt{3}}{2}$ in the domain $0 \leq \theta < 4\pi$



$\theta_R =$ _____

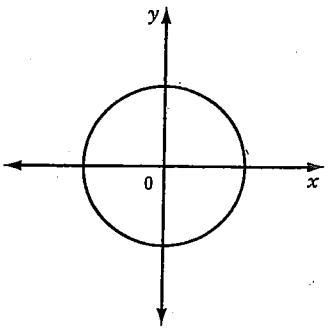
Sine is negative in quadrants _____ and _____.

Therefore, $\sin \theta = -\frac{\sqrt{3}}{2}$ when $\theta =$ _____, _____,

_____, and _____, $0 \leq \theta < 4\pi$.

b) $\csc \theta = 2$ in the domain $-2\pi \leq \theta < 2\pi$

What is the reciprocal of 2?



$\csc \theta$ is the reciprocal of _____.

$\theta_R =$ _____

Cosecant is positive in quadrants _____ and _____.

Therefore, $\csc \theta = 2$ when $\theta =$ _____, _____,

_____, and _____, $-2\pi \leq \theta < 2\pi$.

8. Determine the value of the five other trigonometric ratios if $\csc\theta = \frac{5}{3}$, $90° \leq \theta < 180°$.

The angle is in quadrant _____.

$x =$ _____ $y =$ _____ $r =$ _____

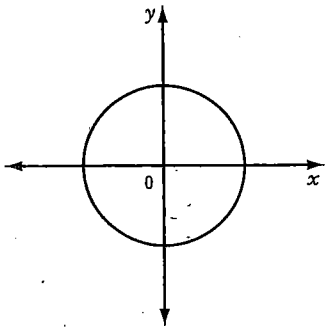
$\sin\theta =$ _____ $\cos\theta =$ _____ $\tan\theta =$ _____

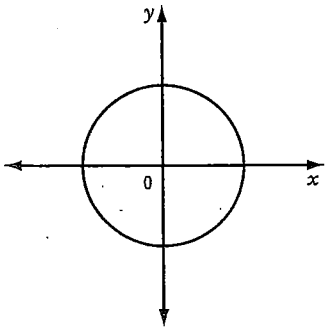
$\sec\theta =$ _____ $\cot\theta =$ _____

This question will help you with #12 on page 202 of *Pre-Calculus 12*.

Connect

9. Choose any two of the special angles $\frac{\pi}{6}$, $\frac{\pi}{4}$, and $\frac{\pi}{3}$. Complete the table below. You may also choose the quadrantal angles (on the axes), but then you will have to change the headings on the table.

$\theta_R =$	Quadrant I	Quadrant II	Quadrant III	Quadrant IV
	sin csc			
	cos sec			
	tan cot			

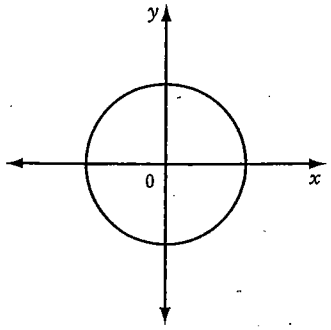
$\theta_R =$	Quadrant I	Quadrant II	Quadrant III	Quadrant IV
	sin csc			
	cos sec			
	tan cot			

Check Your Understanding 4.4

Practise

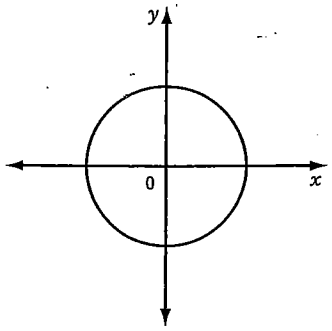
1. Determine the exact solutions for each trigonometric equation in the specified domain.

   a) $4 \sin \theta - 5 = 3, 0° \leq \theta < 360°$



   b) $7 \cot \theta - 4 = 6 \cot \theta - 5, 0 \leq \theta < 4\pi$

   $\cot \theta$ is the reciprocal of _____.

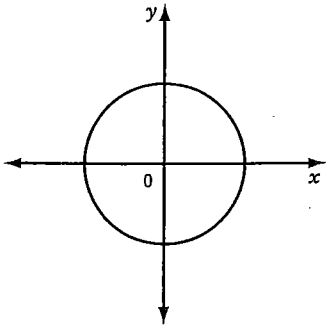


   Therefore, the solutions are

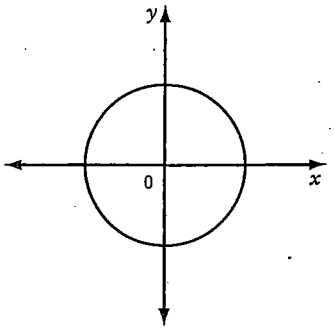
   $\theta =$ _____, $0 \leq \theta < 4\pi$.

2. Solve for θ within the domain $0° \leq \theta < 360°$. Round answers to one decimal place.

   a) $-3(5 - 4 \sec \theta) = \sec \theta$



   b) $\csc \theta + \frac{3}{4} = -\frac{2}{3}$

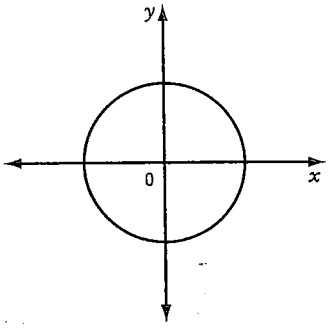


3. Solve for θ within the given domain. Give exact answers where possible. Otherwise, round your answer to two decimal places.

a) $4\cos^2\theta = 3, 0 \leq \theta < 2\pi$

Treat cos θ as a variable.

Determine the reference angles for both solutions.

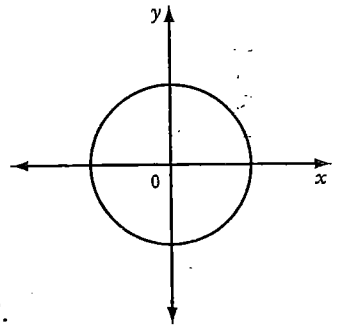


The solutions are _____, $0 \leq \theta < 2\pi$.

b) $\csc^2\theta - 3\csc\theta - 10 = 0, 0 \leq \theta < 360°$

Degrees or radians?

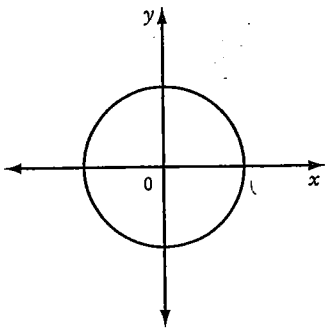
Determine the reference angles for both solutions.



The solutions are _____, $0 \leq \theta < 360°$.

4. The equation $\cos\theta = \frac{\sqrt{3}}{2}, 0 \leq \theta < 2\pi$, has solutions $\frac{\pi}{6}$ and $\frac{11\pi}{6}$. Suppose the domain is not restricted.

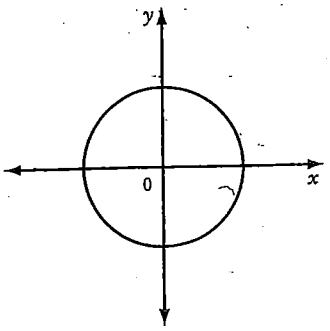
a) Write the general solution corresponding to $\frac{\pi}{6}$.



b) Write the general solution corresponding to $\frac{11\pi}{6}$.

5. The equation $\tan\theta = 1, 0 \leq \theta < 2\pi$, has solutions $\frac{\pi}{4}$ and $\frac{5\pi}{4}$.

a) Write the solutions for $\tan\theta = 1$ if the domain is $0 \leq \theta < 4\pi$.



b) Suppose the domain is unrestricted. Write the general solution.

Chapter 4 Review

4.1 Angles and Angle Measure, pages 109–119

1. Convert each degree measure to radian measure and each radian measure to degree measure. Give exact values.

   a) 270°

   b) $\frac{5\pi}{3}$

   c) 300°

   d) -4

   e) 495°

   f) $\frac{13\pi}{4}$

2. Identify one positive and one negative angle measure that is coterminal with each angle. Then, write a general expression for all the coterminal angles in each case.

   a) $\frac{11\pi}{6}$

   b) $-375°$

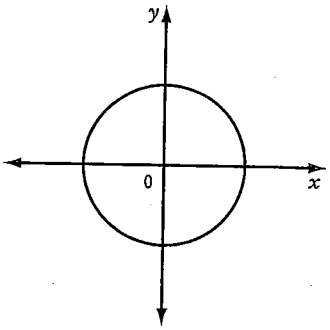
3. Determine the measure of the central angle subtended by each arc to one decimal place.

   a) arc length 31.4 cm, radius 5.0 cm, in radians

   b) arc length 11.3 m, radius 22.6 m, in degrees

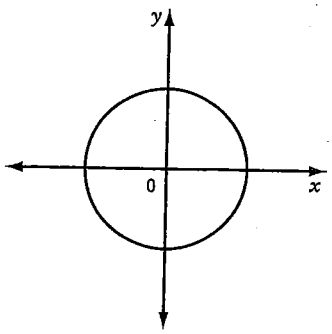
4.2 The Unit Circle, pages 120–128

4. Determine the missing coordinate for point $P\left(x, -\frac{2}{3}\right)$ in quadrant III on the unit circle.

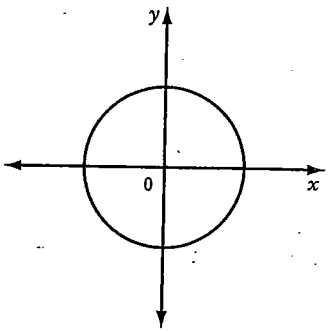


5. Determine the value of angle θ in standard position, $0 \leq \theta < 2\pi$, given the coordinates of $P(\theta)$, the point at which the terminal arm of θ intersects the unit circle.

a) $P\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ $\theta =$

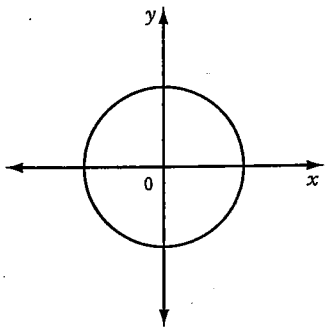


b) $P\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ $\theta =$



4.3 Trigonometric Ratios, pages 129–137

6. Determine the measure of all angles that satisfy $\sec\theta \doteq 1.788$, $0° \leq \theta < 720°$. Round your answers to the nearest degree.



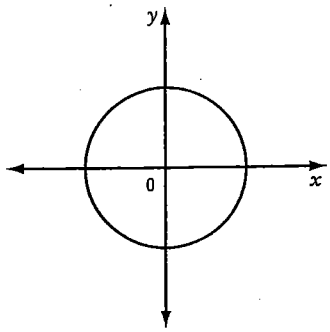
7. Determine the exact value of

a) $\cot\left(\frac{5\pi}{6}\right)$

b) $\csc\left(\frac{5\pi}{3}\right)$

4.4 Introduction to Trigonometric Equations, pages 138–144

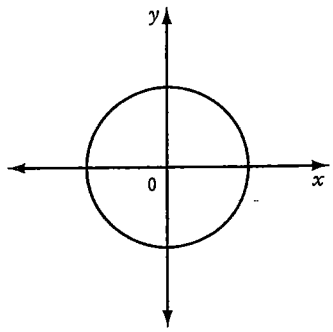
8. Write the general form of the solutions to $\sec\theta + 10 = 2 - 4\sec\theta$ (in degrees).



$\theta_1 \approx$ _____, $n \in I$

$\theta_2 \approx$ _____, $n \in I$

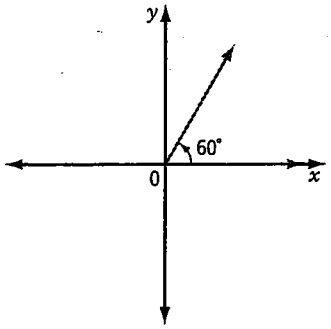
9. Solve $2\sin^2\theta + \sin\theta = 1$, $0 \leq \theta < 2\pi$. Give exact solutions.



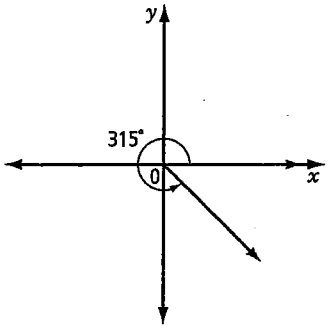
Chapter 4

4.1 Angles and Angle Measure, pages 109–119

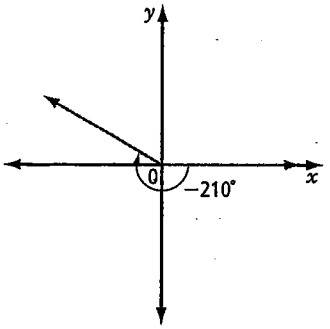
1. a) $\frac{\pi}{3}$



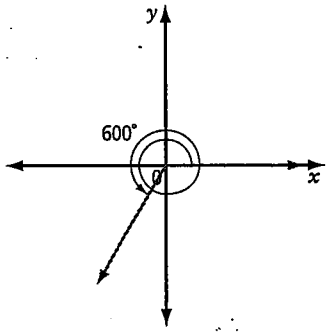
b) $\frac{7\pi}{4}$



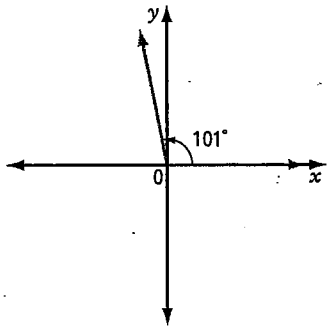
c) $-\frac{7\pi}{6}$



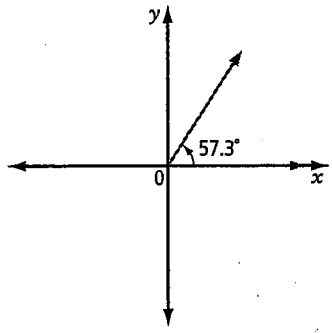
d) $\frac{10\pi}{3}$



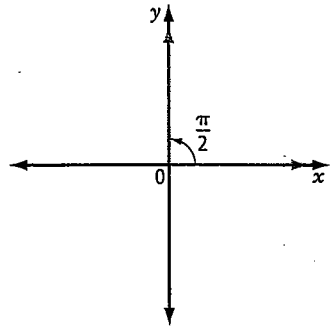
2. a) 1.76



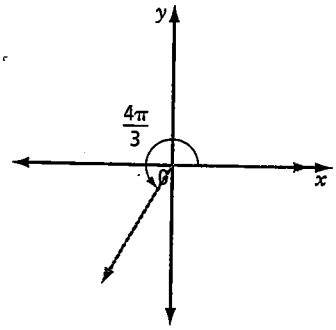
b) 1.00



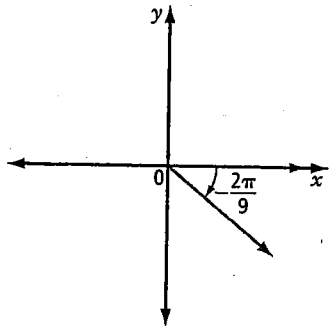
3. a) 90°



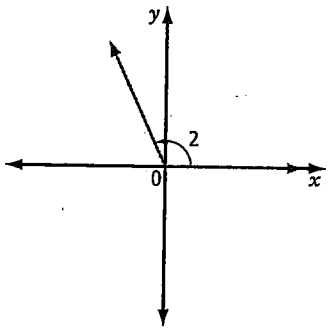
b) 240°



c) −40°



d) 114.59°



4. Examples: **a)** −11°, 709° **b)** −127°, 233°
c) $\frac{8\pi}{3}, -\frac{4\pi}{3}$ **d)** $\frac{\pi}{4}, -\frac{7\pi}{4}$

5. a) $-465°, -105°, 615°; 255° \pm 360°n, n \in \mathbb{N}$
b) $-3\pi, -\pi, 3\pi; \pi \pm 2\pi n, n \in \mathbb{N}$
c) $-\frac{7\pi}{6}, \frac{17\pi}{6}, \frac{29\pi}{6}, \frac{5\pi}{6} \pm 2\pi n, n \in \mathbb{N}$

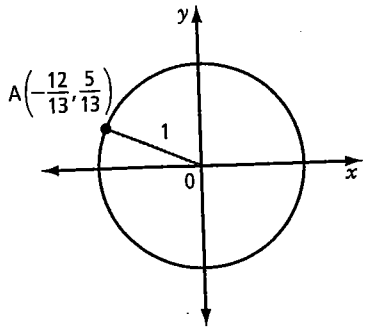
6. a) 41.89 cm **b)** 51.05 mm

4.2 The Unit Circle, pages 120–128

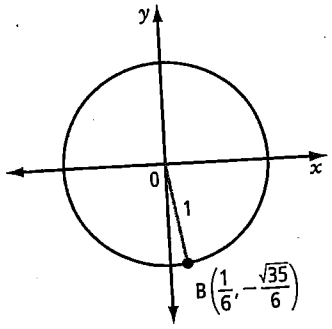
1. a) $x^2 + y^2 = 625$ b) $x^2 + y^2 = 1.21$

2. a) Yes b) Yes c) No

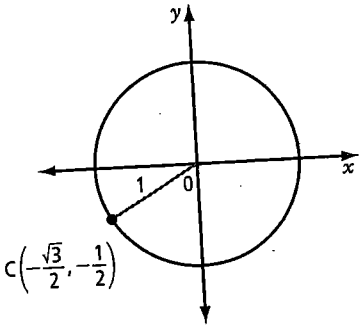
3. a) $-\frac{12}{13}$



b) $-\frac{\sqrt{35}}{6}$



c) $-\frac{\sqrt{3}}{2}$



4. a) $(0, 1)$ b) $(1, 0)$
   c) $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ d) $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$
   e) $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ f) $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

5. a) π b) $\frac{\pi}{6}$
   c) $\frac{3\pi}{4}$ d) $\frac{4\pi}{3}$

6. a) $\frac{\pi}{2}$ b) $\frac{5\pi}{6}$

4.3 Trigonometric Ratios, pages 129–137

1. $\sin\theta = -\frac{24}{25}$, $\cos\theta = \frac{7}{25}$, $\tan\theta = -\frac{24}{7}$,
   $\csc\theta = -\frac{25}{24}$, $\sec\theta = \frac{25}{7}$, $\cot\theta = -\frac{7}{24}$

2. a) − b) +
   c) + d) −

3. a) I, III b) III
   c) I, II d) II

4. a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{2}}$ c) 0
   d) $\frac{1}{\sqrt{3}}$ e) $-\frac{2}{\sqrt{3}}$ f) $-\sqrt{2}$

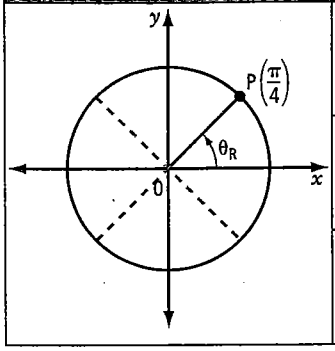
5. a) 3.628 b) −0.249
   c) 2.985 d) −1.701

6. a) 108°, 288°, 468°, 648°
   b) −197°, −163°, 163°, 197°

7. a) $\frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$
   b) $-\frac{11\pi}{6}, -\frac{7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$

8. $\sin\theta = \frac{3}{5}$, $\cos\theta = -\frac{4}{5}$, $\tan\theta = -\frac{3}{4}$,
   $\sec\theta = -\frac{5}{4}$, $\cot\theta = -\frac{4}{3}$

9. Example:

$\theta_R = \frac{\pi}{4}$	Quadrant I	Quadrant II	Quadrant III	Quadrant IV
	$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\csc\frac{\pi}{4} = \sqrt{2}$	$\sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}}$ $\csc\frac{3\pi}{4} = \sqrt{2}$	$\sin\frac{5\pi}{4} = -\frac{1}{\sqrt{2}}$ $\csc\frac{5\pi}{4} = -\sqrt{2}$	$\sin\frac{7\pi}{4} = -\frac{1}{\sqrt{2}}$ $\csc\frac{7\pi}{4} = -\sqrt{2}$
	$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\sec\frac{\pi}{4} = \sqrt{2}$	$\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$ $\sec\frac{3\pi}{4} = -\sqrt{2}$	$\cos\frac{5\pi}{4} = -\frac{1}{\sqrt{2}}$ $\sec\frac{5\pi}{4} = -\sqrt{2}$	$\cos\frac{7\pi}{4} = \frac{1}{\sqrt{2}}$ $\sec\frac{7\pi}{4} = \sqrt{2}$
	$\tan\frac{\pi}{4} = 1$ $\cot\frac{\pi}{4} = 1$	$\tan\frac{3\pi}{4} = -1$ $\cot\frac{3\pi}{4} = -1$	$\tan\frac{5\pi}{4} = 1$ $\cot\frac{5\pi}{4} = 1$	$\tan\frac{7\pi}{4} = -1$ $\cot\frac{7\pi}{4} = -1$

4.4 Introduction to Trigonometric Equations, pages 138–144

1. a) 30°, 150° b) $\frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}$
2. a) 42.8°, 317.2° b) 224.9°, 315.1°
3. a) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ b) 11.54°, 168.46°, 210°, 330°
4. a) $\frac{\pi}{6} + 2\pi n, n \in \mathrm{I}$ b) $\frac{11\pi}{6} + 2\pi n, n \in \mathrm{I}$
5. a) $\frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$ b) $\frac{\pi}{4} + \pi n, n \in \mathrm{I}$

Chapter 4 Review, pages 145–147

1. a) $\frac{3\pi}{2}$ b) 300°
   c) $\frac{5\pi}{3}$ d) $\frac{-720°}{\pi}$
   e) $\frac{11\pi}{4}$ f) 585°
2. Examples:
   a) $\frac{23\pi}{6}, -\frac{\pi}{6}$,
   general form: $\frac{11\pi}{6} \pm 2\pi n, n \in \mathrm{N}$
   b) 345°, −735°,
   general form: $-375° \pm (360°)n, n \in \mathrm{N}$
3. a) 6.3 b) 28.6°
4. $-\frac{\sqrt{5}}{3}$
5. a) $\frac{\pi}{3}$ b) $\frac{7\pi}{4}$
6. 56°, 304°, 416°, 664°
7. a) $-\sqrt{3}$ b) $-\frac{2}{\sqrt{3}}$
8. $\theta_1 \approx 128.7° + 360°n, n \in \mathrm{I}$;
   $\theta_2 \approx 231.3° + 360°n, n \in \mathrm{I}$
9. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$