

Chapter 5

Check Your Understanding

Section 5.1

Practise

1. State the amplitude of each trigonometric function.

a) $y = 2 \cos \theta$

b) $y = \frac{1}{4} \sin \theta$

c) $y = 5 \sin (2\theta)$

d) $y = -3 \cos \left(\frac{1}{2}\theta\right)$

2. State the period of each trigonometric function in degrees and in radians.

a) $y = 3 \sin \theta$

Degrees: $\frac{360^\circ}{|b|} = \underline{\hspace{2cm}}$

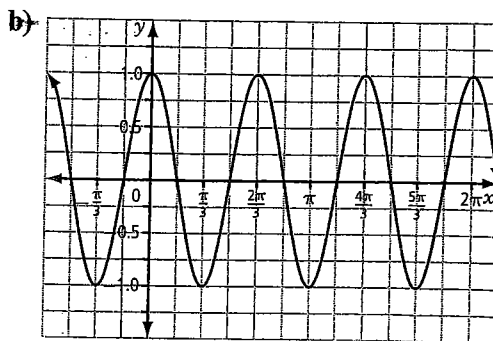
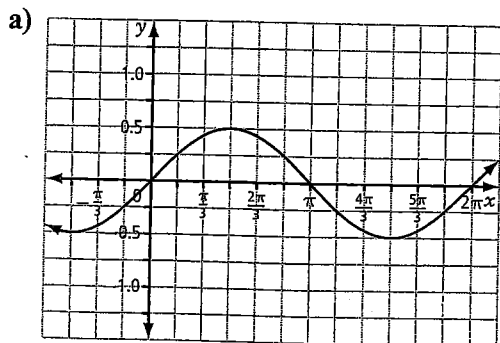
Radians: $\frac{2\pi}{|b|} = \underline{\hspace{2cm}}$

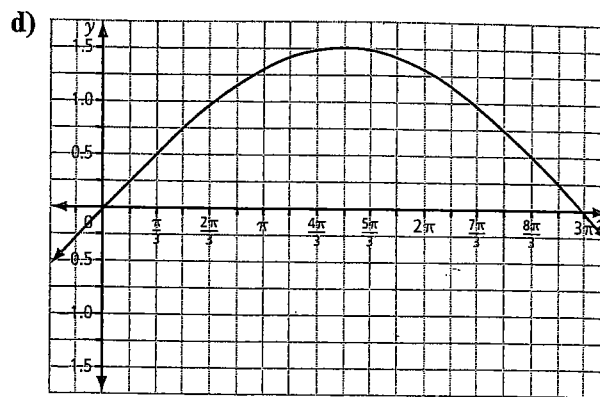
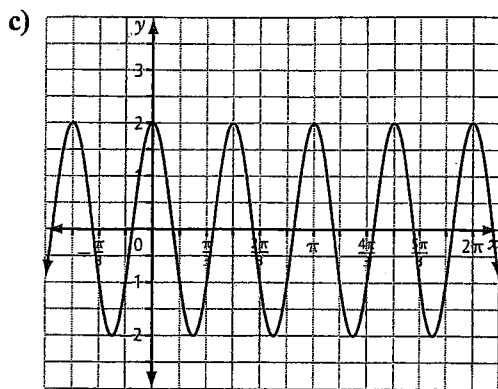
b) $y = \cos (2\theta)$

c) $y = 0.25 \sin (0.25\theta)$

d) $y = -1.5 \cos (1.5\theta)$

3. State the period, in radians, and the amplitude of each trigonometric function.





4. Identify the key features of $y = \sin \theta$ and the transformed sine function. Then, graph at least two cycles of the transformed sine function.

a) $y = \sin\left(\frac{1}{3}\theta\right)$

Identify the key features of $y = \sin \theta$.

$a =$ _____; the amplitude is _____.

Maximum value: _____ Minimum value: _____

$b =$ _____; the period is _____.

θ -intercepts: _____ y -intercept: _____

Identify the key features of $y = \sin\left(\frac{1}{3}\theta\right)$.

$a =$ _____; the amplitude is _____.

The graph _____ reflected in the x -axis.
(is or is not)

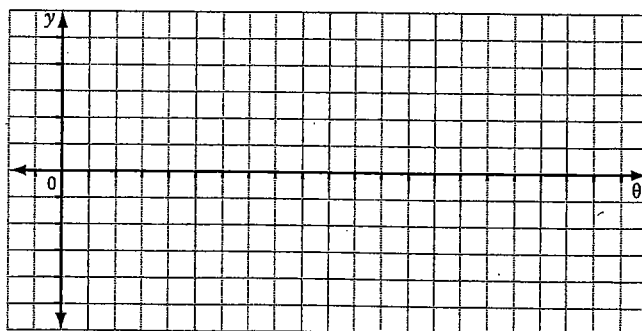
Maximum value: _____ Minimum value: _____

$b =$ _____; the period is _____.

The graph is stretched _____ by a factor of _____.
(horizontally or vertically)

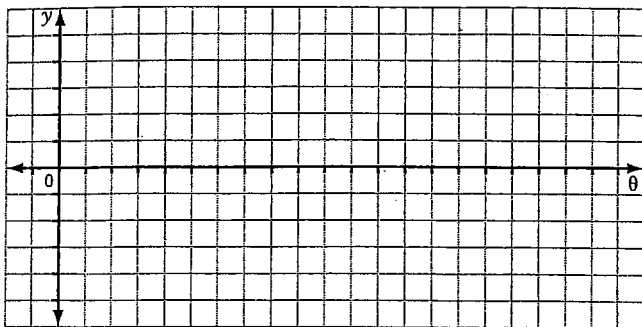
θ -intercepts: _____ y -intercept: _____

Use the key features to sketch the graph of the function.

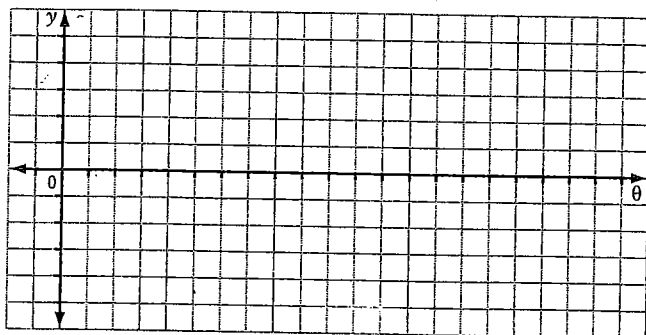


Consider the key features of the function when choosing the scales.

b) $y = 1.5 \sin(2\theta)$

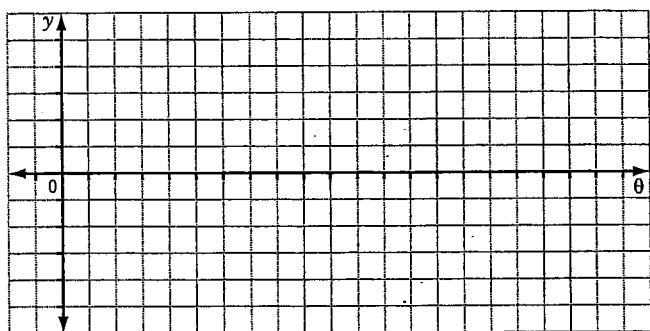


c) $y = -2 \sin(4\theta)$

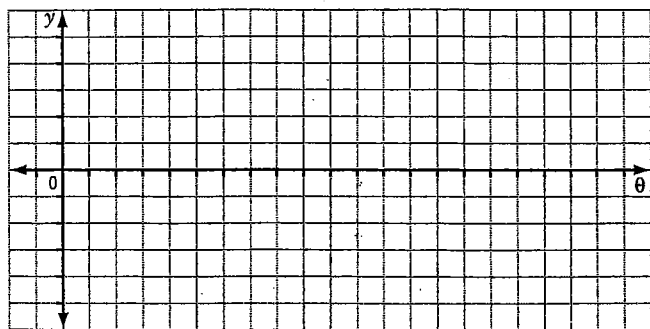


5. Identify the key features of each cosine function. Then, graph at least two cycles of each cosine function.

a) $y = 2 \cos\left(\frac{1}{2}\theta\right)$



b) $y = -\cos(2\theta)$



Check Your Understanding

Section 5.2

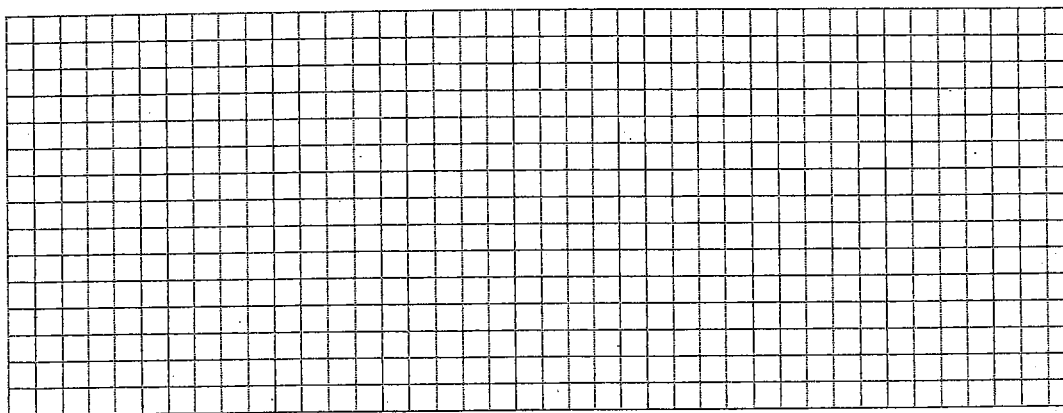
Practise

1. Determine the phase shift and the vertical displacement. Then, graph the function. Choose appropriate scales for the axes.

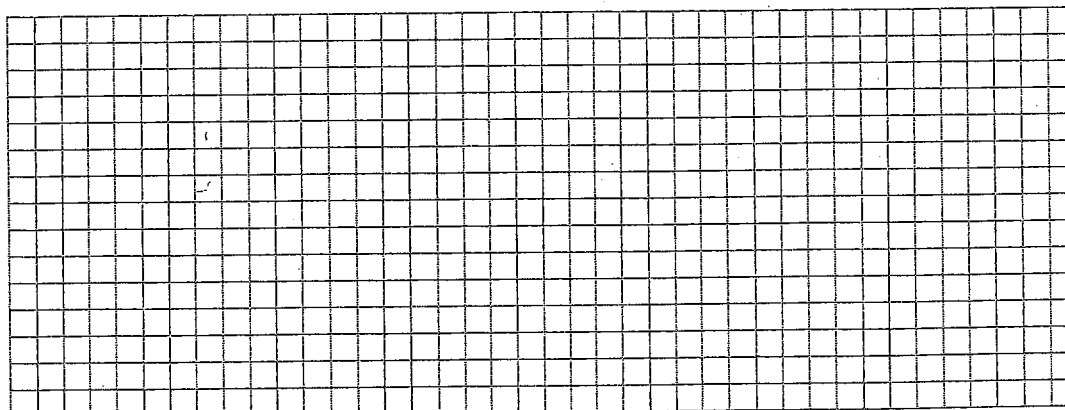
a) $y = \cos\left(\theta - \frac{\pi}{3}\right) - 1$

Phase shift: _____

Vertical displacement: _____



b) $y = \sin\left(\theta + \frac{\pi}{4}\right) + 2$



2. Determine the key features of each sine function.

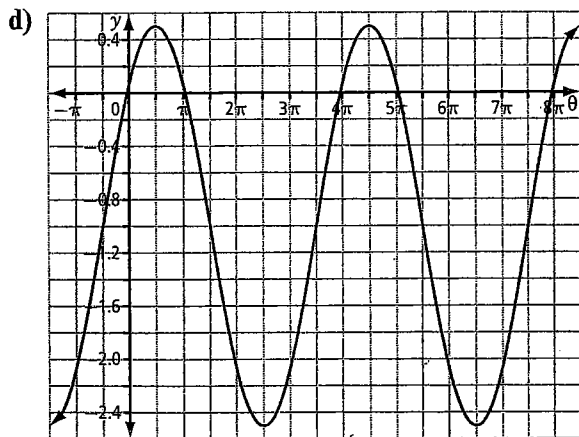
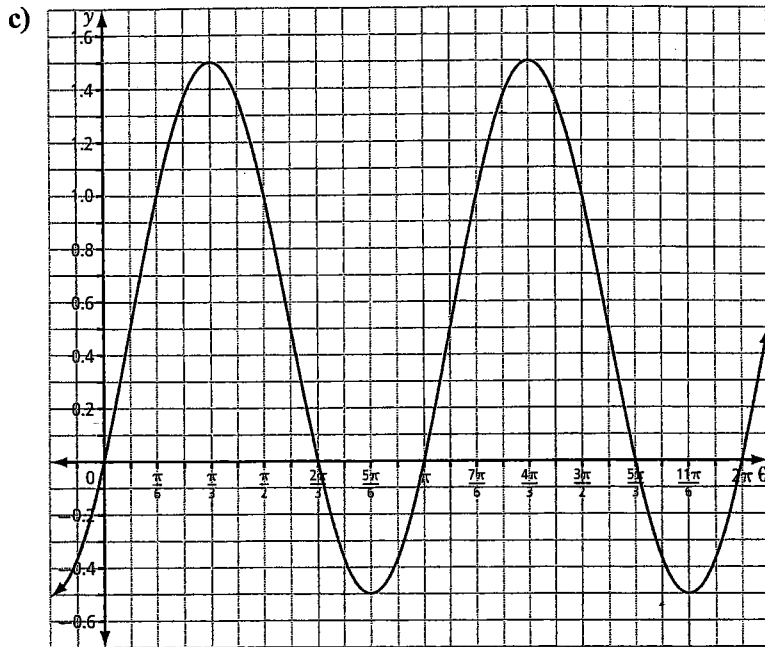
a) $y = -5 \sin\left(\frac{1}{2}(\theta - 90^\circ)\right) + 15$

Amplitude: _____ Period: _____

Phase shift: _____ Vertical displacement: _____

Domain: _____ Range: _____

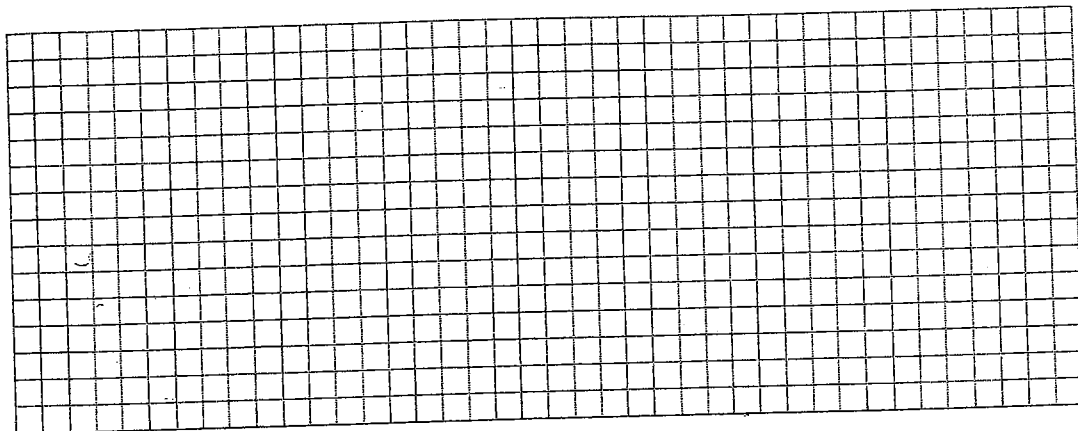
b) $y = 0.1 \sin(2\theta + 90^\circ) - 1$



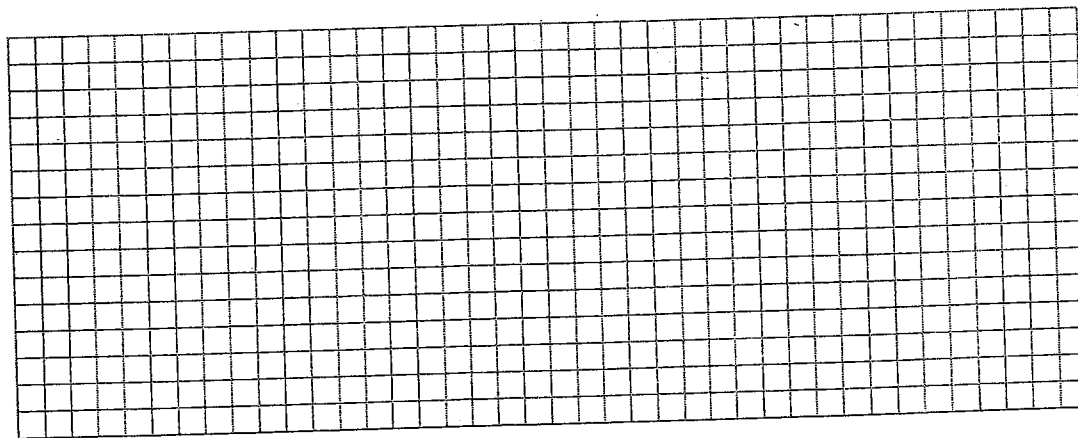
3. Write the equation of each sine function in the form $y = a \sin b(x - c) + d$ given its characteristics.
- amplitude 2, period π , phase shift $\frac{\pi}{3}$ to the left, vertical displacement 1 unit down
 - amplitude $\frac{1}{4}$, period 6π , phase shift π to the left, vertical displacement 2 units up
 - amplitude 4, period 540° , phase shift 60° to the right, no vertical displacement

4. Graph each function in the space provided. Show at least two cycles.

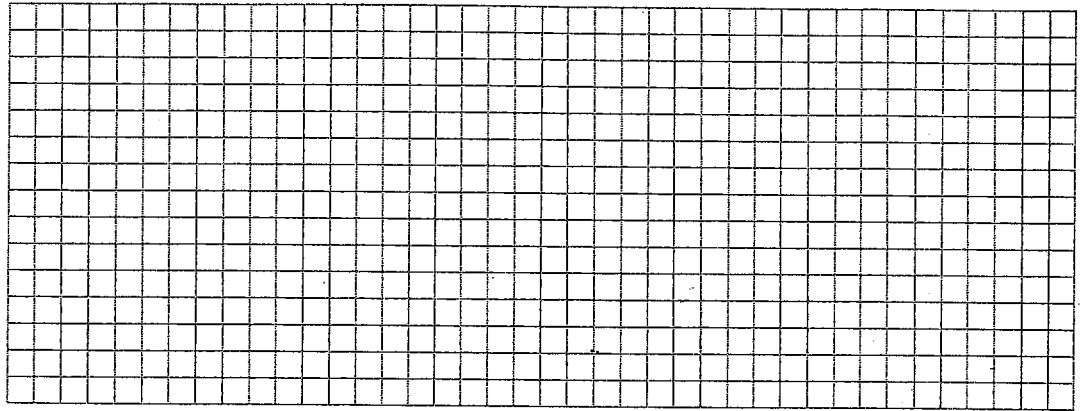
a) $y = 5 \sin 0.5(\theta + \pi) + 3$



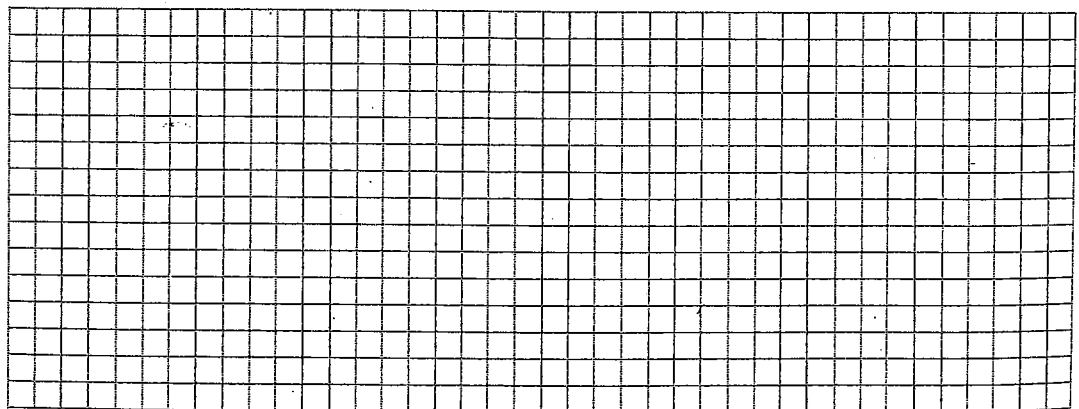
b) $y = -2 \sin 2\left(\theta - \frac{\pi}{3}\right) + 4$



c) $y = 1.5 \cos 3\left(\theta + \frac{\pi}{2}\right) - 1$

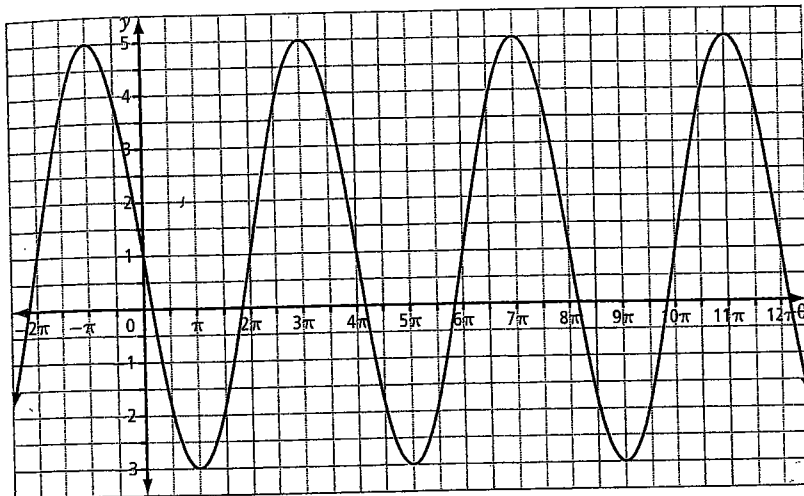


d) $y = -\cos \frac{1}{3}(\theta - \pi) + 3$

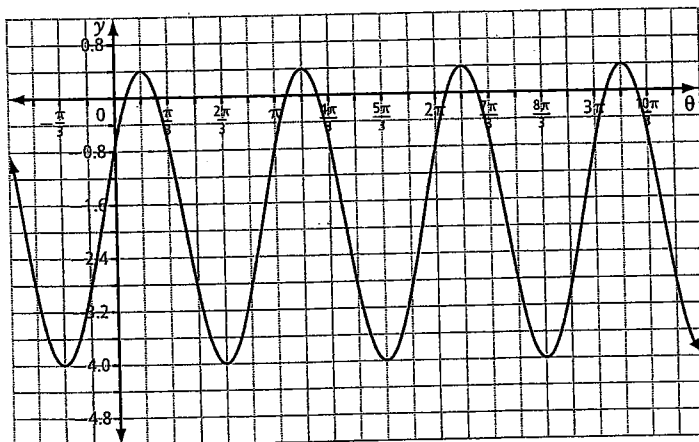


Apply

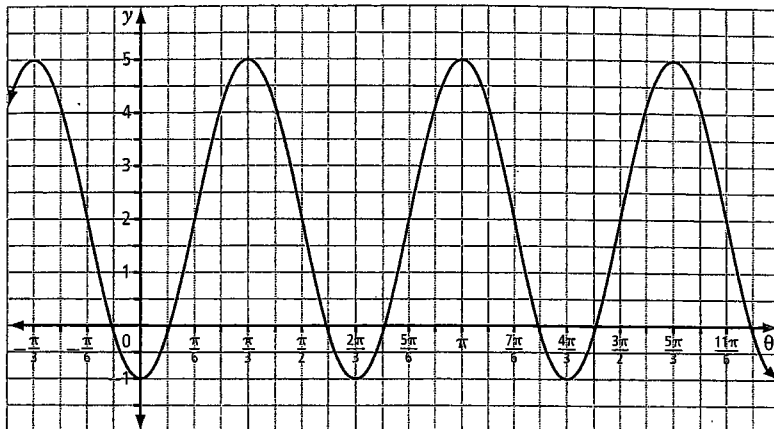
5. Write two different equations of the form $y = a \sin b(\theta - c) + d$ for the function graphed below. Use technology to check that your equations are correct.



6. Write two different equations of the form $y = a \cos b(\theta - c) + d$ to represent the function graphed below. Use technology to verify that your equations are correct.



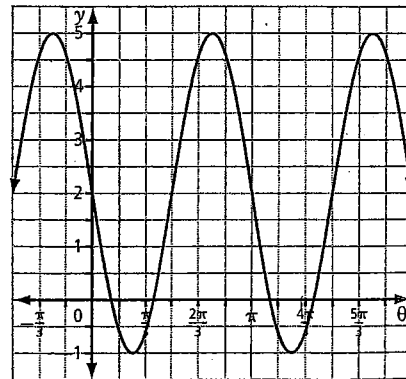
7. Write an equation of the form $y = a \sin b(\theta - c) + d$ and an equation of the form $y = a \cos b(\theta - c) + d$ to represent the function graphed below.



Connect

8. The graphed function is represented by an equation of each of the following forms. Determine the values of a , b , c , and d .

a) $y = a \sin b(\theta - c) + d; a > 0$



b) $y = a \sin b(\theta - c) + d; a < 0$

c) $y = a \cos b(\theta - c) + d; a > 0$

Check Your Understanding Section 5.3

Practise

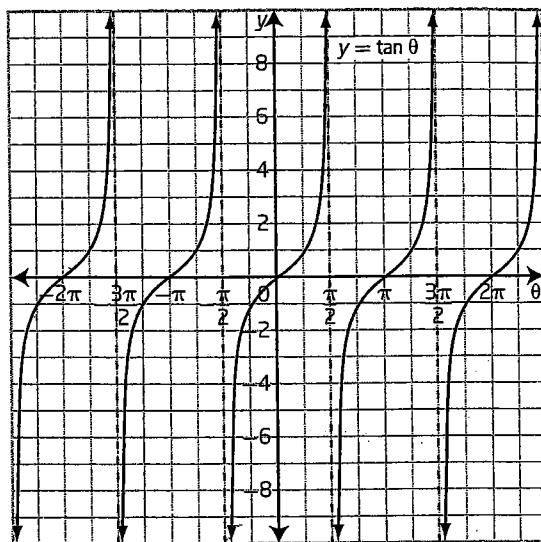
1. Use the graph of $y = \tan \theta$ to determine each value.

a) $\tan 2\pi$

b) $\tan \frac{3\pi}{2}$

c) $\tan \frac{\pi}{4}$

d) $\tan \left(-\frac{\pi}{4}\right)$



2. Use the graph of $y = \tan \theta$ from #1 and your knowledge of the properties of the tangent function to determine each value.

a) $\tan(-\pi)$

b) $\tan(-3\pi)$

c) $\tan(-100\pi)$

3. Use the graph of $y = \tan \theta$ from #1 and your knowledge of the properties of the tangent function to determine each value.

a) $\tan\left(\frac{9\pi}{4}\right)$

b) $\tan\left(\frac{13\pi}{4}\right)$

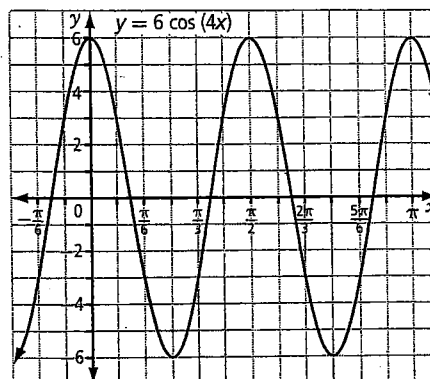
c) $\tan\left(\frac{17\pi}{4}\right)$

Check Your Understanding Section 5.4

Practise

1. Use the graph of $y = 6 \cos(4x)$ to solve each trigonometric equation.

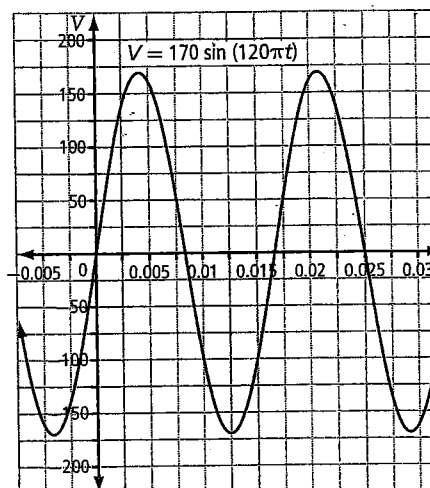
a) $6 \cos(4x) = 3, 0 \leq x \leq \pi$



b) $6 \cos(4x) = -6$, general solution in radians

2. Use the graph of $V = 170 \sin(120\pi t)$ to approximate the solutions to each trigonometric equation.

a) $50 = 170 \sin(120\pi t), 0 \leq x \leq 0.030$



b) $170 \sin(120\pi t) = -120, 0 \leq x \leq 0.030$

c) $170 \sin(120\pi t) = 0$, general solution in radians

3. Sound travels in waves. You can see the sinusoidal patterns of sound waves using a device called an oscilloscope.

a) Orchestra members tune their instruments to $A = 440$ Hz, meaning the sound wave repeats 440 times per second. What is the period of this sound wave, in seconds?

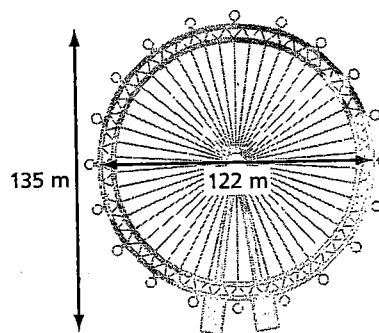
b) Write a simple sine function representing the waveform of the note $A = 440$.

$$\frac{2\pi}{|b|} = \text{period}$$

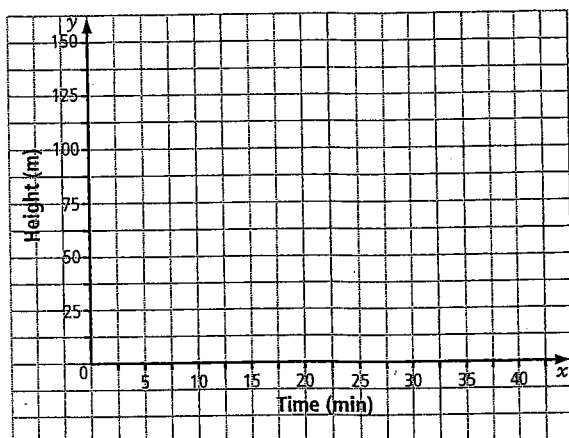
c) "Middle C" has a frequency of 261.63 Hz. What sine function could represent middle C?

Apply

5. The London Eye has diameter 122 m and height 135 m. It takes approximately 30 min for one rotation of the wheel. Passengers board at the bottom of the ride. The ride moves slowly enough that it is usually not necessary for the wheel to stop to let passengers on or off.

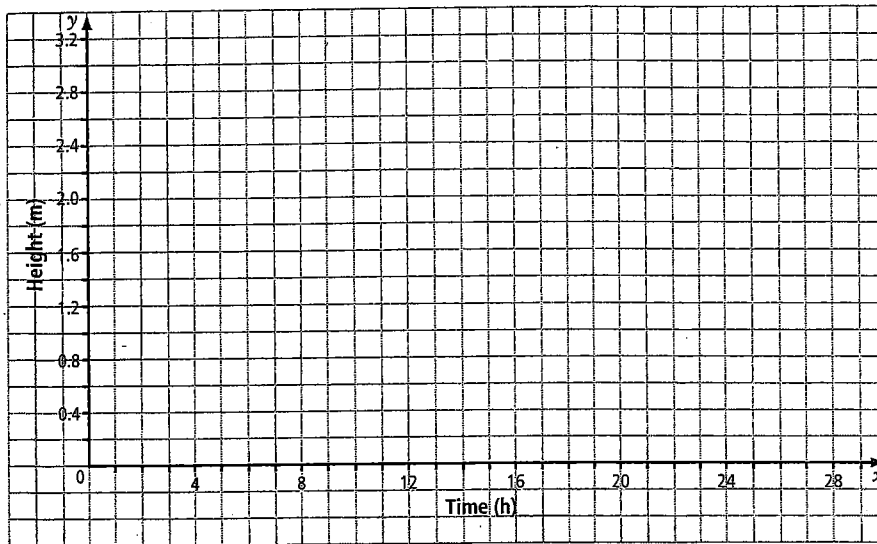


- a) Sketch a sinusoidal function representing the height of a passenger riding the London Eye. What assumptions do you have to make?



- b) Write a sinusoidal function that represents the height of a passenger riding the London Eye. Over what domain is the function valid?

6. One particular afternoon, the tide in Victoria, BC, reached a maximum height of 3.0 m at 2:00 p.m. and a minimum height of 0.2 m at 8:00 p.m.
- a) Sketch a sinusoidal function based on these data. What assumptions do you have to make?

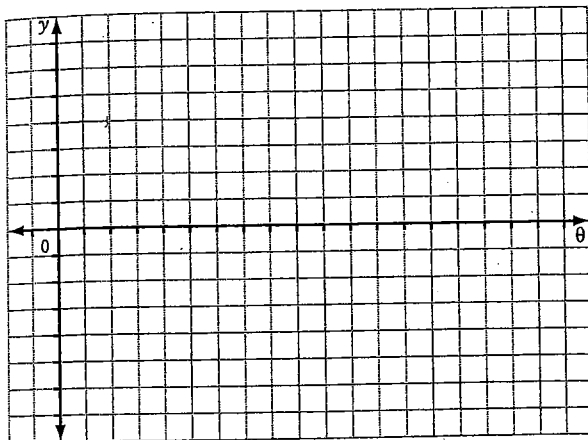


- b) Write a sinusoidal function that represents the tide in Victoria, BC, on this day. Over what domain is the function valid?

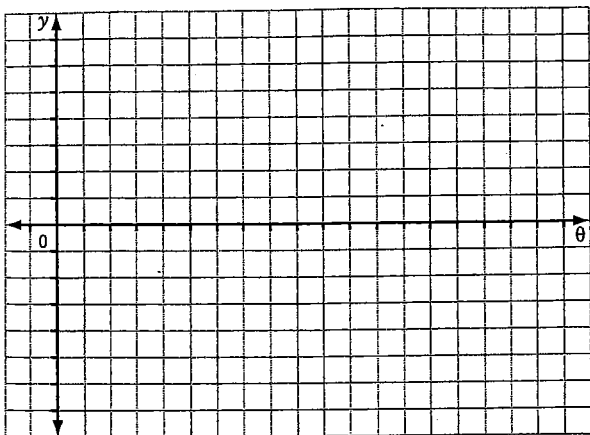
Chapter 5 Review

5.1 Graphing Sine and Cosine Functions, pages 149–157

1. Graph at least two cycles of $y = 3 \cos\left(\frac{1}{2}\theta\right)$. State the amplitude and period in degrees.



2. Graph at least two cycles of $y = -0.5 \sin(2\theta)$. State the amplitude and period in radians.



3. Without graphing, determine the amplitude and period, in radians and in degrees, of each function.

a) $y = 2 \sin 3x$

b) $y = \frac{1}{3} \cos x$

c) $y = \frac{3}{4} \cos 2x$

d) $y = -4 \sin \frac{2}{3}x$

5.2 Transformations of Sinusoidal Functions, pages 158–166

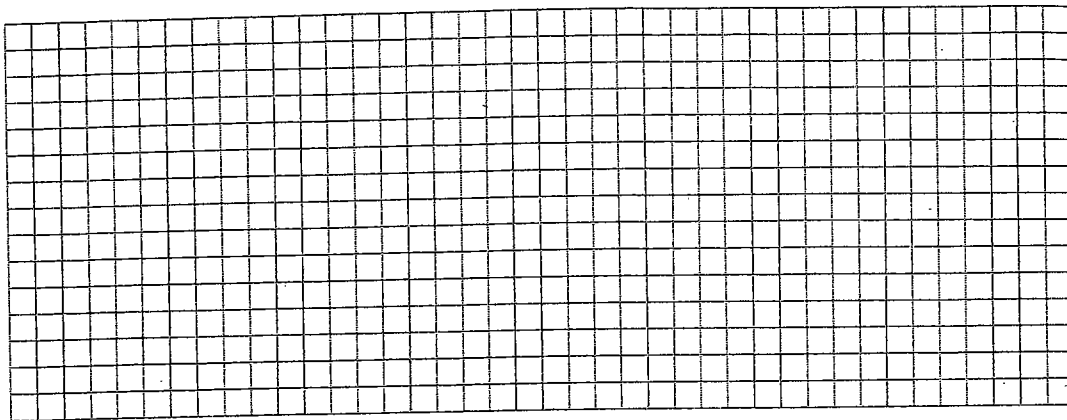
4. Determine the amplitude, period, phase shift, and vertical displacement with respect to $y = \sin x$ or $y = \cos x$ for each function.

a) $y = 5 \sin \frac{1}{4} \left(x + \frac{\pi}{3} \right) - 1$

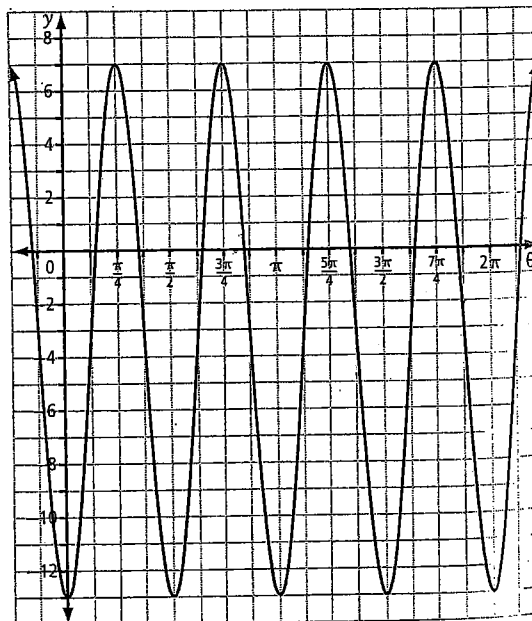
b) $y = -\frac{1}{2} \cos 2(x - \pi) - 3$

c) $y = 3 \cos 4(x + 50^\circ) + 6$

5. Graph at least two cycles of $y = \sin 2\left(x + \frac{\pi}{12}\right) - 0.4$.

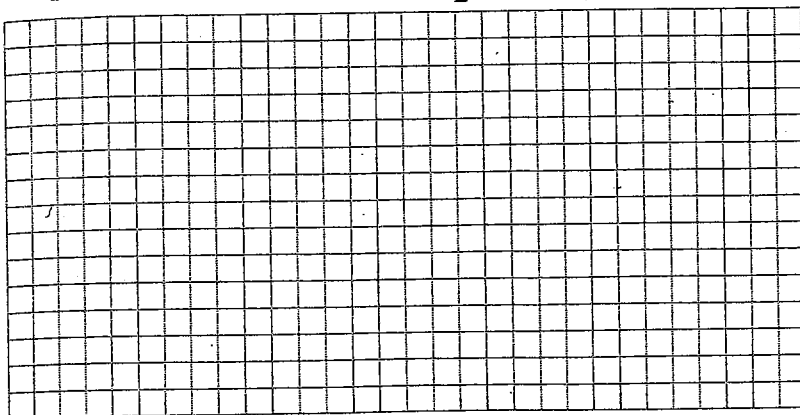


6. Write two equations of the form $y = a \cos b(\theta - c) + d$ that represent the function shown below.



5.3 The Tangent Function, pages 167-174

7. Graph $y = \tan x$ over the domain $-\frac{3\pi}{2} \leq x \leq \frac{5\pi}{2}$.



5.4

10. Solve each equation algebraically.

a) $\sin 2x = 0$ $0 \leq x \leq 2\pi$

b) $\cos(x + \frac{\pi}{2}) + 1 = 0$ $0 \leq x \leq 2\pi$

c) $\sin 2(x - 30^\circ) + 0.5 = 0$ general solution

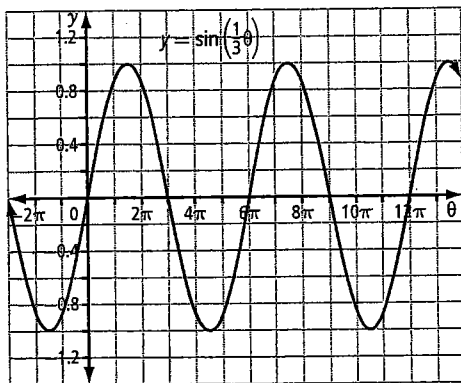
Chapter 5

5.1 Graphing Sine and Cosine Functions, pages 149–157

- a) 2 b) $\frac{1}{4}$
 - c) 5 d) 3
- a) $360^\circ, 2\pi$ b) $180^\circ, \pi$
 - c) $1440^\circ, 8\pi$ d) $240^\circ, \frac{4\pi}{3}$
- a) $2\pi; \frac{1}{2}$ b) $\frac{2\pi}{3}, 1$
 - c) $\frac{\pi}{2}, 2$ d) $6\pi; 1.5$

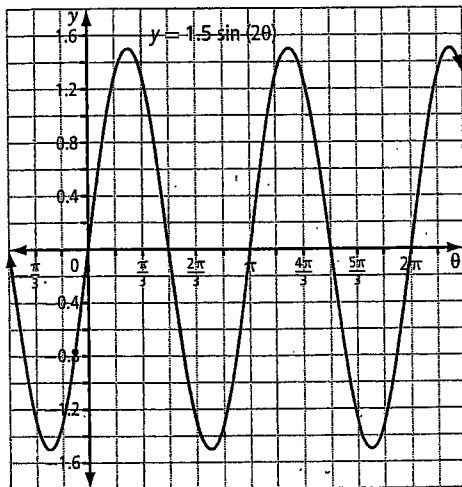
- For $y = \sin \theta$:
amplitude: 1; maximum value: 1; minimum value: -1; period: 2π ; θ -intercepts: $\pi n, n \in \mathbb{I}$; y -intercept: 0

For $y = \sin\left(\frac{1}{3}\theta\right)$:
amplitude: 1; maximum value: 1; minimum value: -1; period: 6π ; θ -intercepts: $3\pi n, n \in \mathbb{I}$; y -intercept: 0



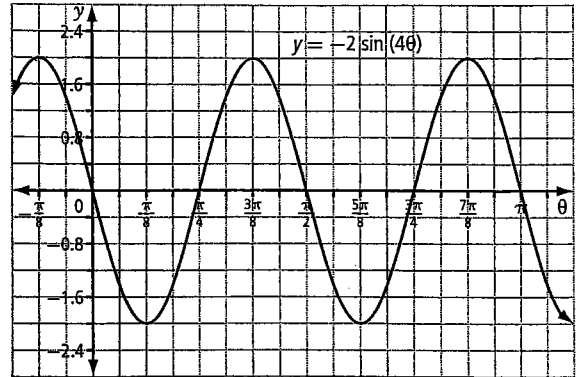
- For $y = \sin \theta$:
amplitude: 1; maximum value: 1; minimum value: -1; period: 2π ; θ -intercepts: $\pi n, n \in \mathbb{I}$; y -intercept: 0

For $y = 1.5 \sin(2\theta)$:
amplitude: 1.5; maximum value: 1.5; minimum value: -1.5; period: π ; θ -intercepts: $\frac{\pi}{2}n, n \in \mathbb{I}$; y -intercept: 0

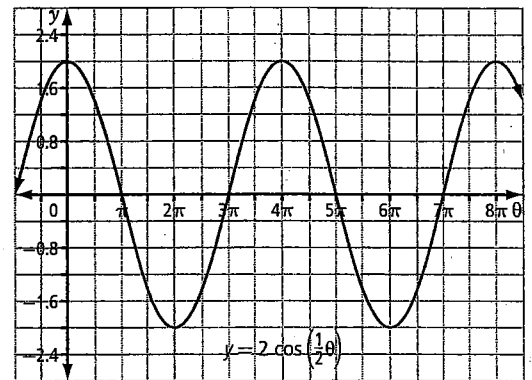


- For $y = \sin \theta$:
amplitude: 1; maximum value: 1; minimum value: -1; period: 2π ; θ -intercepts: $\pi n, n \in \mathbb{I}$; y -intercept: 0

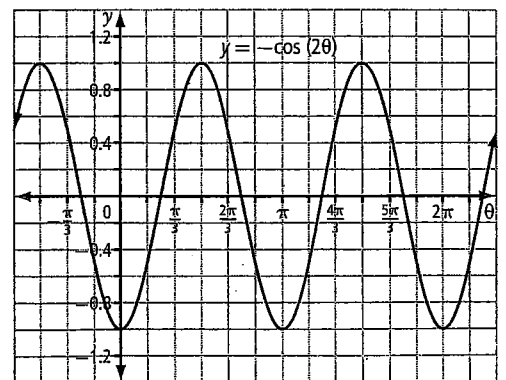
For $y = -2 \sin(4\theta)$:
amplitude: 2; reflected in x -axis; maximum value: 2; minimum value: -2; period: $\frac{\pi}{2}$; θ -intercepts: $\frac{\pi}{4}n, n \in \mathbb{I}$; y -intercept: 0



- amplitude: 2; maximum value: 2; minimum value: -2; period: 4π ; θ -intercepts: $\pi + 2\pi n, n \in \mathbb{I}$; y -intercept: 2

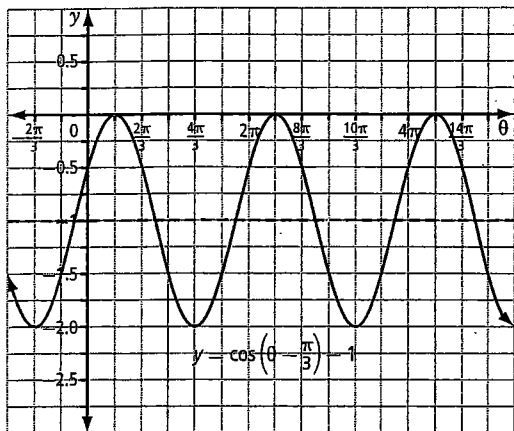


- amplitude: 1; reflected in the x -axis; maximum value: 1; minimum value: -1; period: π ; θ -intercepts: $\frac{\pi}{4} + \frac{\pi}{2}n, n \in \mathbb{I}$; y -intercept: -1

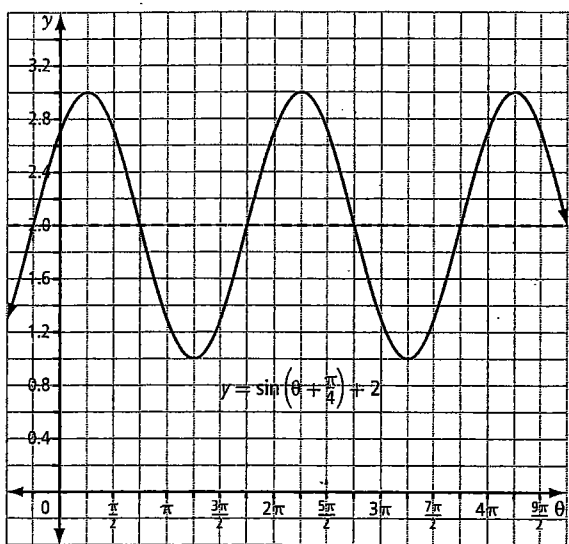


5.2 Transformations of Sinusoidal Functions, pages 158–166

1. a) $\frac{\pi}{3}$ units to the right; 1 unit down

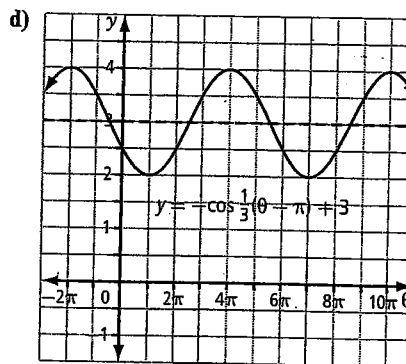
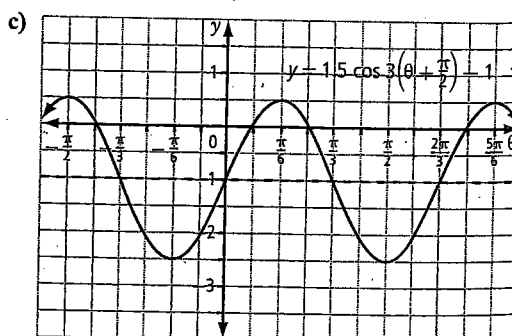
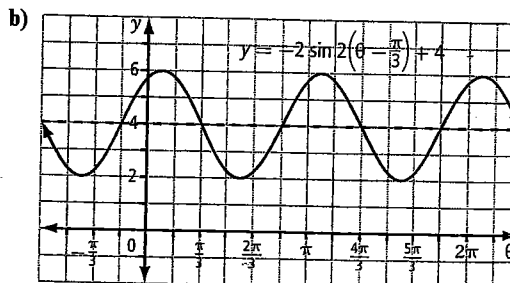
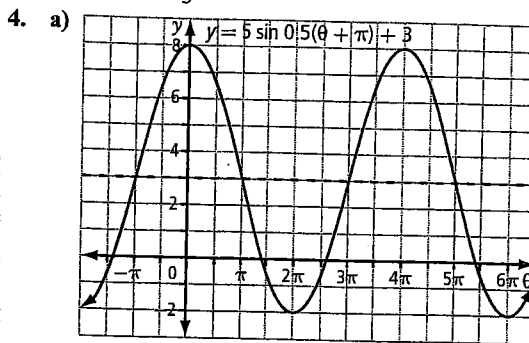


- b) $\frac{\pi}{4}$ units to the left; 2 units up



2. a) amplitude: 5; period: 720° ; phase shift: 90° to the right; vertical displacement: 15 units up; domain: $\{x \mid x \in \mathbb{R}\}$; range: $\{y \mid 10 \leq y \leq 20, y \in \mathbb{R}\}$
- b) amplitude: 0.1; period: 180° ; phase shift: 45° to the left; vertical displacement: 1 unit down; domain: $\{x \mid x \in \mathbb{R}\}$; range: $\{y \mid -1.1 \leq y \leq -0.9, y \in \mathbb{R}\}$
- c) amplitude: 1; period: π ; phase shift: $\frac{\pi}{12}$ units to the right; vertical displacement: 0.5 units up; domain: $\{x \mid x \in \mathbb{R}\}$; range: $\{y \mid -0.5 \leq y \leq 1.5, y \in \mathbb{R}\}$
- d) amplitude: 1.5; period: 4π ; phase shift: $\frac{\pi}{2}$ units to the left; vertical displacement: 1 unit down; domain: $\{x \mid x \in \mathbb{R}\}$; range: $\{y \mid -2.5 \leq y \leq 0.5, y \in \mathbb{R}\}$

3. a) $y = 2 \sin 2\left(x + \frac{\pi}{3}\right) - 1$
 b) $y = \frac{1}{4} \sin \frac{1}{3}(x + \pi) + 2$
 c) $y = 4 \sin \frac{2}{3}(x - 60^\circ)$



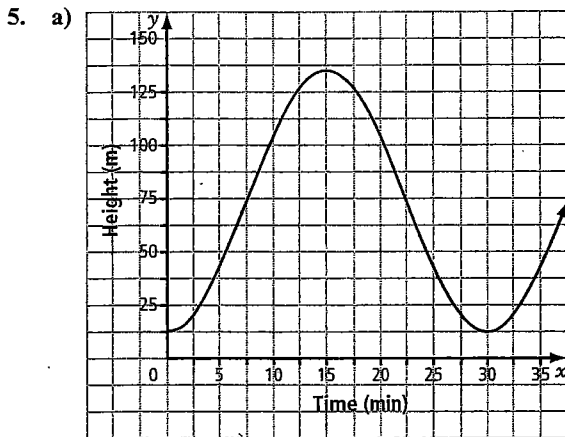
5. Example: $y = -4 \sin(0.5\theta) + 1$;
 $y = 4 \sin(0.5\theta - 2\pi) + 1$
6. Example: $y = 2.2 \cos\left(2\left(\theta - \frac{\pi}{6}\right)\right) - 1.8$;
 $y = 2.2 \cos\left(2\left(\theta + \frac{7\pi}{6}\right)\right) - 1.8$
7. Example: $y = 3 \sin\left(3\left(\theta - \frac{\pi}{6}\right)\right) + 2$;
 $y = -3 \cos(3\theta) + 2$
8. Examples:
- a) $a = 3, b = 2, c = \frac{\pi}{2}, d = 2$
- b) $a = -3, b = 2, c = 0, d = 2$
- c) $a = 3, b = 2, c = -\frac{\pi}{4}, d = 2$

5.3 The Tangent Function, pages 167–174

1. a) 0 b) undefined c) 1 d) -1
2. a) 0 b) 0 c) 0
3. a) 1 b) 1 c) 1

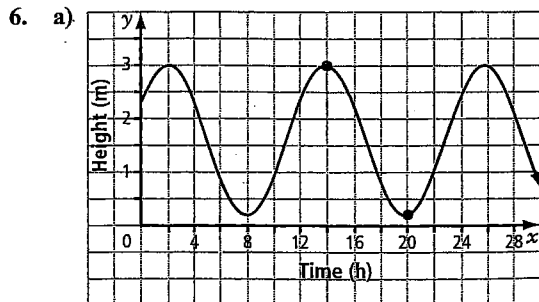
5.4 Equations and Graphs of Trigonometric Functions, pages 175–182

1. a) $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$ b) $x = \frac{\pi}{4} + \frac{\pi}{2}n, n \in \mathbb{I}$
2. Examples:
 - a) $t \approx 0.0008, 0.0075, 0.0175, 0.0242$
 - b) $t \approx 0.0104, 0.0146, 0.0269$
 - c) $t = \frac{n}{120}, n \in \mathbb{I}$
3. a) $\frac{1}{440} \text{ s}$ b) $y = \sin(880\pi x)$
 c) $y = \sin(523.26\pi x)$ or $y \approx \sin(1643.87x)$



Example: I assume that the ride does not stop to let on passengers, and that the wheel is vertical (perpendicular to the ground).

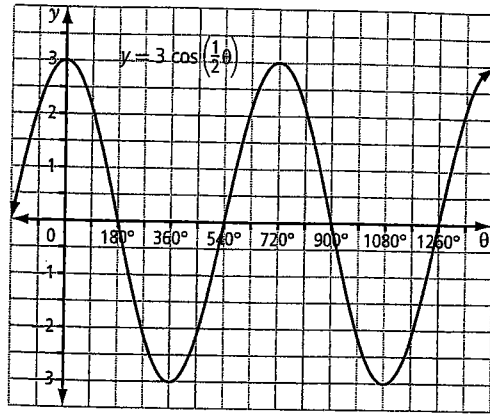
- b) $y = -61 \cos\left(\frac{\pi}{15}x\right) + 74$;
 domain: $\{x \mid 0 \leq x \leq 30, x \in \mathbb{R}\}$ unless the passenger goes around more than once



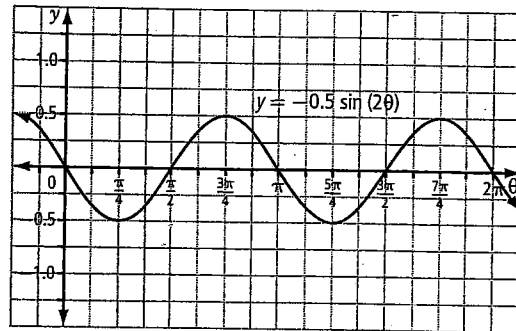
Example: assume that the amplitude of the tide is equal each occurrence, and that the tide comes in every 12 h exactly.

- b) $y = 1.4 \sin\left(\frac{\pi}{6}(x + 1)\right) + 1.6$; The domain should be restricted to some reasonable amount of time such that the assumptions made in part a) are roughly correct.

1. amplitude: 3; period: 720°

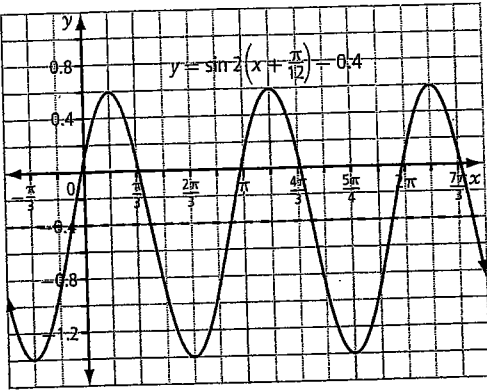


2. amplitude: 0.5; period: π



3. a) amplitude: 2; period: 120° or $\frac{2\pi}{3}$
- b) amplitude: $\frac{1}{3}$; period: 360° or 2π
- c) amplitude: $\frac{3}{4}$; period: 180° or π
- d) amplitude: 4; period: 540° or 3π
4. a) amplitude: 5; period: 8π ; phase shift: $\frac{\pi}{3}$ units to the left; vertical displacement: 1 unit down
- b) amplitude: $\frac{1}{2}$; period: π ; phase shift: π units to the right; vertical displacement: 3 units down
- c) amplitude: 3; period: 90° ; phase shift: 50° to the left; vertical displacement: 6 units up

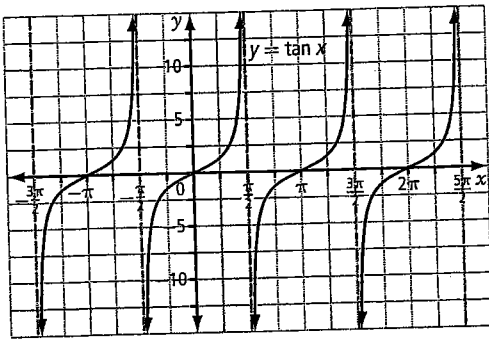
5.



6. Examples: $y = 10 \cos 4\left(\theta - \frac{\pi}{4}\right) - 3$,

$$y = 10 \cos 4\left(\theta + \frac{\pi}{4}\right) - 3$$

7.



10. a) $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ b) $\frac{\pi}{2}$

c) $15^\circ + 180^\circ n$ and $135^\circ + 180^\circ n, n \in \mathbb{I}$