Chapter 6

Check Your Understanding Section 6.1

Practise

1. Determine the non-permissible values of x, in radians, for each expression.

   a) $\frac{\sin x}{\cos x}$

   b) $\frac{\cos x}{\tan x}$

   c) $\frac{\cot x}{1 + \sin x}$

   d) $\frac{\tan x}{\cos x - 1}$

   In parts c) and d), explain whether it is possible to write a single restriction.

2. Simplify each expression to one of the three primary trigonometric functions, $\sin x$, $\cos x$, or $\tan x$.

   a) $\cot x \sin x$

   b) $\frac{\sec^2 x \cos x}{\csc x}$

   c) $\frac{\cot x \tan x}{\csc x}$

3. Simplify. Then, rewrite each expression as one of the three reciprocal trigonometric functions, $\csc x$, $\sec x$, or $\cot x$.

   a) $\frac{\csc x}{\sec x}$

   b) $\csc x \tan x \sec x \cos x$

   c) $\frac{\sin x}{1 - \cos^2 x}$

4. a) Verify that the equation $\frac{\csc x}{\tan x + \cot x} = \cos x$ is true for $x = 60°$ and for $x = \frac{\pi}{6}$.

b) What are the non-permissible values of the equation in the domain $0° \leq x < 360°$.

What determines if a value is permissible when it is in the denominator?

5. Consider the equation $\tan x + \frac{1}{\tan x} = \frac{1}{\cos x \sin x}$.

a) What are the non-permissible values, in radians, for this equation?

c) Verify that the equation is true when $x = \frac{\pi}{4}$. Use exact values for each expression in the equation.

8. Simplify $\frac{\cot x + \tan x}{\sec x}$ to one of the three reciprocal trigonometric ratios. What are the non-permissible values of the original expression in the domain $0 \leq x < 2\pi$?

10. Simplify $(\cos x - \sin x)^2 - (\sin x - \cos x)^2$.

Check Your Understanding Sechon 6.2

Practise

1. Write each expression as a single trigonometric function.

a) $\cos 87° \cos 22° + \sin 87° \sin 22°$

b) $\sin 72° \cos 46° - \cos 72° \sin 46°$

c) $\dfrac{\tan 28° + \tan 33°}{1 - \tan 28° \tan 33°}$

d) $6 \sin \frac{\pi}{10} \cos \frac{\pi}{10}$

e) $1 - 2 \sin^2 \frac{\pi}{8}$

f) $\dfrac{2 \tan \frac{\pi}{3}}{1 - \tan^2 \frac{\pi}{3}}$

2. Simplify. Then, give an exact value for each expression.

a) $2 \sin \frac{\pi}{4} \cos \frac{\pi}{4}$

b) $\cos \frac{\pi}{3} \cos \frac{\pi}{12} + \sin \frac{\pi}{3} \sin \frac{\pi}{12}$

c) $\dfrac{\tan 80° + \tan 40°}{1 - \tan 80° \tan 40°}$

d) $2 \cos^2 \frac{\pi}{2} - 1$

3. Write each as a single trigonometric function.

a) $\sin 80° \cos 40° - \cos 80° \sin 40°$

b) $\cos \frac{2\pi}{3} \cos \frac{\pi}{12} - \sin \frac{2\pi}{3} \sin \frac{\pi}{12}$

c) $\dfrac{\tan \frac{2\pi}{3} - \tan \frac{\pi}{12}}{1 + \tan \frac{2\pi}{3} \tan \frac{\pi}{12}}$

4. Simplify each expression to a single primary trigonometric function.

a) $\dfrac{\cos 2x - 1}{\sin 2x}$

b) $1 - 2\sin^2 \frac{\theta}{4}$

c) $\frac{1}{2} \sin \frac{\theta}{2} \cos \frac{\theta}{2}$

$\frac{1}{2} \sin \frac{\theta}{2} \cos \frac{\theta}{2} = 2$ (________________)

$= \frac{1}{4} \sin 2($________________$)$

$=$ ________________

d) $8\sin^2 2\theta - 4$

$8\sin^2 2\theta - 4 = -4(1 -$ __________$)$

$= -4$__________

5. Consider the expression $\dfrac{1 - \cos 2x}{\sin x}$.

a) State the permissible values.

b) Simplify the expression to one of the three primary trigonometric ratios.

6. Determine the exact value of each trigonometric expression.

a) $\sin 105°$

b) $\cos 165°$

c) $\tan \frac{23\pi}{12}$

d) $\csc \frac{5\pi}{12}$

Apply

7. Simplify $\sin(x + y) + \sin(x - y)$.

8. Angle θ is in quadrant III and $\tan\theta = \frac{7}{24}$. Determine an exact value for each of the following.

a) $\sin 2\theta$

b) $\cos 2\theta$

c) $\tan 2\theta$

9. Angle x is in quadrant II, angle y is in quadrant III, $\cos x = -\frac{5}{13}$, and $\tan y = \frac{4}{3}$. Determine the value of each of the following.

a) $\sin(x + y)$

b) $\cos(x - y)$

c) $\tan(x - y)$

10. Simplify each expression to the equivalent expression shown.

a) $\frac{\sin 2x}{1 - \cos 2x}$; $\cot x$

b) $\sin (x + y) \sin (x - y)$; $\sin^2 x - \sin^2 y$

11. Simplify each of the following.

a) $\cos\left(\frac{3\pi}{4} + x\right) + \sin\left(\frac{3\pi}{4} + x\right)$

b) $\cos\left(\frac{\pi}{4} - x\right) \sec \frac{\pi}{4} - \sin\left(\frac{\pi}{4} - x\right) \csc \frac{\pi}{4}$

Check Your Understanding

Practise

1. Factor and simplify each rational trigonometric expression.

a) $\frac{\cos x - \sin^2 x \cos x}{\cos^2 x}$

b) $\frac{\cos^2 x - 3 \cos x - 10}{8 + 4 \cos x}$

c) $\frac{3 \sec x + 6 \sec x \sin x}{4 \sin^2 x - 1}$

2. Use factoring to help to prove each identity for all permissible values of x.

a) $\dfrac{\sin x + \sin^2 x}{\cos x + \sin x \cos x} = \tan x$

b) $1 - \tan x = \dfrac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x}$

c) $\dfrac{3\cos^2 x + 5\cos x - 2}{9\cos^2 x - 1} = \dfrac{\cos x + 2}{3\cos x + 1}$

3. Use a common denominator to express the rational expressions as a single term.

a) $\dfrac{\cos x}{\sin x} + \sec x$

b) $\dfrac{1}{1 - \cos x} - \dfrac{1}{1 + \cos x}$

c) $\dfrac{\cos x}{1 + \sin x} + \dfrac{\sin x}{\cos x}$

4. Prove each identity.

a) $\frac{1 - \sin^2 x}{\cos x} = \frac{\sin 2x}{2 \sin x}$

b) $\frac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$

Left Side $= \frac{\csc^2 x - 1}{\csc^2 x}$

$= \frac{\boxed{}}{\boxed{}} = \frac{1}{\boxed{}}$

$= 1 -$ ____________

$=$ ____________

Right Side $=$ ____________

c) $(\cos x - \sin x)^2 = 1 - \sin 2x$

5. Match each expression on the left with an equivalent expression on the right. Justify your answer.

a) $\sin x \cot x$	**A** $\sin^2 x + \cos^2 x + \tan^2 x$
b) $1 - 2 \sin^2 x$	**B** $1 + 2 \sin x \cos x$
c) $(\sin x + \cos x)^2$	**C** $\cos x$
d) $\sec^2 x$	**D** $2 \cos^2 x - 1$

7. Prove each identity.

a) $\sec x = \dfrac{2(\cos x \sin 2x - \sin x \cos 2x)}{\sin 2x}$

b) $\sec x = \dfrac{2 \csc 2x \tan x}{\sec x}$

c) $\tan 2x - \sin 2x = 2 \tan 2x \sin^2 x$

d) $\dfrac{1 + \tan x}{1 + \cot x} = \dfrac{1 - \tan x}{\cot x - 1}$

Check Your Understanding Section 6.4

Practise

1. Solve each equation algebraically over the domain $0° \leq x < 360°$.

   a) $2 \sin x = \sqrt{3}$

   b) $2 \cos x - 1 = 0$

   c) $\tan x - 1 = 0$

   d) $\cot x + 1 = 0$

2. Solve each equation algebraically over the domain $0 \leq x < 2\pi$.

   a) $4 \sin^2 x - 1 = 0$

   b) $4 \cos^2 x = 3$

   c) $\tan^2 x - 3 = 0$

   d) $3 \csc^2 x - 4 = 0$

3. Solve each equation algebraically over the domain $0 \leq x < 2\pi$.

   a) $\sin^2 x - \sin x = 0$

   b) $\cos^2 x + \cos x = 0$

   c) $\tan x + \tan^2 x = 0$

   d) $\cos^2 x + 2 \cos x = 0$

4. Solve each equation algebraically over the domain $0 \le x < 2\pi$.

a) $\sin 2x - 1 = \cos 2x$

b) $\sqrt{2}\sin^2 x = \tan x \cos x$

c) $\cos 2x = \cos^2 x$

d) $\cos 2x = 2\sin^2 x$

e) $\sin 2x \tan x = 1$

5. Rewrite each equation in terms of sine or cosine only. Then, solve algebraically for $0 \le x < 2\pi$.

a) $\sin^2 x - \cos^2 x = \frac{1}{2}$

b) $2\sin^2 x - 3\cos 2x = 3$

c) $3\cos 2x + \cos x + 1 = 0$

d) $3 + \sin x = 5\cos 2x$

Apply

6. Solve each equation algebraically over the domain $0 \leq x < 2\pi$.

a) $8 \sin^2 x - 6 \sin x + 1 = 0$

b) $\cos x + 1 = 2 \sin^2 x$

c) $2 \cos^2 x - 3 \cos x + 1 = 0$

d) $\sin^2 x + 2 \sin x - 3 = 0$

e) $2 \tan^2 x = 3 \tan x - 1$

f) $\sin x = -\cos 2x$

8. Solve each equation algebraically over the domain $0 \le x < 2\pi$.

a) $\csc^2 x - \csc x - 2 = 0$

b) $2\sec^2 x + \sec x - 1 = 0$

c) $3\csc^2 x - 5\csc x - 2 = 0$

d) $\sec^2 x + 5\sec x + 6 = 0$

10. Solve $\sin x \cos 2x + \sin x = 0$ algebraically over the domain of real numbers. Give your answer(s) in radians.

11. Solve the equation $\sin 2x = -\sqrt{2}\cos x$ algebraically. Give the general solution expressed in radians.

Chapter 6 Review

6.1 Reciprocal, Quotient, and Pythagorean Identities, pages 188–196

1. Determine the non-permissible values of x, in radians, for each expression.

   a) $\frac{\sec x}{\sin x}$

   b) $\frac{\cos x}{\csc x}$

   c) $\frac{\sec x}{1 + \cos^2 x}$

2. Simplify each expression to one of the three primary trigonometric functions: $\sin x$, $\cos x$, or $\tan x$.

   a) $\frac{\cos x \csc x}{\sec x \cot x}$

   b) $\frac{\cot x \tan x}{\csc x}$

3. Simplify. Then, rewrite each expression as one of the three reciprocal trigonometric functions: $\csc x$, $\sec x$, or $\cot x$.

   a) $\cot x \sec x$

   b) $\frac{\cos x}{(1 - \sin x)(1 + \sin x)}$

4. **a)** Verify that the equation $(\sec x + \tan x) \cos x - 1 = \sin x$ is true for $x = 30°$ and for $x = \frac{\pi}{3}$.

   b) What are the non-permissible values of the equation in part a) in the domain $0° \leq x < 360°$?

6.2 Sum, Difference, and Double-Angle Identities, pages 197–204

5. Write each expression as a single trigonometric ratio. Then, give an exact value for the expression.

a) $\cos^2 15° - \sin^2 15°$

b) $\sin 35° \cos 100° + \cos 35° \sin 100°$

c) $1 - 2\sin^2 75°$

6. Determine the exact value of each trigonometric expression.

a) $\sin\left(-\frac{\pi}{12}\right)$

b) $\cos \frac{\pi}{12}$

c) $\cos 105°$

d) $\sin \frac{23\pi}{12}$

7. Angle θ is in quadrant II and $\sin \theta = \frac{7}{25}$. Determine an exact value for each of the following.

a) $\sin 2\theta$

b) $\cos 2\theta$

c) $\tan 2\theta$

6.3 Proving Identities, pages 205–214

8. Prove $\sin(\pi - x) - \tan(\pi + x) = \frac{\sin x(\cos x - 1)}{\cos x}$.

10. Prove each identity.

a) $\cos x \tan^2 x = \sin x \tan x$

b) $\sin 2x = \tan x + \tan x \cos 2x$

6.4 Solving Trigonometric Equations Using Identities, pages 215–223

11. Rewrite each equation in terms of sine or cosine. Then, solve algebraically for $0 \leq x < 2\pi$.

a) $2\cos^2 x - \sin x = -1$

b) $\sin^2 x = 2\cos x - 2$

c) $\cos x + \cos 2x = 0$

12. Solve each equation algebraically over the domain $0 \leq x < 2\pi$.

a) $2\cos^2 x + 3\cos x + 1 = 0$

b) $\sin^2 x + 3\sin x + 2 = 0$

c) $\sin^2 x + 5\sin x + 6 = 0$

d) $\cos^2 x + 3\cos x + 2 = 0$

13. Solve the equation $2\cos^2 x = 1 - \sin x$ algebraically. Give the general solution expressed in radians.

Chapter 6

6.1 Reciprocal, Quotient, and Pythagorean Identities, pages 188–196

1. a) $x \neq \frac{\pi}{2} + \pi n$, where $n \in I$
   b) $x \neq \frac{\pi}{2}n$, where $n \in I$
   c) $x \neq \pi n$ and $x \neq \frac{3\pi}{2} + 2\pi n$, where $n \in I$
   d) $x \neq \frac{\pi}{2} + \pi n$ and $x \neq 2\pi n$, where $n \in I$

2. a) $\cos x$ b) $\tan x$ c) $\sin x$
3. a) $\cot x$ b) $\sec x$ c) $\csc x$
4. a) When substituted, both values satisfy the equation.
   b) $x \neq 0°, 90°, 180°$, and $270°$
5. a) $x \neq \frac{\pi}{2}n$, where $n \in I$
   b) The graph of both functions, $f(x) = \tan x + \frac{1}{\tan x}$ and $g(x) = \frac{1}{\cos x \sin x}$, look the same, so this may be an identity.
   c) The equation is verified for $x = \frac{\pi}{4}$.

8. $\csc x, x \neq 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

10. 0

6.2 Sum, Difference, and Double-Angle Identities, pages 197–204

1. a) $\cos 65°$ b) $\sin 26°$ c) $\tan 61°$
   d) $3 \sin \frac{\pi}{5}$ e) $\cos \frac{\pi}{4}$ f) $\tan \frac{2\pi}{3}$
2. a) $\sin \frac{\pi}{2} = 1$ b) $\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$
   c) $\tan 120° = -\sqrt{3}$ d) $\cos \pi = -1$
3. a) $\sin 40°$ b) $\cos \frac{3\pi}{4}$ c) $\tan \frac{7\pi}{12}$
4. a) $-\tan x$ b) $\cos \frac{\theta}{2}$ c) $\frac{1}{4} \sin \theta$ d) $-4 \cos 4\theta$
5. a) $x \neq \pi n, n \in I$
   b) $\frac{1 - \cos 2x}{\sin x}$
   $= \frac{1 - (1 - 2\sin^2 x)}{\sin x}$
   $= \frac{2\sin^2 x}{\sin x}$
   $= 2 \sin x$
6. a) $\frac{\sqrt{6} + \sqrt{2}}{4}$ b) $\frac{\sqrt{6} - \sqrt{2}}{4}$ c) $\frac{1 - \sqrt{3}}{1 + \sqrt{3}}$ d) $\frac{4}{\sqrt{6} + \sqrt{2}}$
7. $2 \sin x \cos y$
8. a) $\frac{336}{625}$ b) $\frac{527}{625}$ c) $\frac{336}{527}$
9. a) $-\frac{16}{25}$ b) $-\frac{33}{65}$ c) $\frac{56}{33}$
10. a) $\frac{\sin 2x}{1 - \cos 2x} = \frac{2 \sin x \cos x}{1 - (1 - 2\sin^2 x)}$
    $= \frac{2 \sin x \cos x}{2 \sin^2 x}$
    $= \frac{\cos x}{\sin x}$
    $= \cot x$
    b) $\sin (x + y) \sin (x - y)$
    $= (\sin x \cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y)$
    $= \sin^2 x \cos^2 y - \sin x \cos y \cos x \sin y + \cos x \sin y \sin x \cos y - \cos^2 x \sin^2 y$
    $= \sin^2 x \cos^2 y - \cos^2 x \sin^2 y$
    $= \sin^2 x(1 - \sin^2 y) - (1 - \sin^2 x) \sin^2 y$
    $= \sin^2 x - \sin^2 x \sin^2 y - \sin^2 y + \sin^2 x \sin^2 y$
    $= \sin^2 x - \sin^2 y$
11. a) $-\sqrt{2} \sin x$ b) $2 \sin x$

6.3 Proving Identities, pages 205–214

1. a) $\cos x$ b) $\frac{\cos x - 5}{4}$ c) $\frac{3 \sec x}{2 \sin x - 1}$
2. a) $\frac{\sin x + \sin^2 x}{\cos x + \sin x \cos x} = \frac{\sin x(1 + \sin x)}{(1 + \sin x)\cos x}$
   $= \frac{\sin x}{\cos x}$
   $= \tan x$
   b) $\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x} = \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x(\cos x + \sin x)}$
   $= \frac{\cos x - \sin x}{\cos x}$
   $= 1 - \frac{\sin x}{\cos x}$
   $= 1 - \tan x$
   c) $\frac{3\cos^2 x + 5 \cos x - 2}{9\cos^2 x - 1} = \frac{(3 \cos x - 1)(\cos x + 2)}{(3 \cos x - 1)(3 \cos x + 1)}$
   $= \frac{\cos x + 2}{3 \cos x + 1}$
3. a) $\frac{\cos^2 x + \sin x}{\sin x \cos x}$ b) $2 \cot x \csc x$ c) $\sec x$
4. a) Left Side $= \frac{1 - \sin^2 x}{\cos x}$
   $= \frac{\cos^2 x}{\cos x}$
   $= \cos x$
   Right Side $= \frac{\sin 2x}{2 \sin x}$
   $= \frac{2 \sin x \cos x}{2 \sin x}$
   $= \cos x$
   Left Side = Right Side
   b) Left Side $= \frac{\csc^2 x - 1}{\csc^2 x}$
   $= \frac{\csc^2 x}{\csc^2 x} - \frac{1}{\csc^2 x}$
   $= 1 - \sin^2 x$
   $= \cos^2 x$
   = Right Side
   c) Left Side $= (\cos x - \sin x)^2$
   $= \cos^2 x - 2 \cos x \sin x + \sin^2 x$
   $= \cos^2 x + \sin^2 x - 2 \cos x \sin x$
   $= 1 - 2 \cos x \sin x$
   $= 1 - \sin 2x$
   = Right Side

5. a) C; $\cot x = \frac{\cos x}{\sin x}$, so $\sin x\left(\frac{\cos x}{\sin x}\right) = \cos x$

b) D; Both are forms of the double-angle identity for cos.

c) B; The quadratic expands to $\sin^2 x + \cos^2 x + 2\sin x\cos x$. Applying the Pythagorean identity, $1 + 2\sin x\cos x$.

d) A;

$$\begin{aligned}\sin^2 x + \cos^2 x + \tan^2 x = 1 + \tan^2 x\\ &= \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}\\ &= \frac{1}{\cos^2 x}\\ &= \sec^2 x\end{aligned}$$

7. a)

$$\begin{aligned}\text{Right Side} &= \frac{2(\cos x\sin 2x - \sin x\cos 2x)}{\sin 2x}\\ &= \frac{2(\cos x(2\sin x\cos x) - \sin x(2\cos^2 x - 1))}{2\sin x\cos x}\\ &= \frac{2\cos^2 x\sin x - 2\sin x\cos^2 x + \sin x}{\sin x\cos x}\\ &= \frac{\sin x}{\sin x\cos x}\\ &= \frac{1}{\cos x}\\ &= \sec x\\ &= \text{Left Side}\end{aligned}$$

b)

$$\begin{aligned}\text{Right Side} &= \frac{2\csc 2x\tan x}{\sec x}\\ &= 2\csc 2x\tan x\left(\frac{1}{\sec x}\right)\\ &= 2\left(\frac{1}{\sin 2x}\right)\left(\frac{\sin x}{\cos x}\right)\cos x\\ &= 2\left(\frac{1}{2\sin x\cos x}\right)\sin x\\ &= \frac{1}{\cos x}\\ &= \sec x\\ &= \text{Left Side}\end{aligned}$$

c)

$$\begin{aligned}\text{Left Side} &= \tan 2x - \sin 2x\\ &= \frac{\sin 2x}{\cos 2x} - \sin 2x\\ &= \frac{\sin 2x - \cos 2x\sin 2x}{\cos 2x}\\ &= \frac{\sin 2x(1 - \cos 2x)}{\cos 2x}\\ &= \frac{\sin 2x}{\cos 2x}(1 - \cos 2x)\\ &= \tan 2x(2\sin^2 x)\\ &= 2\tan 2x\sin^2 x\\ &= \text{Right Side}\end{aligned}$$

d)

$$\begin{aligned}\text{Left Side} &= \frac{1 + \tan x}{1 + \cot x}\\ &= \frac{1 + \frac{\sin x}{\cos x}}{1 + \frac{\cos x}{\sin x}}\\ &= \frac{\frac{\cos x + \sin x}{\cos x}}{\frac{\sin x + \cos x}{\sin x}}\\ &= \frac{\cos x + \sin x}{\cos x}\left(\frac{\sin x}{\cos x + \sin x}\right)\\ &= \frac{\sin x}{\cos x}\\ &= \tan x\end{aligned}$$

$$\begin{aligned}\text{Right Side} &= \frac{1 - \tan x}{\cot x - 1}\\ &= \frac{1 - \frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x} - 1}\\ &= \frac{\frac{\cos x - \sin x}{\cos x}}{\frac{\cos x - \sin x}{\sin x}}\\ &= \frac{\cos x - \sin x}{\cos x}\left(\frac{\sin x}{\sin x - \cos x}\right)\\ &= \frac{\sin x}{\cos x}\\ &= \tan x\end{aligned}$$

Left Side = Right Side

6.4 Solving Trigonometric Equations Using Identities, pages 215–223

1. a) 60°, 120° b) 60°, 300°
   c) 45°, 225° d) 135°, 315°

2. a) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ b) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
   c) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ d) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

3. a) $0, \frac{\pi}{2}, \pi$ b) $\frac{\pi}{2}, \pi, \frac{3\pi}{2}$
   c) $0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}$ d) $\frac{\pi}{2}, \frac{3\pi}{2}$

4. a) $\frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2}$ b) $0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi$
   c) $0, \pi$ d) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
   e) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

5. a) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ b) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$
   c) $2.30, 3.98, \frac{\pi}{3}, \frac{5\pi}{3}$ d) $0.41, 2.73, \frac{7\pi}{6}, \frac{11\pi}{6}$

6. a) $0.25, 2.89, \frac{\pi}{6}, \frac{5\pi}{6}$ b) $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$
   c) $0, \frac{\pi}{3}, \frac{5\pi}{3}$ d) $\frac{\pi}{2}$
   e) $0.46, 3.61, \frac{\pi}{4}, \frac{5\pi}{4}$ f) $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

8. a) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ b) π

c) $\frac{\pi}{6}, \frac{5\pi}{6}$ d) $1.91, 4.37, \frac{2\pi}{3}, \frac{4\pi}{3}$

10. $x = \frac{\pi}{2}n$, where $n \in I$

11. The general solution is $x = \frac{\pi}{2} + \pi n$, $x = \frac{5\pi}{4} + 2\pi n$, and $x = \frac{7\pi}{4} + 2\pi n$, where $n \in I$.

Chapter 6 Review, pages 224–227

1. a) $x \neq \frac{\pi}{2}n$, where $n \in I$
   b) $x \neq \pi n$, where $n \in I$
   c) $x \neq \frac{\pi}{2} + \pi n$, where $n \in I$
2. a) $\cos x$ b) $\sin x$
3. a) $\csc x$ b) $\sec x$
4. a) When substituted, both values satisfy the equation.
   b) $x \neq 90°, 270°$
5. a) $\cos 30° = \frac{\sqrt{3}}{2}$
   b) $\sin 135° = \frac{\sqrt{2}}{2}$
   c) $\cos 150° = -\frac{\sqrt{3}}{2}$
6. a) $\frac{\sqrt{2} + \sqrt{6}}{4}$ b) $\frac{\sqrt{6} + \sqrt{2}}{4}$
   c) $\frac{\sqrt{2} - \sqrt{6}}{4}$ d) $\frac{-\sqrt{6} + \sqrt{2}}{4}$
7. a) $-\frac{336}{625}$ b) $\frac{527}{625}$
   c) $-\frac{336}{527}$
8. Left Side $= \sin(\pi - x) - \tan(\pi + x)$

$$= \sin \pi \cos x - \sin x \cos \pi - \frac{\tan \pi + \tan x}{1 - \tan \pi \tan x}$$
$$= \sin x - \tan x$$

Right Side $= \frac{\sin x(\cos x - 1)}{\cos x}$

$$= \frac{\sin x \cos x - \sin x}{\cos x}$$
$$= \frac{\sin x \cos x}{\cos x} - \frac{\sin x}{\cos x}$$
$$= \sin x - \tan x$$

Left Side = Right Side

10. a) Left Side $= \cos x \tan^2 x$

$$= \cos x \frac{\sin^2 x}{\cos^2 x}$$
$$= \frac{\sin^2 x}{\cos x}$$
$$= \sin x \frac{\sin x}{\cos x}$$
$$= \sin x \tan x$$

= Right Side

b) Right Side $= \tan x + \tan x \cos 2x$

$$= \tan x(1 + \cos 2x)$$
$$= \tan x(1 + (2\cos^2 x - 1))$$
$$= \tan x(2\cos^2 x)$$
$$= 2\frac{\sin x}{\cos x}\cos^2 x$$
$$= 2 \sin x \cos x$$
$$= \sin 2x$$

= Left Side

11. a) $\frac{\pi}{2}$ b) 0
    c) $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$
12. a) $\frac{2\pi}{3}, \pi, \frac{4\pi}{3}$ b) $\frac{3\pi}{2}$
    c) no solution d) π
13. The general solution is $x = \frac{7\pi}{6} + 2\pi n$, $x = \frac{11\pi}{6} + 2\pi n$, $x = \frac{\pi}{2} + 2\pi n$, where $n \in I$. These can be combined as $x = \frac{\pi}{2} + \frac{2\pi}{3}n$, where $n \in I$.