Chapter 7

Check Your Understanding Section 7.1

Practise

1. State whether each of the following is an exponential function. Justify your answers.

   a) $y = x^5$

   b) $y = 0.1^x$

   c) $y = 12^x$

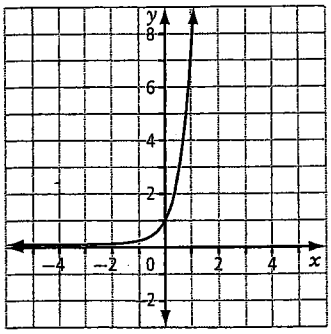
   d) $y = \sqrt[3]{x}$

   e) $y = x^{0.5}$

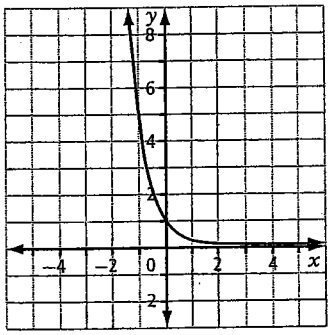
2. Match each exponential function to its graph.

   a) $y = 5^x$ b) $y = 7^x$ c) $y = \left(\frac{3}{4}\right)^x$ d) $y = 0.2^x$

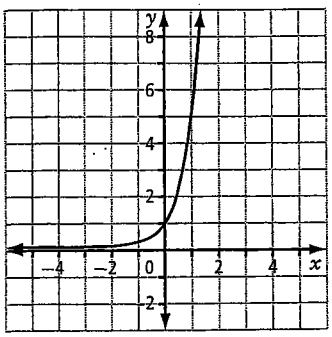
A



B



C



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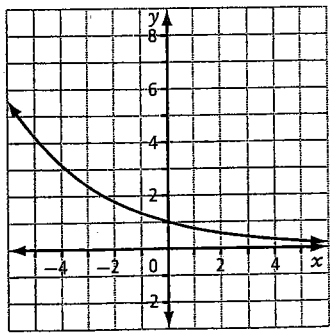
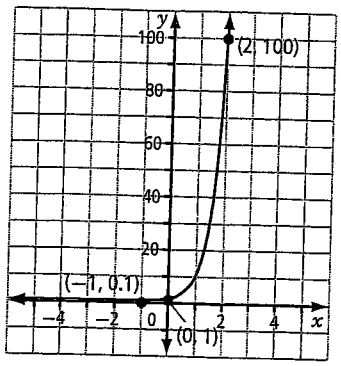


3. Write the equation of each exponential function graphed below.

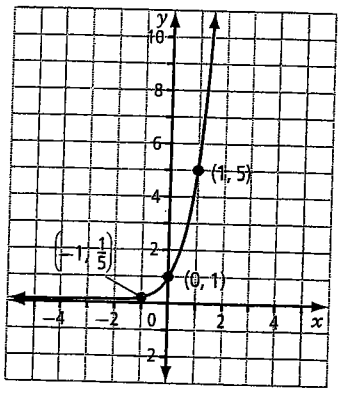
a)



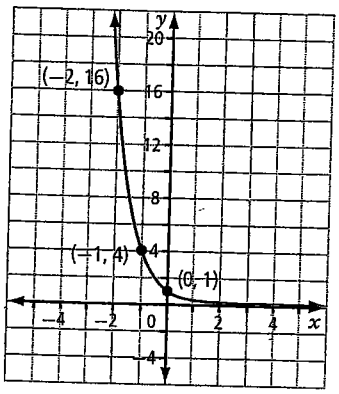
The point (0, 1) does not help determine the equation because _____

However, since you know that _____$^2 = 100$, you can conclude that the base of the exponential function is _____. Thus, the equation is _____.

b)

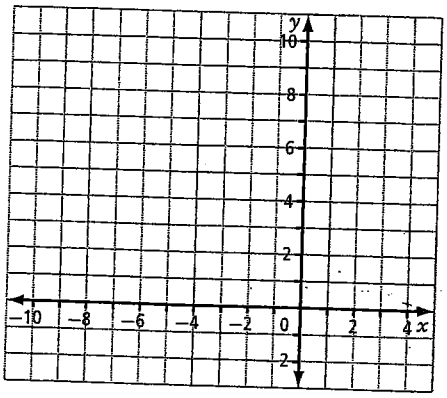


c)

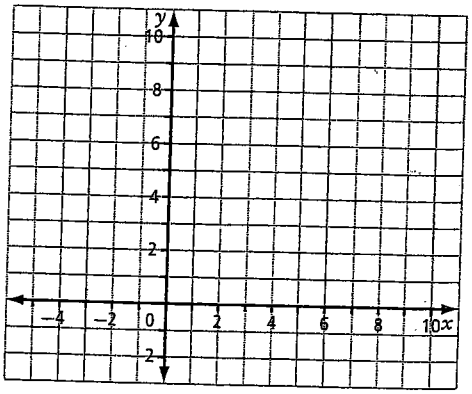


4. Sketch the graph of each exponential function. Identify the domain and range, the y-intercept, whether the graph is increasing or decreasing, and the equation of the horizontal asymptote.

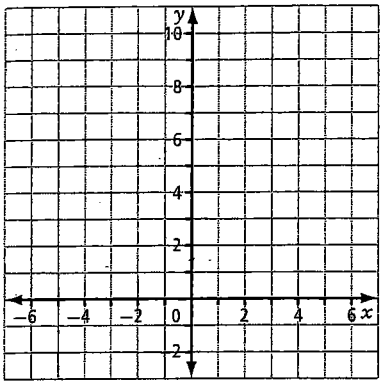
a) $f(x) = 8^x$



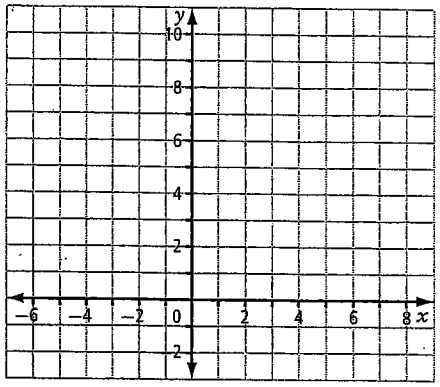
b) $f(x) = 0.5^x$



c) $g(x) = \left(\frac{2}{3}\right)^x$



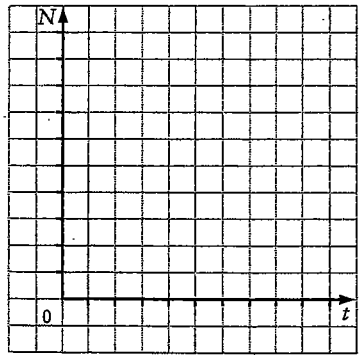
d) $g(x) = \left(\frac{3}{2}\right)^x$



Apply

5. The number of transistors on a computer chip, N, doubles approximately every two years. If originally there is one transistor on a chip, then this can be modelled by the function $N = 2^t$, where t is the number of two-year periods that have passed.

   a) Graph the function. Is the function increasing or decreasing?



   b) What are the domain and range of the function?

   c) How many transistors are on a chip after 2 years? 10 years? 20 years?

How many 2-year periods are there in 10 years? 20 years? How might you use a table of values to solve this question?

Check Your Understanding Section 7.2

Practise

1. State whether each function shows a vertical translation of $y = 5^x$.
   a) $y = 5^{x-2}$ **b)** $y = 5^x - 2$
   c) $y = 2(5)^x$ **d)** $y = 5^{3x}$

2. State whether each function shows a horizontal stretch of $y = 5^x$.
   a) $y = 5^{x-2}$ **b)** $y = 5^x - 2$
   c) $y = 2(5)^x$ **d)** $y = 5^{3x}$

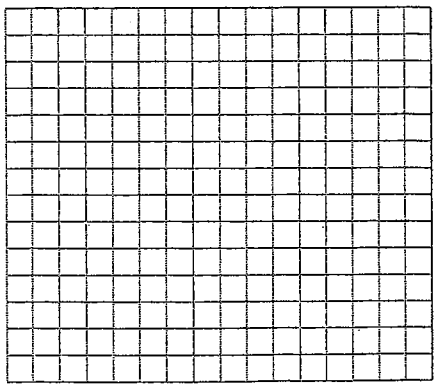
3. State whether each function shows a reflection in the y-axis of $y = 5^x$.
   a) $y = 5^{x-2}$ **b)** $y = -5^x - 2$
   c) $y = 2(5)^{-x}$ **d)** $y = 5^{\frac{x}{3}}$

4. Identify all transformations for each function.
   a) $y = 4^{2(x-5)} - 6$ **b)** $y = \frac{2}{3}\left(\frac{1}{2}\right)^{-x} + 9$

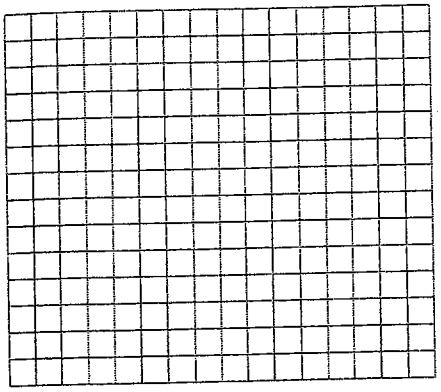
   c) $y = -2(1.06)^{\frac{1}{4}x}$ **d)** $y = 500\left(\frac{5}{2}\right)^{2x+6} - 8$

5. Sketch the graph of each exponential function without using technology. For each function,
   - state the domain and range
   - identify the y-intercept
   - indicate whether the graph is increasing or decreasing
   - write the equation of the horizontal asymptote

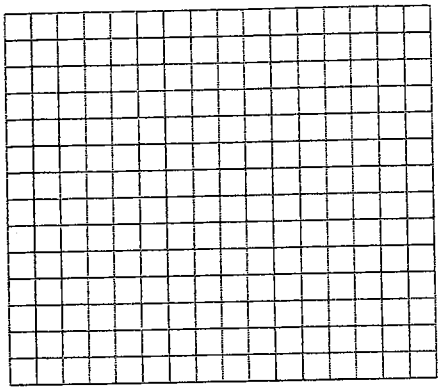
   a) $f(x) = 8^{x-2} + 4$



b) $g(x) = -2^x + 3$



c) $f(x) = 0.5(3)^{x+2} - 5$



Apply

6. Iodine-131 has a half-life of 8 days. This means that after 8 days, half of the original mass of the isotope will have decayed. Suppose a sample of iodine-131 has a mass of 250 grams.

   a) Write an exponential equation that models the amount, M, of iodine-131 remaining after d days. State the transformations that are represented by your function.

Check Your Understanding Section 7.3

Practise

1. Express each of the following with base 3.

a) 81

b) 27^5

c) $3\sqrt{3}$

d) $\sqrt[3]{243}$

e) $9\sqrt[3]{81^2}$

Work with 9 and $\sqrt[3]{81^2}$ separately.

$9 = 3^{\square}$

$\sqrt[3]{81^2} = \left((3^{\square})^{\square}\right)^{\square}$

$= \_\_\_\_\_$

Thus, $9\sqrt[3]{81^2} = 3^{\square}3^{\square}$.

$= \_\_\_\_\_$

f) $\left(\frac{1}{27}\right)^2$

g) $\left(\frac{\sqrt{3}}{81}\right)^{-3}$

2. Rewrite each pair of expressions to have the same base.

a) 8 and 64

b) 3^2 and 9^3

c) 5^{x+6} and 125

d) 2^{3x} and 8^{2x+4}

e) 27^{5x+4} and $\left(\frac{1}{9}\right)^{x+3}$

f) $\left(\frac{1}{4}\right)^{x+7}$ and 8^{-3x}

4. Solve the following.

a) $4^{2x} = 4096$

b) $2^{3x-5} = 128$

c) $6^{x+3} = \frac{1}{216}$

d) $10^{5x+6} = 0.0001$

5. Solve the following.

a) $64^{4x} = 16^{x+5}$

b) $9^{x-7} = 27^{2x-9}$

c) $125^{6x+2} = 25^{8x+1}$

d) $8^{x+2} = \left(\frac{1}{4}\right)^{x+3}$

e) $5(3)^x = 135$

_____ = _____ Divide each side by 5.

3^x = _____ Express as base 3.

_____ = _____ Equate powers and solve.

Apply

7. A type of bacterium doubles each hour.
   a) If there are 4 bacteria in a sample, write a exponential function that models the sample's growth over time.

   b) Use your equation to determine the time it takes for the sample to become 4096 bacteria.

8. A painting doubles in value every 8 years. It is currently worth $1000.
   a) Write an exponential function that models the value of the painting.

   b) Use your equation to determine the time needed for the painting to be worth $3200.

9. The student council of a school notices that their membership is growing by 3% per year.
   a) The membership is currently 350 students. Write an exponential function to model the size of the student council.

Connect

10. Keegan invests $1000 at 3.75% compounded annually.
    a) Write an exponential function to model the growth of Keegan's investment.

Chapter 7 Review

7.1 Characteristics of Exponential Functions, pages 229–237

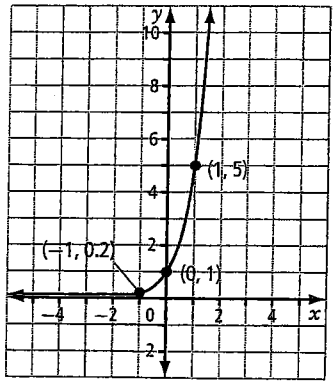
1. For each exponential function, state the domain, range, y-intercept, horizontal asymptote, and whether a graph of the function would be increasing or decreasing. Verify your answers by using technology to graph the functions.

   a) $y = 4.5^x$

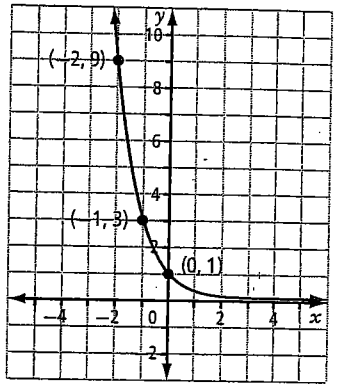
   b) $y = \left(\frac{2}{3}\right)^x$

2. State the exponential function represented by each graph.

   a)



   b)



7.2 Transformations of Exponential Functions, pages 238–248

4. Identify all the transformations in each exponential function below.

   a) $y = 2(3)^x - 3$

   b) $y = 5^{x+3}$

   c) $y = 10^{2x-8} + 1$

   d) $y = 5(8)^{6x+12}$

5. Write the equation for each of the following transformations to the function $y = 4^x$. Then, state the domain and range of the transformed function.

   a) vertically stretched by a factor of $\frac{1}{2}$, translated 2 units left and 6 units down

   b) horizontally stretched by a factor of $\frac{1}{3}$, vertically stretched by a factor of 5

   c) horizontally stretched by a factor of 2, translated 3 units right and 1 unit down

7.3 Solving Exponential Equations, pages 249–255

6. Solve each of the following equations algebraically. ~~Use graphing technology to check your answer~~.

   a) $5^{x+2} = 3125$

   b) $2^{3x-2} = 16^x$

   c) $\left(\frac{1}{9}\right)^{x-6} = 27^{2x-1}$

   d) $(\sqrt{3})^x = 9^{2x+5}$

7. The half-life of a radioactive substance is 4 days.

   a) Write an exponential function that models the proportion, P, of the substance remaining after t days.

   b) Use your function to determine the time that must pass until there is 25% of the substance remaining.

8. The number of bacteria in a sample doubles every 10 h. Initially, there are 64 colonies present.

a) Write an exponential function that models the number of bacteria colonies, N, present after t hours.

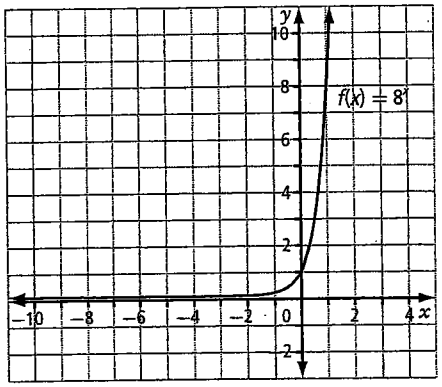
b) Use your function to determine the number of colonies present after 24 h.

c) Determine the time that must pass until there are 1024 colonies present.

Chapter 7

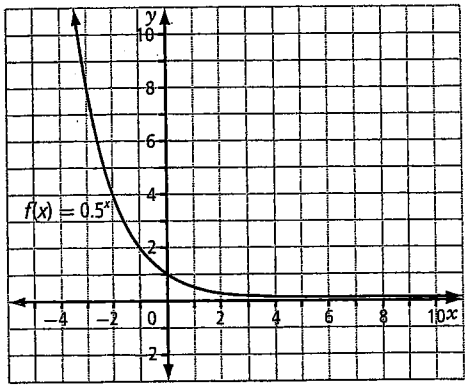
7.1 Characteristics of Exponential Functions, pages 229–237

1. a) No, the variable is not the exponent.
   b) Yes, the base is greater than 0 and the variable is the exponent.
   c) Yes, the base is greater than 0 and the variable is the exponent.
   d) No, the variable is not the exponent.
   e) No, the variable is not the exponent.
2. a) C b) A c) D d) B
3. a) $y = 10^x$ b) $y = 5^x$ c) $y = \left(\frac{1}{4}\right)^x$
4. a)



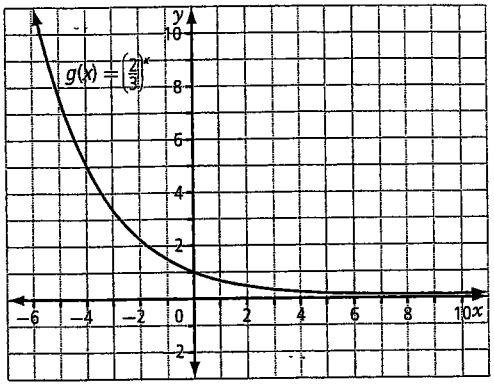
domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$; y-intercept 1; function increasing; horizontal asymptote $y = 0$

b)



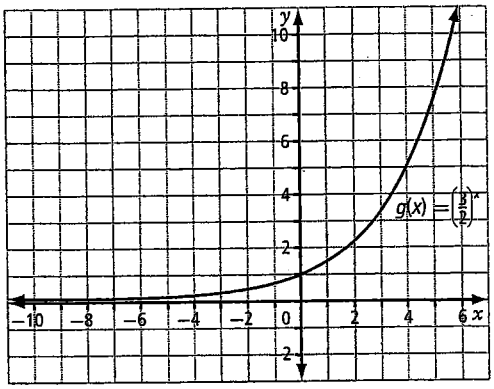
domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$; y-intercept 1; function decreasing; horizontal asymptote $y = 0$

c)



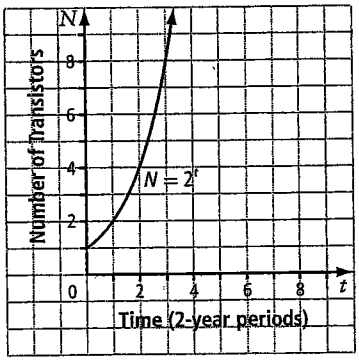
domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$; y-intercept 1; function decreasing; horizontal asymptote $y = 0$

d)



domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$; y-intercept 1; function increasing; horizontal asymptote $y = 0$

5. a)



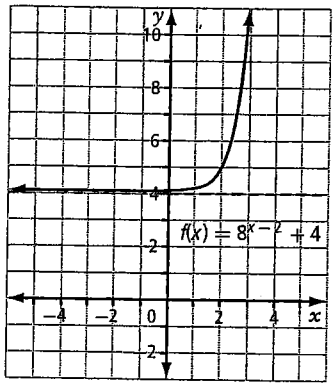
function increasing

b) domain: $\{t \mid t \geq 0, t \in \mathrm{R}\}$; range: $\{N \mid N \geq 1, N \in \mathrm{N}\}$

c) 2 transistors; 32 transistors; 1024 transistors

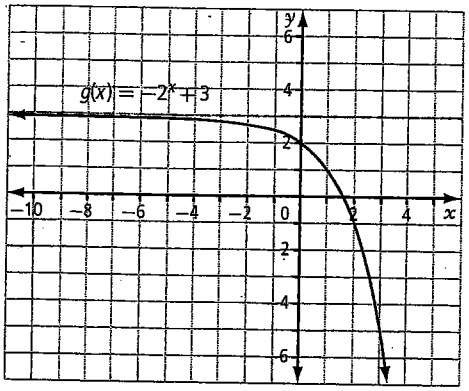
7.2 Transformations of Exponential Functions, pages 238–248

1. a) No b) Yes
   c) No d) No
2. a) No b) No
   c) No d) Yes
3. a) No b) No
   c) Yes d) No
4. a) horizontally stretched by a factor of $\frac{1}{2}$, translated 5 units right and 6 units down
   b) vertically stretched by a factor of $\frac{2}{3}$, reflected in the y-axis, translated 9 units up
   c) vertically stretched by a factor of 2, reflected in the x-axis, horizontally stretched by a factor of 4
   d) vertically stretched by a factor of 500, horizontally stretched by a factor of $\frac{1}{2}$, translated 3 units left and 8 units down
5. a)



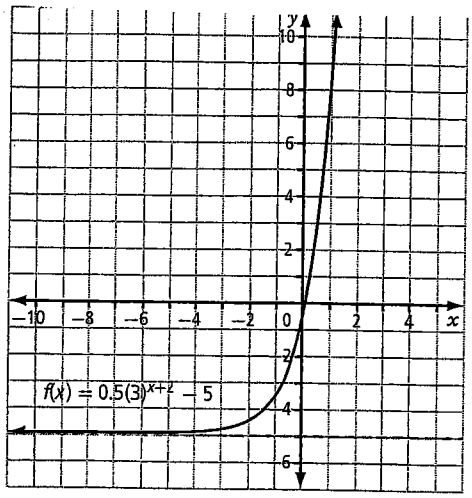
domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 4, y \in \mathrm{R}\}$; y-intercept $\frac{257}{64}$ or ≈ 4.02; function increasing; horizontal asymptote $y = 4$

b)



domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y < 3, y \in \mathrm{R}\}$; y-intercept 2; function decreasing; horizontal asymptote $y = 3$

c)



domain: $\{x \mid x \in \mathrm{R}\}$; range $\{y \mid y > -5, y \in \mathrm{R}\}$; y-intercept $-\frac{1}{2}$; function increasing; horizontal asymptote $y = -5$

6. a) $M(d) = 250\left(\frac{1}{2}\right)^{\frac{d}{8}}$; vertical stretch by a factor of 250; horizontal stretch by a factor of 8

7.3 Solving Exponential Equations, pages 249–255

1. a) 3^4 b) 3^{15}
   c) $3^{\frac{3}{2}}$ d) $3^{\frac{5}{3}}$
   e) $3^{\frac{14}{3}}$ f) 3^{-6}
   g) $3^{\frac{21}{2}}$
2. a) $2^3, 2^6$ b) $3^2, 3^6$
   c) $5^{x+6}, 5^3$ d) $2^{3x}, 2^{6x+12}$
   e) $3^{15x+12}, 3^{-2x-6}$ f) $2^{-2x-14}, 2^{-9x}$
4. a) 3 b) 4
   c) −6 d) −2
5. a) 1 b) $\frac{13}{4}$
   c) −2 d) $-\frac{12}{5}$
   e) 3
7. a) $N = 4(2)^t$, where N is the number of bacteria, and t is the time, in hours.
   b) 10 h
8. a) $V = 1000(2)^{\frac{t}{8}}$, where V is the value of the painting and t is the time, in years.
   b) approximately 13.5 years
9. a) $M = 350(1.03)^t$, where M is the number of members and t is the time, in years.
10. a) $V = 1000(1.0375)^t$, where V is the value of the investment and t is the time, in years.

Chapter 7 Review, pages 256–258

1. **a)** domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$; y-intercept 1; horizontal asymptote $y = 0$; function increasing

   b) domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$; y-intercept 1; horizontal asymptote $y = 0$; function decreasing

2. **a)** $y = 5^x$ **b)** $y = \left(\frac{1}{3}\right)^x$

4. **a)** vertically stretched by a factor of 2, translated 3 units down

   b) translated 3 units left

   c) horizontally stretched by a factor of $\frac{1}{2}$, translated 4 units right and 1 unit up

   d) vertically stretched by a factor of 5, horizontally stretched by a factor of $\frac{1}{6}$, translated 2 units left

5. **a)** $y = \frac{1}{2}(4)^{x+2} - 6$; domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > -6, y \in \mathrm{R}\}$

   b) $y = 5(4)^{3x}$; domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$

   c) $y = (4)^{\frac{1}{2}(x-3)} - 1$; domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > -1, y \in \mathrm{R}\}$

6. **a)** 3 **b)** -2

   c) $\frac{15}{8}$ **d)** $-\frac{20}{7}$

7. **a)** $P = \left(\frac{1}{2}\right)^{\frac{t}{4}}$ **b)** 8 days

8. **a)** $N = 64(2)^{\frac{t}{10}}$

   b) 337 colonies

   c) 40 h