Pre-Calculus 12 Name : _____ Block : ____

Chapter 8 Assignment

1. Graph the function using transformations. $y = -\log_2(2(x+3)) - 1$

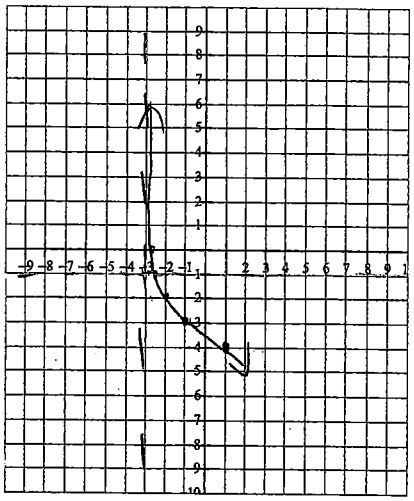
$y = \log_2 x$

$2^y = x$

y	x
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

divide x's by 2, mult y's by -1

x	y
$\frac{1}{4}$	1
$\frac{1}{2}$	0
1	-1
2	-2
4	-3



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2. The function $y = \log_2 x$ is transformed to $y = a\log_2(b(x-h)) + k$. Write the new equation if the original function was reflected over the x-axis, horizontally stretched by a factor of 5, vertically translated up 2 and horizontally translated left 3.

$a = -1$

$b = \frac{1}{5}$

$y = -\log\left(\frac{1}{5}(x+3)\right) + 2$

1

3. Evaluate without a calculator. Show all your steps

a) $\log_4 64$

$\log_4 4^3$

3

b) $\log_2 \frac{1}{32}$

$\log_2 2^{-5}$

-5

c) $\log_5 \sqrt{125}$

$\log_5 (5^3)^{\frac{1}{2}}$

$\log_5 5^{3/2}$

$3/2$

3

6

4. Find the inverse. Write your function in explicit form. Y=

a) $y = 20^x$	b) $y = log_3 x$
$x = 20^y$ swith x and y then rewrite as y= $\log_{20} x = y$	$x = \log_3 y$ $3^x = y$

5. Find the domain, range, equation of asymptote, x-intercept and y-intercept for:

$y = 2\log_2(2x + 3) - 1$

$2x+3>0$
$2x>-3$
$x>-3/2$

$y=2\log_2(2(x+3/2))-1$

Domain $\{x \mid x > -3/2, x \in \mathbb{R}\}$

Range $\{y \mid y \in \mathbb{R}\}$

Aymptote $x = -3/2$

x-intercept $y=0$

$0 = 2\log_2(2x+3) - 1$
$1 = 2\log_2(2x+3)$
$\frac{1}{2} = \log_2(2x+3)$
$2^{1/2} = 2x+3$
$2^{1/2} - 3 = 2x$
$x = \frac{2^{1/2}-3}{2}$
$x = -0.79$

y-intercept

$x=0$
$y = 2\log_2(2(0)+3) - 1$
$y = 2\log_2(3) - 1$ — change of base
$y = 2\left(\frac{\log 3}{\log 2}\right) - 1$
$y = 2.17$

6. Use the formula $pH = -\log(H^+)$ to solve the problem

a) Find the pH of beer if $[H^+] = 3.16 \times 10^{-3}$ moles per liter

$pH = -\log(3.16 \times 10^{-3})$
$pH = 2.5$

b) Find the $[H^+]$ of vinegar if the pH is 3.1

$3.1 = -\log[H^+]$
$-3.1 = \log[H^+]$
$10^{-3.1} = [H^+]$
$[H^+] = 7.9 \times 10^{-4}$

7. The green solution has a pH of 6.9 and is 15 times more acidic than the blue solution. What is the pH of the blue solution?

green
$6.9 = -\log[H^+]$
$-6.9 = \log[H^+]$
$10^{-6.9} = [H^+]$

blue
$x = -\log[H^+]$
$-x = \log[H^+]$
$10^{-x} = [H^+]$

$\frac{\text{green}}{\text{blue}} = 15$
$\frac{10^{-6.9}}{10^{-x}} = 15$
$10^{-6.9+x} = 15$
$\log 15 = -6.9 + x$
$\log 15 + 6.9 = x \quad x = 8.1$

8. Simplify

a) $log_2 12 - log_2 3$

$\log_2 \frac{12}{3}$
$= \log_2 4$
$= \log_2 2^2$
$= 2$

b) $log_5 10 + log_5 75 - (log_5 2 + log_5 3)$

$\log_5 (10 \cdot 75) - \log_5 (2 \cdot 3)$
$\log_5 750 - \log_5 6$
$\log_5 \frac{750}{6}$
$\log_5 125$
$\log_5 5^3 = 3$

c) $\frac{1}{2} log_2 16 - \frac{1}{3} log_2 8$

$\log_2 16^{1/2} - \log_2 8^{1/3}$
$\log_2 4 - \log_2 2$
$\log_2 \frac{4}{2}$
$\log_2 2$
1

d) $2log_4 2 - 2log_4 4 - log_4 \frac{1}{4}$

$\log_4 2^2 - \log_4 4^2 - \log_4 \frac{1}{4}$
$\log_4 4 - \log_4 16 - \log_4 \frac{1}{4}$
$\log_4 \frac{4}{16} - \log_4 \frac{1}{4}$
$\log_4 \frac{\frac{1}{4}}{1/4}$

$\log_4 \frac{1}{4} \cdot \frac{4}{1}$
$\log_4 1$
0

9. Write each expression as a single logarithm in simplest form. State any restrictions on the variable.

$x > 0$

a) $\log_7 x^2 + \log_7 x - \frac{5\log_7 x}{2}$ $5/2$

$\log_7 x^2 + \log_7 x - \log_7 x^{5/2}$

$\log_7 \frac{x^2 \cdot x^1}{x^{5/2}}$

$\log_7 \frac{x^3}{x^{5/2}} = \log_7 x^{1/2}$

b) $\log_5(2x-2) - \log_5(x^2+2x-3)$

$\log_5 \frac{2x-2}{x^2+2x-3}$

$\log_5 \frac{2(x-1)}{(x+3)(x-1)}$

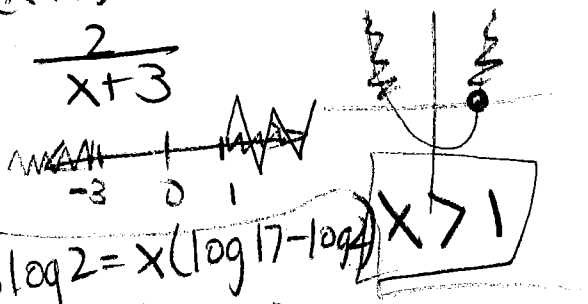
$\log_5 \frac{2}{x+3}$

$2x-2 > 0$

$x > 1$

$x^2+2x-3 > 0$

$(x+3)(x-1) > 0$



10. Solve (3 decimal places) $2^{x+3} = 17^x$

$\log 2^{x+3} = \log 17^x$

$(x+3)\log 2 = x \log 17$

$x\log 2 + 3\log 2 = x\log 17$

$3\log 2 = x\log 17 - x\log 2$

$3\log 2 = x(\log 17 - \log 2)$

$x = \frac{3\log 2}{\log 17 - \log 2}$

$x = .972$

11. Solve (3 decimal places) $4^{x+1} = 5^{x-2}$

$\log 4^{x+1} = \log 5^{x-2}$

$(x+1)\log 4 = (x-2)\log 5$

$x\log 4 + \log 4 = x\log 5 - 2\log 5$

$\log 4 + 2\log 5 = x\log 5 - x\log 4$

$\log 4 + 2\log 5 = x(\log 5 - \log 4)$

$x = \frac{\log 4 + 2\log 5}{\log 5 - \log 4}$

$x = 20.638$

12. What is the half-life, to the nearest month, of a radioactive isotope if it takes 7 years for 560 grams to decay to 35 grams?

$A = 35$

$A_0 = 560$

$c = 1/2$

$t = 7$

$T = ?$

$A = A_0(c)^{t/T}$

$35 = 560\left(\frac{1}{2}\right)^{7/T}$

$\frac{35}{560} = \left(\frac{1}{2}\right)^{7/T}$

$\log \frac{35}{560} = \log\left(\frac{1}{2}\right)^{7/T}$

$\log \frac{35}{560} = \frac{7}{T}\log \frac{1}{2}$

$T \log \frac{35}{560} = 7 \log \frac{1}{2}$

$T = \frac{7\log \frac{1}{2}}{\log \frac{35}{560}}$

$T = 1.75$ years

$T = 1.75(12)$

$T = 21$ months

13. Solve $\log_4(x+2)+\log_4(x-1)=1$

$\log_4[(x+2)(x-1)] = \log_4 4$

$(x+2)(x-1)=4$

$x^2+x-2=4$

$x^2+x-6=0$

$(x+3)(x-2)=0$

$x=-3$ ~~$x=-3$~~ $\boxed{x=2}$

$x+2>0$

$x>-2$

$x-1>0$

$x>1$

14. Solve $\log(x-3)+\log(x-2)=\log(2x-6)$

$\log[(x-3)(x-2)] = \log(2x-6)$

$(x-3)(x-2)=2x-6$

$x^2-5x+6=2x-6$

$x^2-7x+12=0$

$(x-4)(x-3)=0$

$\boxed{x=4}$ ✓ $x=3$

$x-3>0$

$x>3$

$x-2>0$

$x>2$

$2x-6>0$

$2x>6$

$x>3$

15. Solve $\log_3(3x-1)-\log_3(x-1)=4$

$\log_3 \frac{3x-1}{x-1} = \log_3 3^4$

$\frac{3x-1}{x-1}=81$

$3x-1=81(x-1)$

$3x-1=81x-81$

$80=78x$

$x=80/78$ $\boxed{x=\frac{40}{39}}$ ✓

$3x-1>0$

$3x>1$

$x>\frac{1}{3}$

$x-1>0$

$x>1$