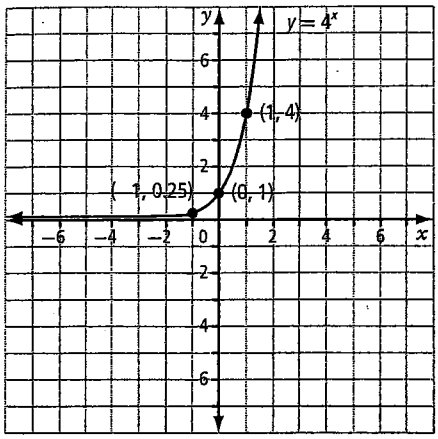
Chapter 8

Check Your Understanding — Section 8.1

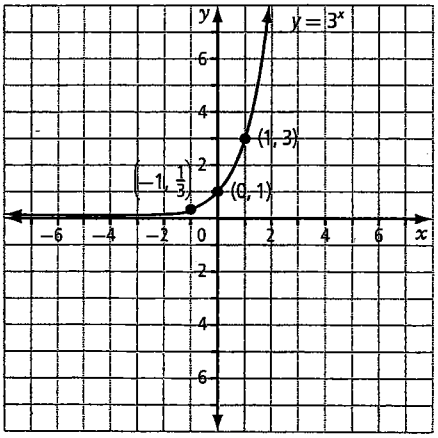
Practise

1. For each exponential graph below, sketch the inverse function and state the domain, range, x-intercept, and equation of the vertical asymptote. Then, write the equation of the inverse function.

a)



b)



2. Express in logarithmic form.

a) $3^5 = 243$

b) $10^4 = 10\,000$

c) $16^{\frac{1}{2}} = 4$

d) $8^{-2} = \frac{1}{64}$

e) $10^{-2} = 0.01$

f) $27^{\frac{2}{3}} = 9$

g) $12^x = 2y$

h) $2^{2x-5} = y - 1$

3. Express in exponential form.

a) $\log_2 32 = 5$

b) $\log_8 512 = 3$

c) $\log_5 625 = 4$

d) $\log 1000 = 3$

e) $\log 0.0001 = -4$

f) $\log_{\frac{1}{2}} 8 = -3$

g) $\log_3 (x + 1) = y$

h) $\log_4 2x = y + 1$

4. Evaluate.

a) $\log_6 36$

b) $\log_2 64$

c) $\log 100$

d) $\log_3 \frac{1}{9}$

This expression asks for the exponent of 3 that gives a value of $\frac{1}{9}$.

e) $\log_5 \frac{1}{125}$

f) $\log_7 7$

g) $\log_5 5^4$

h) $\log_2 (8\sqrt{32})$

Writing as powers of 2,

$8 =$ _____ and $\sqrt{32} =$ _____.

By the exponent laws, $(8\sqrt{32}) = 2^{\square}$.

$\log_2 (8\sqrt{32}) =$ _____

i) $\log_{12} 1$

j) $\log_{25} 5$

Completing these will help you to complete #1 - #4 on page 380 of *Pre-Calculus 12*.

5. Put the following in ascending order: $\log_6 400$, $\log_2 100$, $\log_{10} 300$.

Apply

6. Determine the value of x in each of the following.

a) $4^x = 64$

b) $10^{2x} = 1\,000\,000$

c) $\log_2 x = 4$

d) $\log_5 x = -2$

e) $\log_4 256 = x$

f) $\log_{16} 4 = x$

g) $\log_x 81 = 4$

h) $\log_x 6 = \frac{1}{2}$

i) $\log_x \frac{1}{25} = -2$

j) $\log_x \frac{1}{64} = -3$

7. Evaluate each expression.

a) 10^y, where $y = \log_{10} 216$

b) 8^y, where $y = \log_8 4$

c) 6^y, where $y = \log_6 12$

d) $\log_2 2^7$

e) $\log_5 5^{-8}$

f) $\log_7 7^{10}$

8. The intensity of sound is measured in decibels (dB). The level of a sound, L, in decibels, is given by $L = 10\log\left(\frac{I}{I_0}\right)$, where I is the intensity of the sound and I_0 is the faintest sound detectable to humans.

a) Determine the level of a sound that is 20 times more intense than I_0, correct to the nearest decibel.

b) The level of sound in a quiet bedroom at night might be 30 dB, while normal conversation has a sound level of about 60 dB. How many times more intense is normal conversation than the quiet room?

Check Your Understanding Section 8.2

Practise

1. State the transformations, in order of application, to transform $y = \log_e x$ to each of the following.

a) $y = \log_4 (x + 1) - 8$

b) $y = 2\log (4x)$

c) $y = -\log_2 (3x)$

d) $y = 5\log_6 (-2(x + 4))$

2. Write the equations that correspond to the following transformations of $y = \log_5 x$.

a) vertically stretched by a factor of 3 and translated 2 units to the right

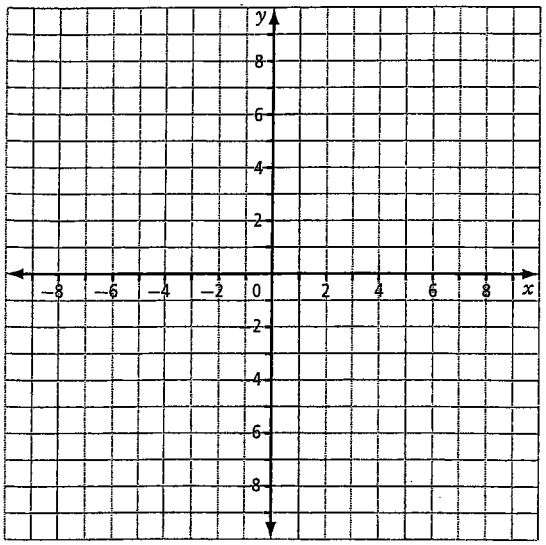
b) reflected in the x-axis and translated 1 unit down and 4 units left

c) vertically stretched by a factor of $\frac{1}{2}$ and horizontally stretched by a factor of $\frac{1}{2}$

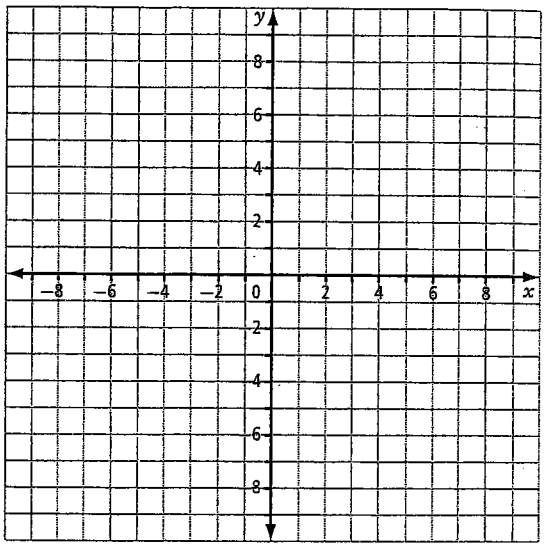
d) vertically stretched by a factor of 4, reflected in the y-axis, and translated 2.5 units down

3. Sketch each of the following transformations of $y = \log_c x$.

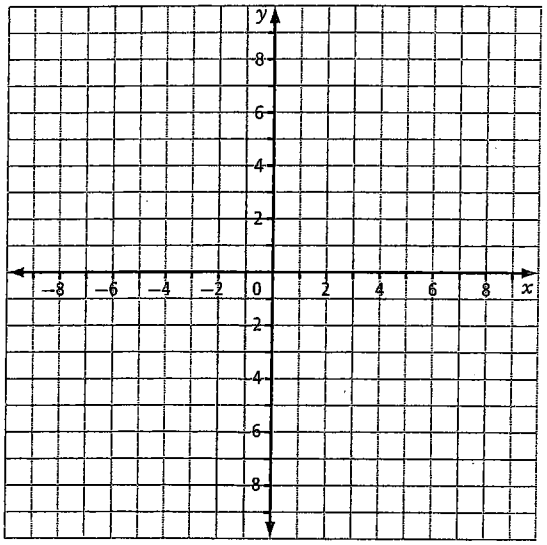
a) $y = \log_2 (x + 2) - 3$



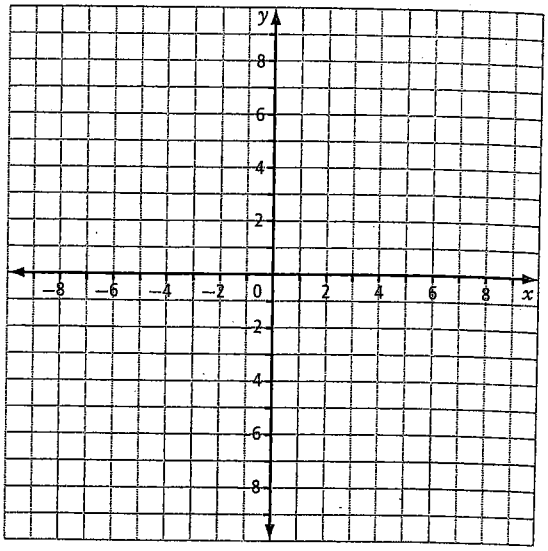
b) $y = 2 \log_8 x + 4$



c) $y = -\log_5 (x - 1) - 2$



d) $y = \log (2x) + 5$



4. Describe how the graph of each logarithmic function could be obtained from the graph of its base function, $y = \log_c x$.

a) $y = \log_6 (2x + 6)$

b) $y = \log_2 (3x - 12)$

c) $y = \log\left(\frac{1}{2}x - 3\right)$

d) $y = \log_3\left(\frac{1}{3}x + 6\right)$

6. For each of the following, state the domain, range, intercepts to the nearest tenth (if they exist), and equation of the vertical asymptote.

a) $y = \log_5 (x - 8) - 12$

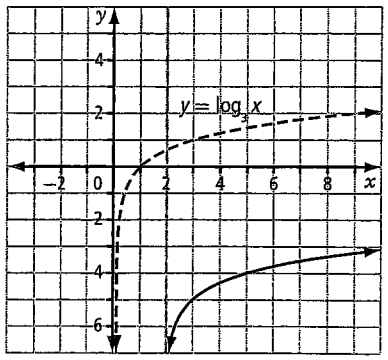
b) $y = -3 \log_9 (4(x - 1)) + 2$

c) $y = \frac{1}{2} \log_{12} (6x) - 4$

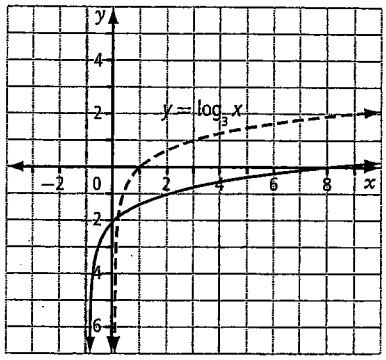
d) $y = \log_2 \left(\frac{1}{4}(x + 3)\right) - 5$

7. Each graph has been translated from $y = \log_3 x$. State the translation(s) in each case.

a)

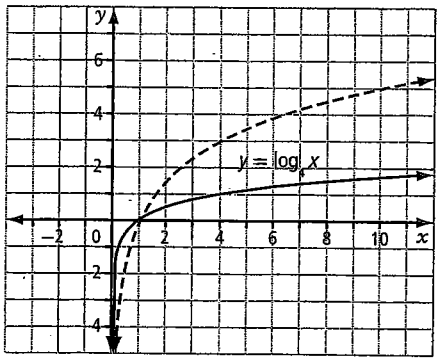


b)



8. Each graph below has been vertically stretched from $y = \log_4 x$. State the stretch in each case.

a)



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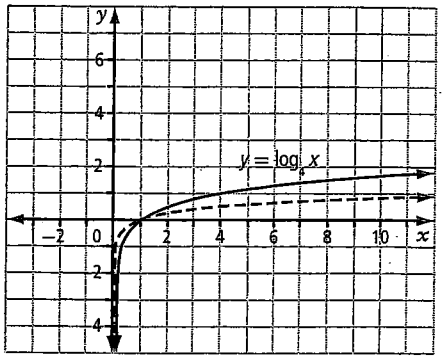


Check Your Understanding Section 8.3

Practise

1. Use the laws of logarithms to evaluate.

a) $\log_4 8 + \log_4 2$

b) $\log_5 250 - \log_5 2$

c) $\log_2 96 - \log_2 3$

d) $\log_6 3 + \log_6 12$

e) $\log_5 10 + \log_5 10 - \log_5 4$

f) $\log 25 + 2\log 4 + \log 5 - \log 2$

g) $\log_4 4^5$

h) $\log_9 9^{11}$

i) $2^{\log_2 16}$

Since $\log_2 16 =$ _____, this means that $2^{\log_2 16} =$ _____

j) $10^{\log 1000}$

2. Expand each expression using the laws of logarithms.

a) $\log_7 x^4\sqrt{y^3}$

Use the product law to rewrite $\log_7 x^4\sqrt{y^3}$.

$\log_7$ _____ + $\log_7$ _____

Rewrite $\sqrt{y^3}$ using exponents: _____

Use the power law to rewrite $\log_7 x^4$ and $\log_7 \sqrt{y^3}$.

$\log_7 x^4 =$ _____ and $\log_7 \sqrt{y^3} =$ _____ $\log_7 y$

Thus, the expanded expression for $\log_7 x^4\sqrt{y^3}$ is _____.

b) $\log_{12}(xy^2z^5)^3$

c) $\log_8 \frac{x^3}{\sqrt{yz^5}}$

d) $\log \sqrt{\frac{x}{y^3}}$

3. Expand each expression using the laws of logarithms. Then, evaluate and simplify where possible.

a) $\log_7 49\sqrt[3]{x^5}$

b) $\log \frac{100}{x^2y^2}$

c) $\log_5 \frac{\sqrt[3]{y^7}}{125x}$

d) $\log_2 \frac{3x^6}{96y^2}$

4. Write each expression as a single logarithm in simplest form.

a) $\log_6 2x^7 + \log_6 3x^2 + \log_6 \frac{9}{x^5}$

b) $\log_2 5x^2y^3 - \log_2 20x^4y + \log_2 2xy^6$

c) $\log_4 (x^2y)^2 + 5\log_4 x^3y^4 + \log_4 \left(\frac{1}{x^3y^2}\right)$

d) $6\log_3 xy - \log_3 xy^2 - \log_3 \sqrt[3]{x^4y}$

e) $\frac{1}{2}\log 4x\sqrt{y} - \log 25x^2\sqrt{y}$

f) $\log_7 x^4 + \frac{1}{3}(\log_7 x^2 - \log_7 \sqrt{5x})$

g) $\frac{\log 16x^8}{4} - \frac{\log 27x}{3}$

h) $\frac{\log_9 x^4y^8}{2} + \frac{\log_9 x^{12}y^{15}}{3}$

7. Decide whether each of the following is true or false. Justify your answer.

a) $\log_5 (x + 10) = \log_5 x + \log_5 10$

b) $\frac{\log_2 18}{\log_2 9} = \log_2 2$

c) $4^{\log_4 y} = y$

d) $\log_c 1 = 0$

e) $\log_c xy^2 = 2\log_c xy$

8. Use the laws of logarithms to isolate x in each expression.

a) $\log_6 36x = 1$

b) $\log_3 \frac{27}{x} = 2$

c) $3 \log_x 4 = 2$

9. Let $\log_5 12 = P$. Write each of the following expressions in terms of P.

a) $\log_5 12^7$

b) $\log_5 60$

c) $\log_5 144$

d) $\log_5 \frac{12}{5}$

e) $\log_5 \frac{1}{12}$

f) $\log_5 \sqrt{12}$

Check Your Understanding Section 8.4

Practise

1. Solve. Give exact answers.

a) $\log_4 x = 5$

b) $\log_5 x + 6 = 8$

c) $2 \log_2 x = 10$

d) $\log_6 (x + 3) + 2 = 5$

e) $3 \log_5 x = \log_5 125$

f) $2 \log (x - 5) = 6$

2. Solve. Round your answers to two decimal places.

a) $12^{3x} = 1000$

b) $7^{x+2} = 441$

c) $2^{3-x} = 100$

d) $3^{\frac{2x}{3}} = 350$

3. Solve. Express your answers as exact values.

a) $5^x = 205$

Take the logarithm of each side of the equation: _____ = _____

Then, use the power law of logarithms: _____ = _____

Divide each side by log 5: $x =$ _____

b) $4^{x-3} = 311$

c) $10^{2x+1} = 7539$

d) $5(4)^{x+2} = 200$

e) $6^{\frac{x}{2}} = 85$

4. Solve.

a) $3 \log_6 x = \log_6 9 + \log_6 24$

b) $\log_2 x^2 - \log_2 5 = \log_2 20$

c) $\log_4 x + 2 \log_4 x = 6$

d) $5 \log_3 x - \log_3 x = 8$

Completing #1–#4 will help you with #1–#3 on page 412 of *Pre-Calculus 12*.

5. Identify the values of x for which each equation is defined.

a) $\log_9 (x + 4) = \log_9 (2x)$

Since logarithms are only defined for positive values, $x + 4 > 0$, or $x >$ _____.

Similarly, since $2x > 0$, $x >$ _____.

If $x > 0$, both statements are true. So, the equation is defined when $x >$ _____.

b) $\log_7 (3x + 1) - \log_7 (x - 2) = 1$

c) $\log_6 (3 - x) + \log_6 (x - 3) = 2$

Apply

6. Solve. Express your answers as exact values and as decimal values correct to the nearest hundredth.

a) $5^{x-3} = 10^x$

b) $8^{2x+3} = 12^{2x}$

c) $2^{2x-5} = 6^{x+2}$

d) $2(6)^{x+2} = 3^{2x-3}$

7. Solve.

a) $\log_2 (4x + 10) - \log_2 x = 3$

b) $\log_3 (x + 7) - \log_3 (x - 3) = 2$

c) $\log (2x + 6) = 1 + \log (x - 1)$

d) $\log_5 (4x - 6) - 3 = \log_5 (2x - 3)$

8. Solve.

a) $\log x + \log(x + 3) = 1$

b) $\log_4(x - 4) + \log_4(x + 2) = 2$

c) $\log_6(x + 3) - 2 = -\log_6(x - 2)$

d) $\log(x + 2) = 2 - \log(7x - 1)$

9. The half-life of plutonium-238 is 88 years. Suppose that a sample of plutonium has a mass of 65 grams.

a) Write an exponential equation to model the mass of plutonium, m, present after t years.

b) Determine the mass of plutonium in the sample after 50 years. Round your answer to two decimal places.

c) Determine the time needed for the sample to decay to a mass of 20 grams, to the nearest tenth of a year.

10. The population of a high school is growing by 1.5% per year. Currently there are 974 students in the school.

a) Write an exponential equation to model the population of the school, p, after t years.

b) What population should be expected at the high school in five years?

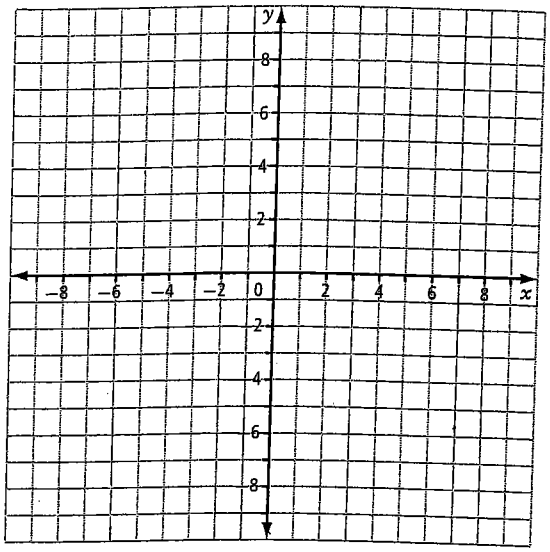
c) When will the population of the school reach 1200 students?

Chapter 8 Review

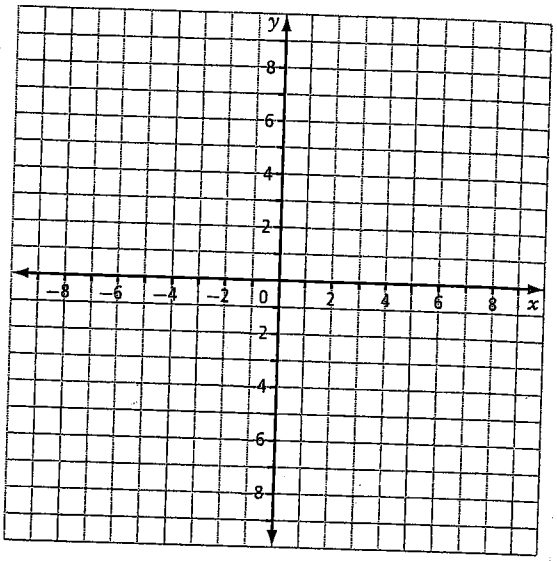
8.1 Understanding Logarithms, pages 260–266

1. Sketch the graph of each logarithmic function. Then, state the domain, range, x-intercept, and vertical asymptote.

a) $y = \log_2 x$



b) $y = \log_5 x$



2. Write each expression in logarithmic form.

a) $6^3 = 216$

b) $2^{10} = 1024$

c) $10^{-3} = 0.001$

d) $5^x = 125$

3. Write each expression in exponential form.

a) $\log_3 81 = 4$

b) $\log_{25} 5 = \frac{1}{2}$

c) $\log 1 = 0$

d) $\log_2 (3x - 4) = 9$

8.2 Transformations of Logarithmic Functions, pages 267–274

4. Identify the transformations in each logarithmic function. State the domain, range, and intercepts of the graph of each. Round your answers to one decimal place if necessary.

a) $y = 2\log_4(x + 1)$

b) $y = \log_7(x - 3) + 5$

5. Write the equation for each of the following transformations to the function $y = \log x$. Then, state the domain and range of the transformed function.

a) translation 5 units right and 4 units down

b) vertical stretch by a factor of 3, translation 2 units left and 6 units down

c) horizontal stretch by a factor of $\frac{1}{3}$, translation 1 unit up

8.3 Laws of Logarithms, pages 275–281

6. Use the laws of logarithms to evaluate each of the following.

a) $\log_6 9 + \log_6 4$

b) $\log 2000 - \log 2$

c) $\log_{12} 9 + \log_{12} 2 + \log_{12} 8$

d) $\log_7 100 - \log_7 25 - \log_7 4$

7. Expand each of the following.

a) $\log_5\left(25x^4\sqrt[4]{y^3}\right)$

b) $\log\frac{\sqrt{x}y^5}{100x}$

8. Write each of the following as a single logarithm.

a) $\log_4 x^2y^5 + \log_4 xy^{-2}$

b) $\log \frac{x^4}{\sqrt{y}} - \log \frac{y^2}{x}$

8.4 Logarithmic and Exponential Equations, pages 282–291

9. Solve. Express each answer as an exact value and as a decimal rounded to two places.

a) $3^x = 100$

b) $7^{x-3} = 517$

c) $10^{2x+1} = 5500$

d) $5^x = 2^{x-4}$

10. Solve.

a) $\log_2 x = 7$

b) $\log_3 (4x + 9) = 5$

c) $\log_2 (6x - 3) - \log_2 x = 4$

d) $\log_8 (6x + 2) + \log_8 (x - 3) = 2$

12. A strain of bacteria doubles every 4 hours. A sample contains 40 bacteria.

a) Write an exponential equation to determine the number of bacteria present, N, after t hours.

b) Determine the time needed until 1000 bacteria are present. Round your answer to two decimal places.

c) Determine the time needed for the number of bacteria in the sample to triple. Does your answer depend on the number of bacteria present at the beginning?

13. A water filter removes 40% of the impurities in a sample of water.

a) Write an exponential equation to determine the percent of impurities remaining, P, after the water has passed through n filters.

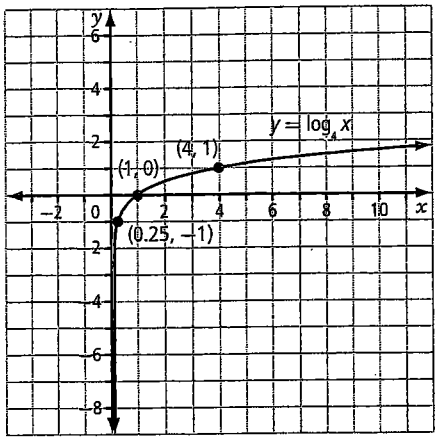
b) What percent of impurities will remain after the water has passed through 3 filters?

c) How many filters are needed to remove at least 99% of impurities in the water?

Chapter 8

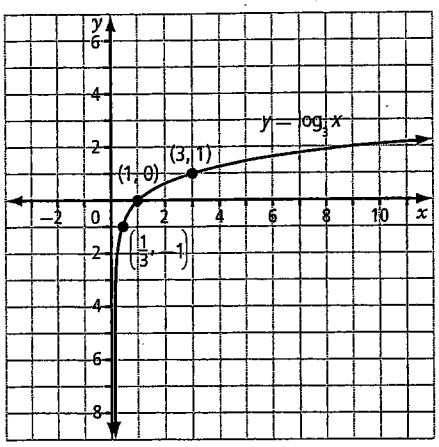
8.1 Understanding Logarithms, pages 260–266

1. a)



domain: $\{x \mid x > 0, x \in \mathrm{R}\}$;
range: $\{y \mid y \in \mathrm{R}\}$; x-intercept 1;
vertical asymptote $x = 0$; $y = \log_4 x$

b)



domain: $\{x \mid x > 0, x \in \mathrm{R}\}$;
range: $\{y \mid y \in \mathrm{R}\}$; x-intercept 1;
vertical asymptote $x = 0$; $y = \log_3 x$

2. a) $\log_3 243 = 5$ **b)** $\log 10\,000 = 4$

c) $\log_{16} 4 = \frac{1}{2}$ **d)** $\log_8 \frac{1}{64} = -2$

e) $\log 0.01 = -2$ **f)** $\log_{27} 9 = \frac{2}{3}$

g) $\log_{12} 2y = x$ **h)** $\log_2 (y - 1) = 2x - 5$

3. a) $2^5 = 32$ **b)** $8^3 = 512$

c) $5^4 = 625$ **d)** $10^3 = 1000$

e) $10^{-4} = 0.0001$ **f)** $\left(\frac{1}{2}\right)^{-3} = 8$

g) $3^y = x + 1$ **h)** $4^{(y+1)} = 2x$

4. a) 2 **b)** 6 **c)** 2

d) −2 **e)** −3 **f)** 1

g) 4 **h)** $\frac{11}{2}$ **i)** 0

j) $\frac{1}{2}$

5. $\log_{10} 300$, $\log_6 400$, $\log_2 100$

6. a) 3 **b)** 3 **c)** 16

d) $\frac{1}{25}$ **e)** 4 **f)** $\frac{1}{2}$

g) 3 **h)** 36 **i)** 5

j) 4

7. a) 216 **b)** 4 **c)** 12

d) 7 **e)** −8 **f)** 10

8. a) 13 dB **b)** 1000 times

8.2 Transformations of Logarithmic Functions, pages 267–274

1. a) translated 1 unit left and 8 units down

b) vertically stretched by a factor of 2 and horizontally stretched by a factor of $\frac{1}{4}$

c) reflected in the x-axis and horizontally stretched by a factor of $\frac{1}{3}$

d) vertically stretched by a factor of 5, horizontally stretched by a factor of $\frac{1}{2}$, reflected in the y-axis, translated 4 units left

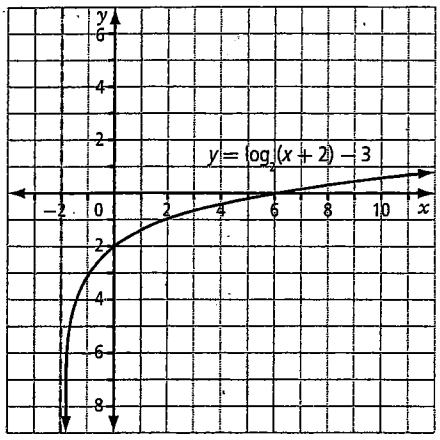
2. a) $y = 3 \log_5 (x - 2)$

b) $y = -\log_5 (x + 4) - 1$

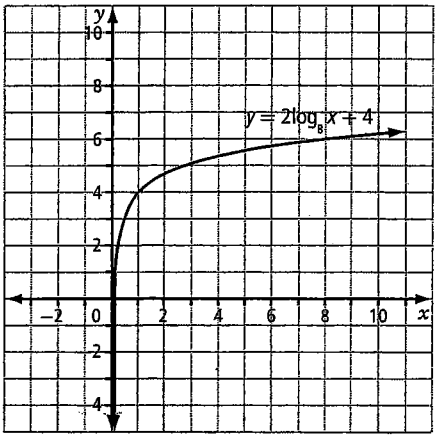
c) $y = \frac{1}{2} \log_5 (2x)$

d) $y = 4 \log_5 (-x) - 2.5$

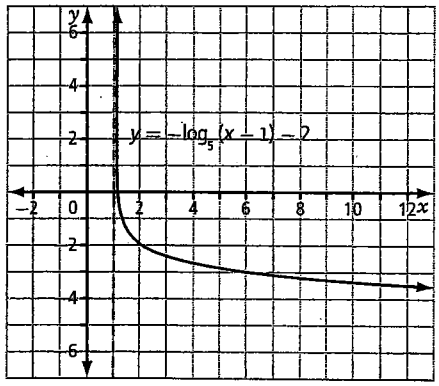
3. a)



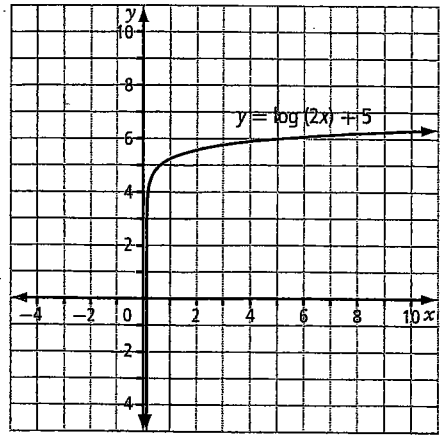
b)



c)



d)



4. a) horizontally stretched by a factor of $\frac{1}{2}$, translated 3 units to the left
   b) horizontally stretched by a factor of $\frac{1}{3}$, translated 4 units to the right
   c) horizontally stretched by a factor of 2, translated 6 units to the right
   d) horizontally stretched by a factor of 3, translated 18 units to the left

6. a) domain: $\{x \mid x > 8, x \in \mathbb{R}\}$; range: $\{y \mid y \in \mathbb{R}\}$; x-intercept: 244 140 633; no y-intercept; vertical asymptote $x = 8$
   b) domain: $\{x \mid x > 1, x \in \mathbb{R}\}$; range: $\{y \mid y \in \mathbb{R}\}$; x-intercept: 2.1; no y-intercept; vertical asymptote $x = 1$
   c) domain: $\{x \mid x > 0, x \in \mathbb{R}\}$; range: $\{y \mid y \in \mathbb{R}\}$; x-intercept: 71 663 616; no y-intercept; vertical asymptote $x = 0$
   d) domain: $\{x \mid x > -3, x \in \mathbb{R}\}$; range: $\{y \mid y \in \mathbb{R}\}$; x-intercept 125; y-intercept −5.4; vertical asymptote $x = -3$
7. a) translated 2 units right and 5 units down
   b) translated 1 unit left and 2 units down
8. a) vertically stretched by a factor of 3
   b) vertically stretched by a factor of $\frac{1}{2}$

8.3 Laws of Logarithms, pages 275–281

1. a) 2 b) 3 c) 5 d) 2 e) 2
   f) 3 g) 5 h) 11 i) 16 j) 1000
2. a) $4 \log_7 x + \frac{3}{2} \log_7 y$
   b) $3 \log_{12} x + 6 \log_{12} y + 15 \log_{12} z$
   c) $3 \log_8 x - \frac{1}{2} \log_8 y - \frac{5}{2} \log_8 z$
   d) $\frac{1}{2} \log x - \frac{3}{2} \log y$
3. a) $2 + \frac{5}{3} \log_7 x$ b) $2 - 2 \log x - 2 \log y$
   c) $\frac{7}{3} \log_5 y - 3 - \log_5 x$ d) $6 \log_2 x - 5 - 2 \log_2 y$
4. a) $\log_6 54x^4$ b) $\log_2 \frac{y^8}{2x}$ c) $\log_4 x^{16}y^{20}$
   d) $\log_3 (xy)^{\frac{11}{3}}$ e) $\log \frac{2}{25x^{\frac{3}{2}}y^{\frac{1}{4}}}$ f) $\log_7 \frac{x^{\frac{9}{2}}}{\sqrt[6]{5}}$
   g) $\log \frac{2x^{\frac{5}{3}}}{3}$ h) $\log_9 x^6y^9$

7. a) False; it must be a multiplication inside the logarithm.
   b) False; the division must take place inside the logarithm.
   c) True d) True
   e) False; the exponent must apply to the entire argument of the logarithm.
8. a) $\frac{1}{6}$ b) 3 c) 8
9. a) $7P$ b) $P + 1$ c) $2P$
   d) $P - 1$ e) $-P$ f) $\frac{P}{2}$

8.4 Logarithmic and Exponential Equations, pages 282–291

1. **a)** 1024 **b)** 25 **c)** 32
d) 213 **e)** 5 **f)** 1005

2. **a)** 0.93 **b)** 1.13 **c)** −3.64 **d)** 8.00

3. **a)** $\frac{\log 205}{\log 5}$ **b)** $\frac{\log 311}{\log 4} + 3$
c) $\frac{\log 7539 - 1}{2}$ **d)** $\frac{\log 40}{\log 4} - 2$
e) $\frac{2 \log 85}{\log 6}$

4. **a)** 6 **b)** 10 or −10
c) 16 **d)** 9

5. **a)** $x > 0$ **b)** $x > 2$
c) undefined for all x

6. **a)** $\frac{3 \log 5}{\log 5 - 1} \approx -6.97$
b) $\frac{-3 \log 8}{2 \log \frac{2}{3}} \approx 7.69$
c) $\frac{2 \log 6 + 5 \log 2}{2 \log 2 - \log 6} \approx -17.39$
d) $\frac{3 \log 3 + 2 \log 6 + \log 2}{2 \log 3 - \log 6} \approx 18.68$

7. **a)** $\frac{5}{2}$ **b)** $\frac{17}{4}$ **c)** 2 **d)** no soluion

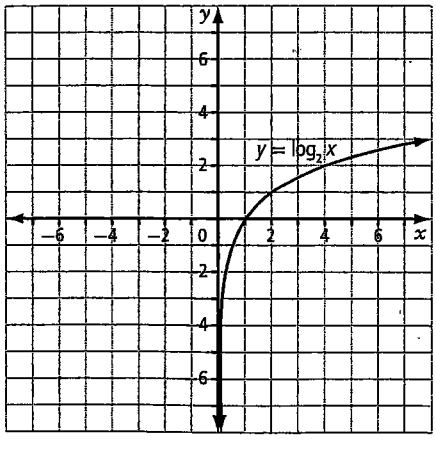
8. **a)** 2 **b)** 6 **c)** 6 **d)** 3

9. **a)** $m = 65\left(\frac{1}{2}\right)^{\frac{t}{88}}$ **b)** 43.84 g **c)** 149.6 years

10. **a)** $p = 974(1.015)^t$ **b)** 1049 **c)** 14 years

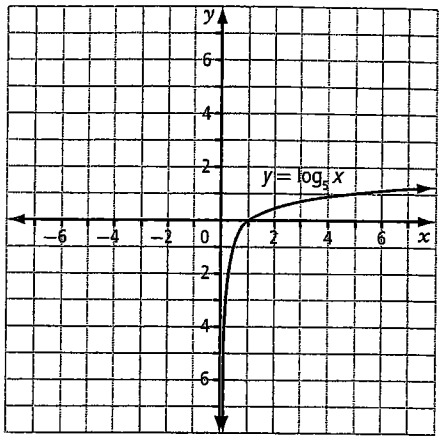
Chapter 8 Review, pages 292–295

1. **a)**



domain: $\{x \mid x > 0, x \in \mathbb{R}\}$;
range: $\{y \mid y \in \mathbb{R}\}$; x-intercept 1;
vertical asymptote $x = 0$

b)



domain: $\{x \mid x > 0, x \in \mathbb{R}\}$;
range: $\{y \mid y \in \mathbb{R}\}$; x-intercept 1;
vertical asymptote $x = 0$

2. **a)** $\log_6 216 = 3$ **b)** $\log_2 1024 = 10$
c) $\log 0.001 = -3$ **d)** $\log_5 125 = x$

3. **a)** $3^4 = 81$ **b)** $25^{\frac{1}{2}} = 5$
c) $10^0 = 1$ **d)** $2^9 = 3x - 4$

4. **a)** vertically stretched by a factor of 2, translated 1 unit left; domain: $\{x \mid x > -1, x \in \mathbb{R}\}$; range: $\{y \mid y \in \mathbb{R}\}$; x-intercept: 0; y-intercept: 0
b) translated 3 units right and 5 units up; domain: $\{x \mid x > 3, x \in \mathbb{R}\}$; range: $\{y \mid y \in \mathbb{R}\}$; x-intercept: 3.0; no y-intercept

5. **a)** $y = \log(x - 5) - 4$; domain: $\{x \mid x > 5, x \in \mathbb{R}\}$; range: $\{y \mid y \in \mathbb{R}\}$
b) $y = 3 \log(x + 2) - 6$; domain: $\{x \mid x > -2, x \in \mathbb{R}\}$; range: $\{y \mid y \in \mathbb{R}\}$
c) $y = \log(3x) + 1$; domain: $\{x \mid x > 0, x \in \mathbb{R}\}$; range: $\{y \mid y \in \mathbb{R}\}$

6. **a)** 2 **b)** 3 **c)** 2 **d)** 0

7. **a)** $2 + 4 \log_5 x + \frac{3}{4} \log_5 y$
b) $5 \log y - 2 - \frac{1}{2} \log x$

8. **a)** $3 \log_4 xy$ **b)** $\log \frac{x^5}{\sqrt{y^5}}$

9. **a)** $\frac{2}{\log 3} \approx 4.19$ **b)** $\frac{\log 517}{\log 7} + 3 \approx 6.21$
c) $\frac{\log 5500 - 1}{2} \approx 1.37$ **d)** $\frac{4 \log 2}{\log 2 - \log 5} \approx -3.03$

10. **a)** 128 **b)** $\frac{117}{2}$ **c)** no solution **d)** 5

12. **a)** $N = 40(2)^{\frac{t}{4}}$ **b)** 18.58 h
c) 6.34 h; does not depend on the number of bacteria present at the beginning

13. **a)** $P = 100(0.6)^n$ **b)** 21.6%
c) 9 filters