

Derivatives Problem

Thursday, April 26, 2018

11:22 AM

AP Calculus
Review

Free Response Problem Using Derivatives

Consider the differential equation $\frac{dy}{dx} = \frac{3-x}{y}$. Let $y = f(x)$ be a particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f . Find the x coordinate of the point of tangency and determine whether f has a local maximum, local minimum, or neither at this point.

What information are you given

$$\frac{dy}{dx} = \frac{3-x}{y}$$

$y = -2$ tangent line

What information can you find

$$\frac{d^2y}{dx^2}$$

Find x when
 $y = -2$

How will you answer the question

Use the 2nd
derivative test

$y = -2$ is a tangent line
 $\frac{dy}{dx} = 0$ when $y = -2$

$$\frac{dy}{dx} = \frac{3-x}{y}$$

$$0 = \frac{3-x}{-2}$$

$$x = 3 \quad \text{point } (3, -2)$$

$$\frac{d^2y}{dx^2} = \frac{y(-1) - 1 \frac{dy}{dx}(3-x)}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{-2(-1) - (0)(3-3)}{(-2)^2}$$

$$\frac{d^2y}{dx^2} = \frac{2}{4} = \frac{1}{2} > 0$$

concave up

$\therefore (3, -2)$ is a local minimum.

$$\begin{aligned} x &= 3 \\ y &= -2 \\ \frac{dy}{dx} &= 0 \end{aligned}$$