## Section II, Part A
## Time – 30 Minutes
## Number of Problems - 2
## A graphing calculator is required for some problems or parts of problems.

| $t$ (hours) | 0 | 0.4 | 0.8 | 1.2 | 1.6 | 2.0 | 2.4 |
|---|---|---|---|---|---|---|---|
| $v(t)$ (miles per hour) | 0 | 11.8 | 9.5 | 17.2 | 16.3 | 16.8 | 20.1 |

1. Ruth rode her bicycle on a straight trail. She recorded her velocity $v(t)$, in miles per hour, for selected values of $t$ over the interval $0 \le t \le 2.4$ hours, as shown in the table above. For $0 < t \le 2.4$, $v(t) > 0$.

   (a) Use the data in the table to approximate Ruth's acceleration at time $t = 1.4$ hours. Show the computations that lead to your answer. Indicate units of measure.

   (b) Using correct units, interpret the meaning of $\int_0^{2.4} v(t)\,dt$ in the context of the problem. Approximate $\int_0^{2.4} v(t)\,dt$ using a midpoint Riemann sum with three subintervals of equal length and values from the table.

   (c) For $0 \le t \le 2.4$ hours, Ruth's velocity can be modeled by the function $g$ given by $g(t) = \dfrac{24t + 5\sin(6t)}{t + 0.7}$. According to the model, what was Ruth's average velocity during the time interval $0 \le t \le 2.4$ ?

   (d) According to the model given in part (c), is Ruth's speed increasing or decreasing at time $t = 1.3$ ? Give a reason for your answer.

2. A store is having a 12-hour sale. The total number of shoppers who have entered the store $t$ hours after the sale begins is modeled by the function $S$ defined by $S(t) = 0.5t^4 - 16t^3 + 144t^2$ for $0 \le t \le 12$. At time $t = 0$, when the sale begins, there are no shoppers in the store.

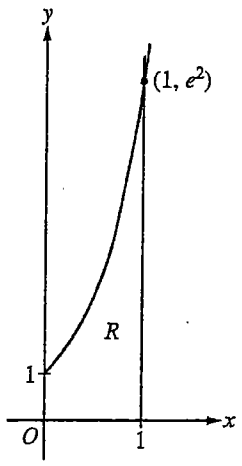
   (a) At what rate are shoppers entering the store 3 hours after the start of the sale?

   (b) Find the value of $\frac{1}{3}\int_6^9 S'(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{3}\int_6^9 S'(t)\,dt$ in the context of this problem.

   (c) The rate at which shoppers leave the store, measured in shoppers per hour, is modeled by the function $L$ defined by $L(t) = -80 + \dfrac{4400}{t^2 - 14t + 55}$ for $0 \le t \le 12$. According to the model, how many shoppers are in the store at the end of the sale (time $t = 12$)? Give your answer to the nearest whole number.

   (d) Using the given models, find the time $t$, $0 \le t \le 12$, at which the number of shoppers in the store is the greatest. Justify your answer.

Section II, Part B
Time – 60 Minutes
Number of Problems - 4
No calculator is allowed for these problems.

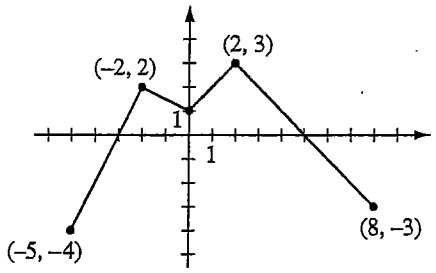



3. Let $f(x) = e^{2x}$. Let $R$ be the region in the first quadrant bounded by the graph of $y = f(x)$ and the vertical line $x = 1$, as shown in the figure above.

(a) Write an equation for the line tangent to the graph of $f$ at $x = 1$.

(b) Find the area of $R$.

(c) Region $R$ forms the base of a solid whose cross sections perpendicular to the $y$-axis are squares. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.




Graph of $f$

4. The continuous function $f$ is defined on the interval $-5 \le x \le 8$. The graph of $f$, which consists of four line segments, is shown in the figure above. Let $g$ be the function given by $g(x) = 2x + \int_{-2}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g(-5)$.

(b) Find $g'(x)$ in terms of $f(x)$. For each of $g''(4)$ and $g''(-2)$, find the value or state that it does not exist.

(c) On what intervals, if any, is the graph of $g$ concave down? Give a reason for your answer.

(d) The function $h$ is given by $h(x) = g(x^3 + 1)$. Find $h'(1)$. Show the work that leads to your answer.

5. Particle $X$ moves along the positive $x$-axis so that its position at time $t \geq 0$ is given by $x(t) = 5t^3 - 9t^2 + 7$.

   (a) Is particle $X$ moving toward the left or toward the right at time $t = 1$? Give a reason for your answer.

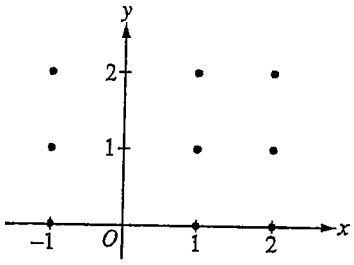
   (b) At what time $t \geq 0$ is particle $X$ farthest to the left? Justify your answer.

   (c) A second particle, $Y$, moves along the positive $y$-axis so that its position at time $t$ is given by $y(t) = 7t + 3$. At any time $t$, $t \geq 0$, the origin and the positions of the particles $X$ and $Y$ are the vertices of a triangle in the first quadrant. Find the rate of change of the area of the triangle at time $t = 1$. Show the work that leads to your answer.

6. Consider the differential equation $\frac{dy}{dx} = \left(1 - \frac{2}{x^2}\right)(y - 1)$, where $x \neq 0$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(1) = 2$.

   (a) Find the slope of the line tangent to the graph of $f$ at the point $(1, 2)$.

   (b) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.




   (c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = \left(1 - \frac{2}{x^2}\right)(y - 1)$ with initial condition $f(1) = 2$.

2014 AP Calculus AB Free-Response Questions

Question 1

a) -2.25 miles/hr$^2$
b) 36.64 miles
c) 14.064 miles/hr
d) Increasing

Question 2

a) 486 shoppers/hr
b) 301.5 shoppers/hr
c) 196 shoppers
d) t = 5.545 (you need to show that you checked the endpoints)

Question 3

a) $y = 2e^2x - e^2$
b) $\frac{1}{2}(e^2 - 1)$
c) $V = 1 + 2\int_1^{e^2}\left(1 - \frac{1}{2}\ln y\right)^2 dy$

Question 4

a) $g(0) = 3 \quad g(-5) = -7$
b) $g''(4) = -1 \quad g''(-2)$ *does not exist*
c) On the intervals (-2,0) and (2,8)
d) 15

Question 5

a) Left as $x'(1) < 0$
b) t = 6/5
c) -9/2

Question 6

a) m = -1
b) Draw your slope field
c) $y = e^{x+\frac{2}{x}-3} + 1 \quad for\ x > 0$