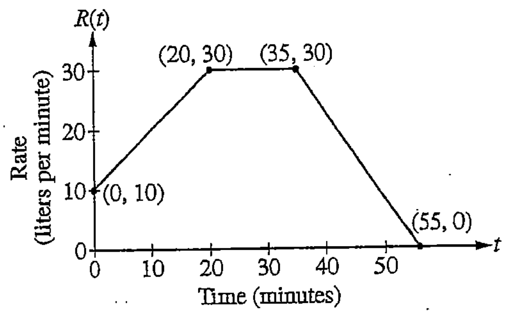
Section II, Part A
Time – 30 Minutes
Number of Problems - 2
A graphing calculator is required for some problems or parts of problems.



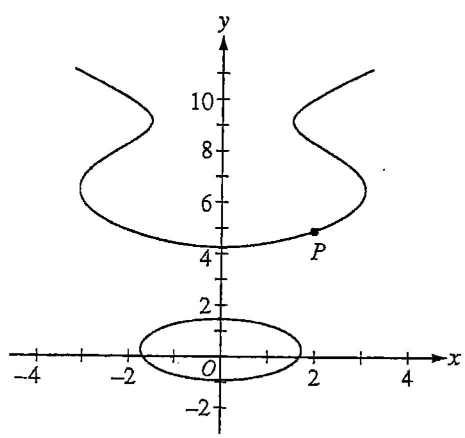
1. At time $t = 0$ minutes, a tank contains 100 liters of water. The piecewise-linear graph above shows the rate $R(t)$, in liters per minute, at which water is pumped into the tank during a 55-minute period.

   (a) Find $R'(45)$. Using appropriate units, explain the meaning of your answer in the context of this problem.

   (b) How many liters of water have been pumped into the tank from time $t = 0$ to time $t = 55$ minutes? Show the work that leads to your answer.

   (c) At time $t = 10$ minutes, water begins draining from the tank at a rate modeled by the function D, where $D(t) = 10e^{(\sin t)/10}$ liters per minute. Water continues to drain at this rate until time $t = 55$ minutes. How many liters of water are in the tank at time $t = 55$ minutes?

   (d) Using the functions R and D, determine whether the amount of water in the tank is increasing or decreasing at time $t = 45$ minutes. Justify your answer.



2. The graph of the equation $x^2 = -2 + y + 5\cos y$ is shown above for $y \le 11$. It is known that $\frac{dy}{dx} = \frac{2x}{1 - 5\sin y}$. The x-coordinate of point P shown on the graph is 2.

   (a) Write an equation for the line tangent to the graph at point P.

   (b) For $y \le 11$, find the y-coordinate of each point on the graph where the line tangent to the graph at that point is vertical.

   (c) Find the average value of the x-coordinates of the points on the graph in the first quadrant between $y = 5$ and $y = 9$.

Section II, Part B
Time – 60 Minutes
Number of Problems - 4
No calculator is allowed for these problems.

t (seconds)	0	3	5	8	12
$k(t)$ (feet per second)	0	5	10	20	24

3. Kathleen skates on a straight track. She starts from rest at the starting line at time $t = 0$. For $0 < t \le 12$ seconds, Kathleen's velocity k, measured in feet per second, is differentiable and increasing. Values of $k(t)$ at various times t are given in the table above.

   (a) Use the data in the table to estimate Kathleen's acceleration at time $t = 4$ seconds. Show the computations that lead to your answer. Indicate units of measure.

   (b) Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate $\int_0^{12} k(t)\,dt$. Indicate units of measure. Is this approximation an overestimate or an underestimate for the value of $\int_0^{12} k(t)\,dt$? Explain your reasoning.

   (c) Nathan skates on the same track, starting 5 feet ahead of Kathleen at time $t = 0$. Nathan's velocity, in feet per second, is given by $n(t) = \dfrac{150}{t+3} - 50e^{-t}$. Write, but do not evaluate, an expression involving an integral that gives Nathan's distance from the starting line at time $t = 12$ seconds.

   (d) Write an expression for Nathan's acceleration in terms of t.

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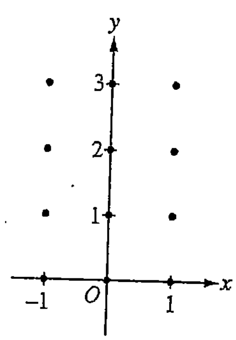
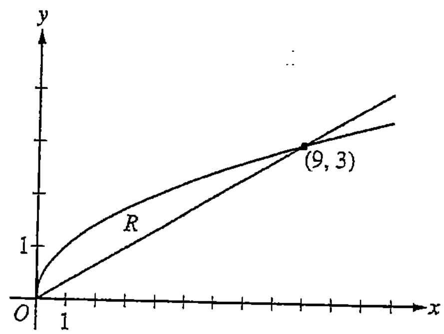
4. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{x(y-1)}{4}$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



   (b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 3$. Write an equation for the line tangent to the graph of f at the point $(1, 3)$ and use it to approximate $f(1.4)$.

   (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = 3$.



5. Let R be the region in the first quadrant enclosed by the graphs of $g(x) = \sqrt{x}$ and $h(x) = \frac{x}{3}$, as shown in the figure above.

(a) Find the area of region R.

(b) Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid generated when R is revolved about the horizontal line $y = 4$.

(c) Find the maximum vertical distance between the graph of g and the graph of h between $x = 0$ and $x = 16$. Justify your answer.

6. Let $g(x) = 4(x+1)^{-2/3}$ and let f be the function defined by $f(x) = \int_0^x g(t)\,dt$ for $x \geq 0$.

(a) Find $f(26)$.

(b) Determine the concavity of the graph of $y = f(x)$ for $x > 0$. Justify your answer.

(c) Let h be the function defined by $h(x) = x - f(x)$. Find the minimum value of h on the interval $0 \leq x \leq 26$.

2015 AP Calculus AB Free-Response Questions

Question 1

a) $\frac{-3}{2}$ liters/min^2
b) 1150 liters
c) 799.725 liters
d) Increasing $R(45) > D(45)$

Question 2

a) $y = .6798(x - 2) + 4.9283$
b) 0.201, 6.485, 9.223
c) 2.550

Question 3

a) $\frac{5}{2}$ ft/sec^2
b) 191 ft overestimate
c) The correct equation gives distance =196.416 ft
d) $\frac{-150}{(t+3)^2} + 50e^{-t}$

Question 4

a) Draw your slope field
b) 3.2
c) $y = 2e^{\frac{x^2-1}{8}} + 1$

Question 5

a) 9/2
b) $V = \pi \int_0^9 \left(4 - \frac{x}{3}\right)^2 - \left(4 - \sqrt{x}\right)^2 dx$
c) 4/3 at x=16

Question 6

a) 24
b) Concave down
c) Minimum of -5 when x=7