Infinite Geometric Series Worksheet

1. Which infinite geometric series have a sum? What is the sum if it exists?

   (a) $8 + 4 + 2 + 1 + \ldots$

   (b) $27 + 18 + 12 + 8 + \ldots$

   (c) $-16 + 12 - 9 + 6.75 - \ldots$

   (d) $2 + 6 + 18 + 54 + \ldots$

2. Express each decimal as an infinite series. Express each decimal as a fraction.

   (a) $2.33333\ldots$

   (b) $0.521521521\ldots$

   (c) $0.2151515\ldots$

   (d) $6.0858585\ldots$

3. The sum of an infinite geometric series is 81, and its common ratio is $\frac{2}{3}$. What is the value of the first term?

4. The first term of an infinite geometric series is -8, and its sum is $\frac{-40}{3}$. What is the common ratio?

5. The sum of an infinite geometric series is $\frac{24}{7}$ and the common ratio is $\frac{-3}{4}$. What is the value of the first term?

6. A ball is dropped from a height of 15m, on each bounce the ball rises to 40% of its previous height. Calculate the total vertical distance the ball travels before coming to rest.

7. In its first month of production an oil well produces 24000 barrels of oil. Each successive month the well produces 93% of the previous month's production. What is the lifetime production of the well?

8. Determine the values of x such that each series has a finite sum.
   (a) $5 + 5x + 5x^2 + 5x^3 + \ldots$

   (b) $1 + \frac{x}{3} + \frac{x^2}{9} + \ldots$

   (c) $2 + 4x + 8x^2 + 16x^3 + \ldots$

Answers:

1. (a) convergent; 16   (b) convergent; 81   (c) convergent; $\frac{-64}{7}$

   (d) divergent; no sum

2. (a) $\frac{21}{9}$ $\frac{7}{3}$ (b) $\frac{521}{999}$ (c) $\frac{213}{990}$ $\frac{71}{330}$ (d) $\frac{1205}{198}$

3. 27

4. $\frac{2}{5}$

5. 6

6. ~~25~~ 35

7. 342857

8. (a) $-1 < x < 1$ (b) $-3 < x < 3$ (c) $-\frac{1}{2} < x < \frac{1}{2}$