Questions #1-24 Handout

① $\sin\theta \cot\theta = \cos\theta$

$\sin\theta \cdot \frac{\cos\theta}{\sin\theta}$

$\cos\theta = \cos\theta$

② $\cos A \tan A = \sin A$

$\cos A \cdot \frac{\sin A}{\cos A}$

$\sin A = \sin A$

③ $\frac{\sin^2\theta + \cos^2\theta}{\cos\theta} = \sec\theta$

$\frac{1}{\cos\theta}$

$\sec\theta = \sec\theta$

④ $\frac{1+\sin\alpha}{\sin\alpha} = 1 + \csc\alpha$

$\frac{1}{\sin\alpha} + \frac{\sin\alpha}{\sin\alpha}$

$\csc\alpha + 1 = 1 + \csc\alpha$

⑤ $\frac{\sin\alpha - 1}{\cos\alpha} = \tan\alpha - \sec\alpha$

$\frac{\sin\alpha}{\cos\alpha} - \frac{1}{\cos\alpha}$

$\frac{\sin\alpha - 1}{\cos\alpha}$

⑥ $1 - \sin B \cos B \tan B = \cos^2 B$

$1 - \sin B \cos B \frac{\sin B}{\cos B}$

$1 - \sin^2 B$

$\cos^2 B$

⑦ $\sin\alpha + \cos\alpha \cot\alpha = \csc\alpha$

$\sin\alpha + \cos\alpha \frac{\cos\alpha}{\sin\alpha}$

$\frac{\sin\alpha}{\sin\alpha} \cdot \sin\alpha + \frac{\cos^2\alpha}{\sin\alpha}$

$\frac{\sin^2\alpha}{\sin\alpha} + \frac{\cos^2\alpha}{\sin\alpha}$

$\frac{\sin^2\alpha + \cos^2\alpha}{\sin\alpha}$

$\frac{1}{\sin\alpha}$

$\csc\alpha$

⑧ $1 - 2\sin^2 x = 2\cos^2 x - 1$

$1 - 2(1 - \cos^2 x)$

$1 - 2 + 2\cos^2 x$

$2\cos^2 x - 1$

⑨ $\cos\alpha(\csc\alpha - \sec\alpha) = \cot\alpha - 1$

$\cos\alpha \csc\alpha - \cos\alpha \sec\alpha$

$\cos\alpha \cdot \frac{1}{\sin\alpha} - \cos\alpha \frac{1}{\cos\alpha}$

$\frac{\cos\alpha}{\sin\alpha} - 1$

$\cot\alpha - 1$

⑩ $\csc B(\csc B + \cot B) = \frac{1}{1 - \cos B}$

$= \frac{1}{1 - \cos B} \cdot \frac{1 + \cos B}{1 + \cos B}$

$= \frac{1 + \cos B}{1 - \cos^2 B}$

$= \frac{1 + \cos B}{\sin^2 B}$

$= \frac{1}{\sin^2 B} + \frac{\cos B}{\sin^2 B}$

$= \csc^2 B + \cot B \csc B$

$= \csc B(\cot B + \csc B)$

(11) $\sin^4\alpha - \cos^4\alpha = 2\sin^2\alpha - 1$

$(\sin^2\alpha - \cos^2\alpha)(\sin^2\alpha + \cos^2\alpha)$

$\sin^2\alpha - (1 - \sin^2\alpha)(1)$

$\sin^2\alpha + \sin^2\alpha - 1$

$2\sin^2\alpha - 1$

(12) $\tan^4\rho - \sec^4\rho = 1 - 2\sec^2\rho$

$(\tan^2\rho - \sec^2\rho)(\tan^2\rho + \sec^2\rho)$

$(1)(\sec^2\rho - 1 + \sec^2\rho)$

$2\sec^2\rho - 1$

(13) $\dfrac{\sin\beta + \tan\beta}{1 + \cos\beta} = \tan\beta$

$\dfrac{\cos\beta}{\cos\beta} \cdot \dfrac{\sin\beta + \frac{\sin\beta}{\cos\beta}}{1 + \cos\beta}$

$\dfrac{\frac{\cos\beta\sin\beta + \sin\beta}{\cos\beta}}{1 + \cos\beta}$

$\dfrac{\frac{\sin\beta(\cos\beta + 1)}{\cos\beta}}{1 + \cos\beta}$

$\dfrac{\sin\beta(\cos\beta + 1)}{\cos\beta} \cdot \dfrac{1}{1 + \cos\beta}$

$\dfrac{\sin\beta}{\cos\beta}$

$\tan\beta$

(14) $\sec\theta + \tan\theta = \dfrac{\cos\theta}{1-\sin\theta}$

$= \dfrac{\cos\theta}{1-\sin\theta} \cdot \dfrac{1+\sin\theta}{1+\sin\theta}$

$= \dfrac{\cos\theta + \cos\theta\sin\theta}{1-\sin^2\theta}$

$= \dfrac{\cos\theta + \cos\theta\sin\theta}{\cos^2\theta}$

$= \dfrac{\cos\theta}{\cos^2\theta} + \dfrac{\cos\theta\sin\theta}{\cos^2\theta}$

$= \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= \sec\theta + \tan\theta$

(15) $(1+\csc\alpha)(1-\sin\alpha) = \cot\alpha\cos\alpha$

$1 - \sin\alpha + \csc\alpha - \sin\alpha\csc\alpha$

$1 - \sin\alpha + \csc\alpha - \sin\alpha \cdot \dfrac{1}{\sin\alpha}$

$1 - \sin\alpha + \csc\alpha - 1$

$-\sin\alpha + \csc\alpha$

$\dfrac{\sin\alpha}{\sin\alpha} \cdot -\sin\alpha + \dfrac{1}{\sin\alpha}$

$\dfrac{-\sin^2\alpha + 1}{\sin\alpha}$

$\dfrac{1-\sin^2\alpha}{\sin\alpha}$

$\cot\alpha\cos\alpha$

$\dfrac{\cos\alpha}{\sin\alpha} \cdot \cos\alpha$

$\dfrac{\cos^2\alpha}{\sin\alpha}$

L.S. = R.S.

⑯ $(1+\tan\theta+\sec\theta)^2 = 2(1+\sec\theta)(\tan\theta+\sec\theta)$

$(1+\tan\theta+\sec\theta)(1+\tan\theta+\sec\theta)$

$1+\tan\theta+\sec\theta+\tan\theta+\tan^2\theta+\tan\theta\sec\theta+\sec\theta+\sec\theta\tan\theta+\sec^2\theta$

$1+2\tan\theta+2\sec\theta+2\tan\theta\sec\theta+\sec^2\theta+\tan^2\theta$

L.S. $2\tan\theta+2\sec\theta+2\tan\theta\sec\theta+\sec^2+\sec^2\theta$

$2\tan\theta+2\sec\theta+2\tan\theta\sec\theta+2\sec^2\theta$

R.S. $2(1+\sec\theta)(\tan\theta+\sec\theta)$

$2(\tan\theta+\sec\theta+\sec\theta\tan\theta+\sec^2\theta)$

$2\tan\theta+2\sec\theta+2\tan\theta\sec\theta+2\sec^2\theta)$

(17) $(1+\sec\beta)(\sec\beta-1) = \dfrac{\sin\beta\sec\beta}{\cos\beta\csc\beta}$

$\sec\beta - 1 + \sec^2\beta - \sec\beta$

$\sec^2\beta - 1$

$\tan^2\beta$

$\tan^2\beta$

$\dfrac{\sin\beta \cdot \frac{1}{\cos\beta}}{\cos\beta \cdot \frac{1}{\sin\beta}}$

$\dfrac{\frac{\sin\beta}{\cos\beta}}{\frac{\cos}{\sin\beta}}$

$\dfrac{\sin\beta}{\cos\beta} \cdot \dfrac{\sin\beta}{\cos\beta}$

$\dfrac{\sin^2\beta}{\cos^2\beta}$

$\tan^2\beta$

(18) $(\csc\beta - 1)(1+\csc\beta) = \dfrac{\csc\beta\cos\beta}{\sec\beta\sin\beta}$

$\csc\beta + \csc\beta - 1 - \csc\beta$

$\csc^2\beta - 1$

$\cot^2\beta$

$\cot^2\beta$

$= \dfrac{\frac{1}{\sin\beta}\cdot\cos\beta}{\frac{1}{\cos\beta}\cdot\sin\beta}$

$= \dfrac{\frac{\cos\beta}{\sin\beta}}{\frac{\sin\beta}{\cos\beta}}$

$= \dfrac{\cos^2\beta}{\sin^2\beta}$

$= \cot^2\beta$

(19) $\dfrac{\sin\theta\cos\theta}{1+\cos\theta} - \dfrac{\sin\theta}{1-\cos\theta} = -(\cot\theta\cos\theta + \csc\theta)$

$\dfrac{1-\cos\theta}{1-\cos\theta} \cdot \dfrac{\sin\theta\cos\theta}{1+\cos\theta} - \dfrac{\sin\theta}{1-\cos\theta} \cdot \dfrac{1+\cos\theta}{1+\cos\theta}$

$\dfrac{\sin\theta\cos\theta - \sin\theta\cos^2\theta}{1-\cos^2\theta} - \dfrac{(\sin\theta + \sin\theta\cos\theta)}{1-\cos^2\theta}$

$\dfrac{-\sin\theta\cos^2\theta - \sin\theta}{1-\cos^2\theta}$

$\dfrac{-\sin\theta(\cos^2\theta + 1)}{1-\cos^2\theta}$

$= \dfrac{-\sin\theta(\cos^2\theta+1)}{\sin^2\theta}$

$= \dfrac{-(\cos^2\theta+1)}{\sin\theta}$

$-(\cot\theta\cos\theta + \csc\theta$

$-\left(\dfrac{\cos\theta}{\sin\theta}\cdot\cos\theta + \dfrac{1}{\sin}\right.$

$\dfrac{-(\cos^2\theta+1)}{\sin\theta}$

$\dfrac{-(\cos^2\theta+1)}{\sin\theta}$

L.S = R.S.

(20) $\dfrac{\sin\theta + \cos\theta}{\sec\theta + \tan\theta} + \dfrac{\cos\theta - \sin\theta}{\sec\theta - \tan\theta} = 2 - 2\sin^2\theta \sec\theta$

$\left(\dfrac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta}\right) \cdot \dfrac{\sin\theta + \cos\theta}{\sec\theta + \tan\theta} + \dfrac{\cos\theta - \sin\theta}{\sec\theta - \tan\theta} \cdot \dfrac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}$

$\dfrac{\sec\theta\sin\theta + \sec\theta\cos\theta - \sin\theta\tan\theta - \cos\theta\tan\theta}{\sec^2\theta - \tan^2\theta} + \dfrac{\cos\theta\sec\theta + \cos\theta\tan\theta - \sin\theta\sec\theta - \sin\theta\tan\theta}{\sec^2\theta - \tan^2\theta}$

$\dfrac{2\sec\theta\cos\theta - 2\sin\theta\tan\theta}{1}$

$2 \dfrac{1}{\cos\theta} \cdot \cos\theta - 2\sin\theta \dfrac{\sin\theta}{\cos\theta}$

$2 - 2\sin^2\theta \cdot \dfrac{1}{\cos\theta}$

$2 - 2\sin^2\theta \cdot \sec\theta$

LS = RS $2 - 2\sin^2\theta \sec\theta$

(21) $\dfrac{\sec\theta}{1-\cos\theta} = \dfrac{\sec\theta + 1}{\sin^2\theta}$

$\dfrac{(1+\cos\theta)}{(1+\cos\theta)} \cdot \dfrac{\sec\theta}{1-\cos\theta}$

$\dfrac{\sec\theta + \sec\theta\cos\theta}{1-\cos^2\theta}$

$\dfrac{\sec\theta + \sec\theta\cos\theta}{\sin^2\theta}$

$\dfrac{\sec\theta + \frac{1}{\cos\theta} \cdot \cos\theta}{\sin^2\theta}$

$\dfrac{\sec\theta + 1}{\sin^2\theta}$ $\qquad$ $\dfrac{\sec\theta + 1}{\sin^2\theta}$

L.S $\qquad$ R.S

(22) $\dfrac{\tan\theta}{\tan\theta + \sin\theta} = \dfrac{1-\cos\theta}{\sin^2\theta}$

$\dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta}{\cos} + \sin\theta \cdot \frac{\cos\theta}{\cos\theta}}$

$\dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta + \sin\theta\cos\theta}{\cos\theta}}$

$\dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta(1+\cos\theta)}{\cos\theta}}$

$\dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{\cos\theta}{\sin\theta(1+\cos\theta)}$

$\dfrac{1}{1+\cos\theta} \cdot \dfrac{1-\cos\theta}{1-\cos\theta}$

$\dfrac{1-\cos\theta}{1-\cos^2\theta}$

$\dfrac{1-\cos\theta}{\sin^2\theta}$

L.S = R.S

$\dfrac{1-\cos\theta}{\sin^2\theta}$

(23) $\dfrac{1+\sec\theta}{\sec\theta-1} + \dfrac{1+\cos\theta}{\cos\theta-1} = 0$

$\dfrac{(\cos\theta-1)}{(\cos\theta-1)} \cdot \dfrac{1+\sec\theta}{\sec\theta-1} + \dfrac{1+\cos\theta}{\cos\theta-1} \cdot \dfrac{(\sec\theta-1)}{(\sec\theta-1)}$

$\dfrac{\cos\theta+\cos\theta\sec\theta-1-\sec\theta}{\cos\theta\sec\theta-\cos\theta-\sec\theta+1} + \dfrac{\sec\theta-1+\cos\theta\sec\theta-\cos\theta}{\cos\theta\sec\theta-\cos\theta-\sec\theta+1}$

$\dfrac{2\cos\theta\sec\theta-2}{\cos\theta\sec\theta-\cos\theta-\sec\theta+1}$

$2\cos\theta \cdot \dfrac{1}{\cos\theta} - 2$

$\dfrac{2-2}{\cos\theta\sec\theta-\cos\theta-\sec\theta+1}$

$\dfrac{0}{\cos\theta\sec\theta-\cos\theta-\sec\theta+1}$

0

0

L.S. R.S

24 $$\frac{\sec^2\theta(1+\csc\theta)-\tan\theta(\sec\theta+\tan\theta)}{\csc\theta(1+\sin\theta)}-1=0$$

$$\frac{\sec^2\theta+\sec^2\theta\csc\theta-\tan\theta\sec\theta-\tan^2\theta}{\csc\theta(1+\sin\theta)}-1=0$$

$$\frac{\sec^2\theta-\tan^2\theta+\sec^2\theta\csc\theta-\tan\theta\sec\theta}{\csc\theta(1+\sin\theta)}-1=0$$

$$\frac{1+\frac{1}{\cos^2\theta}\cdot\frac{1}{\sin\theta}-\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}}{\csc\theta(1+\sin\theta)}-1=0$$

$$\frac{1+\frac{1}{\cos^2\theta\sin\theta}-\frac{\sin\theta}{\cos^2\theta}\cdot\frac{\sin\theta}{\sin\theta}}{\csc\theta(1+\sin\theta)}-1=0$$

$$\frac{1+\frac{1-\sin^2\theta}{\cos^2\theta\sin\theta}}{\csc\theta(1+\sin\theta)}-1=0$$

$$\frac{1+\frac{\cos^2\theta}{\cos^2\theta\sin\theta}}{\csc\theta(1+\sin\theta)}-1=0$$

$$\frac{1+\frac{1}{\sin\theta}}{\csc\theta(1+\sin\theta)}-1=0$$

$$\frac{\frac{\sin\theta}{\sin\theta}+\frac{1}{\sin\theta}}{\csc\theta(1+\sin\theta)}-1=0$$

$$\frac{\frac{1}{\sin\theta}(\sin\theta+1)}{\csc\theta(1+\sin\theta)}-1=0$$

$$\frac{\csc\theta}{\csc\theta}-1=0$$

$$1-1=0$$

$$0=0$$